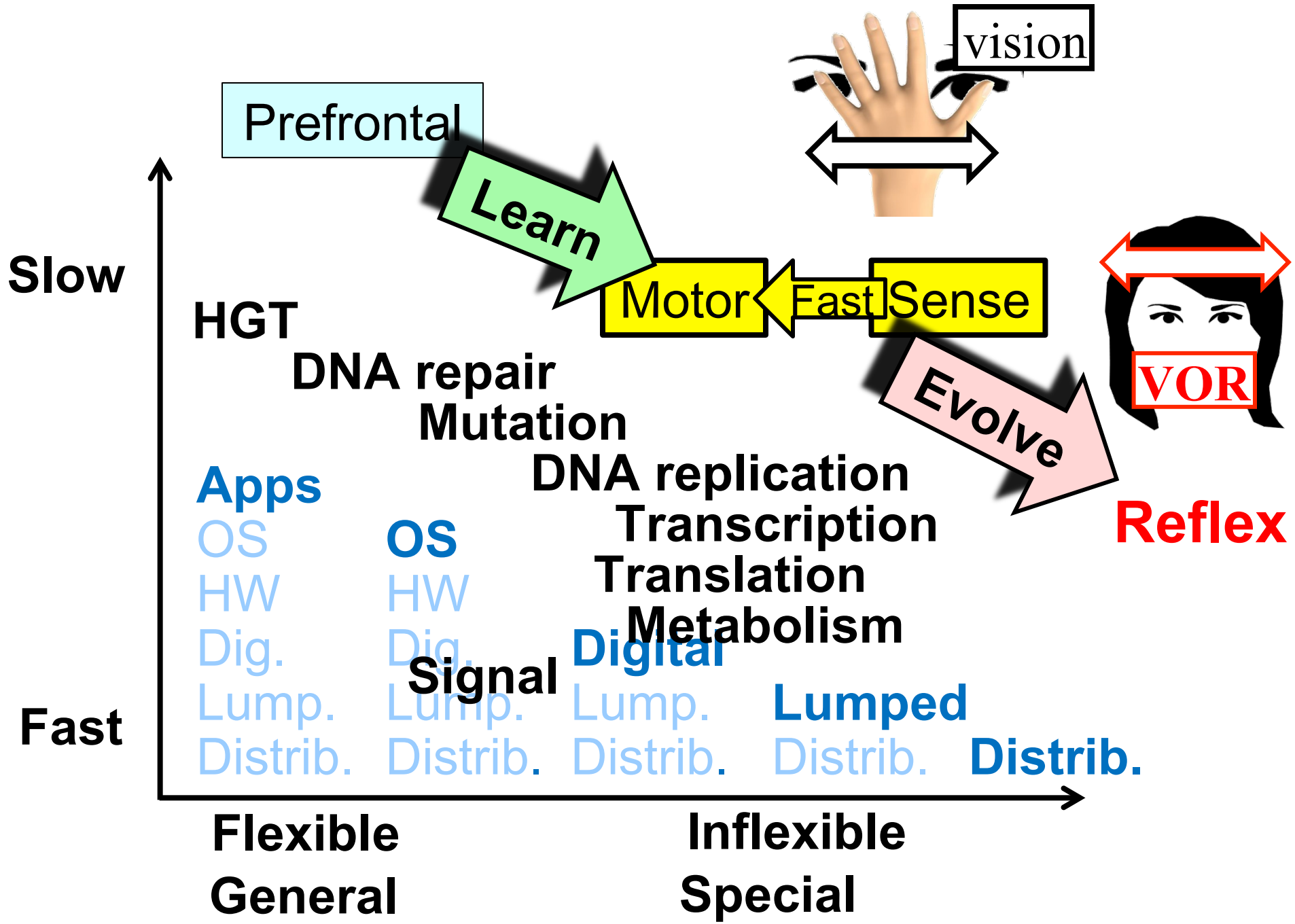

vision
Prefrontal
Learn
Motor
Fast
Sense
VOR
Evolve
Reflex
Slow
HGT
DNA repair
Mutation
DNA replication
Transcription
Translation
Metabolism
Apps
OS
HW
Dig.
Lump.
Distrib.
OS
HW
Dig.
Lump.
Distrib.
Signal
Digital
Lump.
Distrib.
Lumped
Distrib.
Distrib.
Fast
Flexible
General
Inflexible
Special

Compute

Comms for Comp/Cntrl/Bio

Info Thry

Optimization

Statistics

Theory

Control, OR

Orthophysics (Eng/Bio/Math)

Physics

Localized (distributed) control

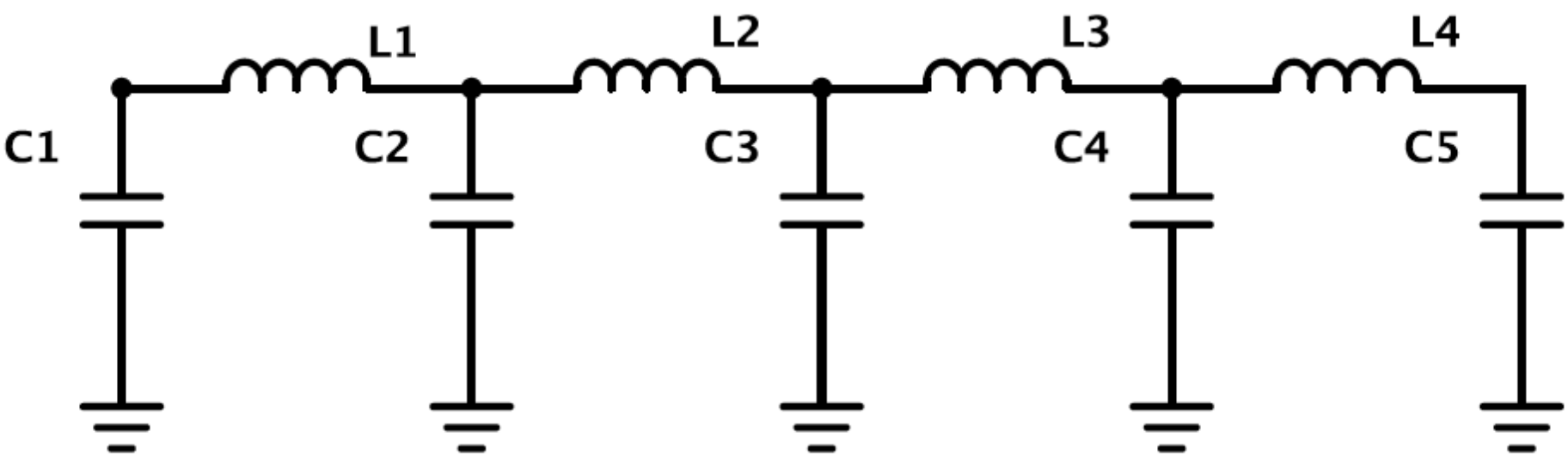
- Localizable control: Wang, Matni, You and Doyle ACC ’14
- Localized LQR control: Wang, Matni, and Doyle CDC’14
- Output feedback? Allerton?

Another *extremely* toy model

- Concretely illustrate important new ideas
- Minimal complexity otherwise
- Familiar, intuitive circuit dynamics
- Emphasize role of delays

- Instability mechanism is artificial
- Comparable to biological instabilities
- … but (so far) rare in tech infrastructure

- LC circuit
- Each node = grounded capacitance
- Each link = inductance



System Model

- Assuming each L and each C has unit value, the dynamics of the system are

$$\dot{x}(t) = Ax(t)$$

$$A = \begin{bmatrix} 0 & M \\ -M' & 0 \end{bmatrix}$$

where x(t) is states of node voltage and link current, M is the incidence matrix of the circuit graph.

(Will reorder for plotting later.)

Discrete Time System Model

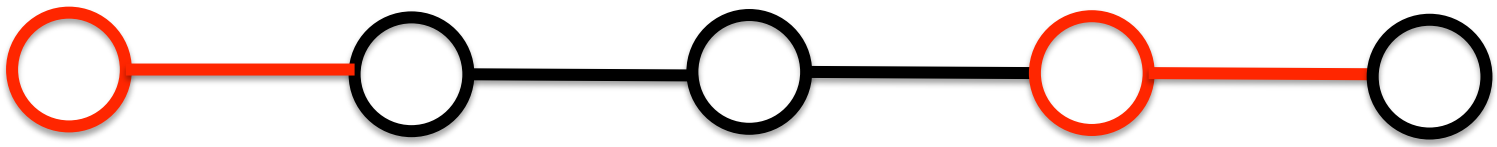
- A first order (Euler) approximation is

$$Ad = \begin{bmatrix} I & step * M \\ -step * M' & I \end{bmatrix}$$

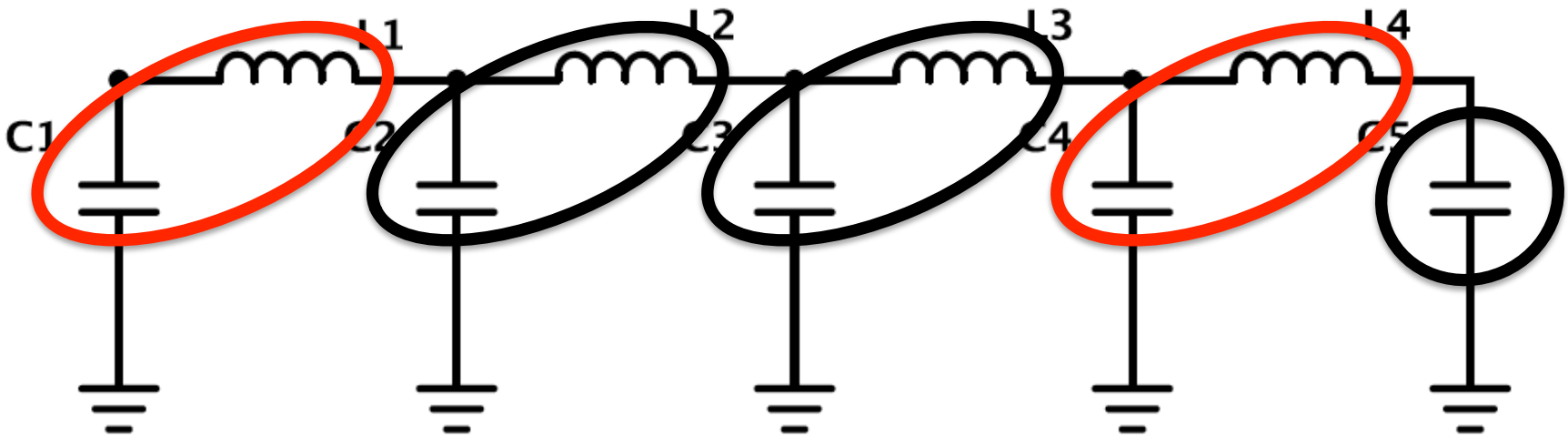
- With step = 0.2, the maximum eigenvalue of Ad is 1.0768
- Artificially create a ***very*** unstable system

- Only biology is systematically this unstable, so far.

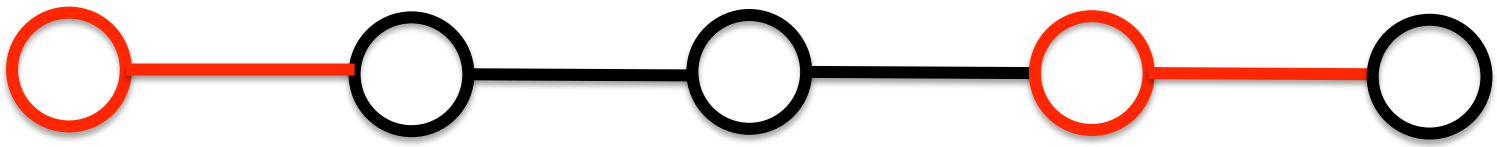
Simplified diagram (2 states per node)



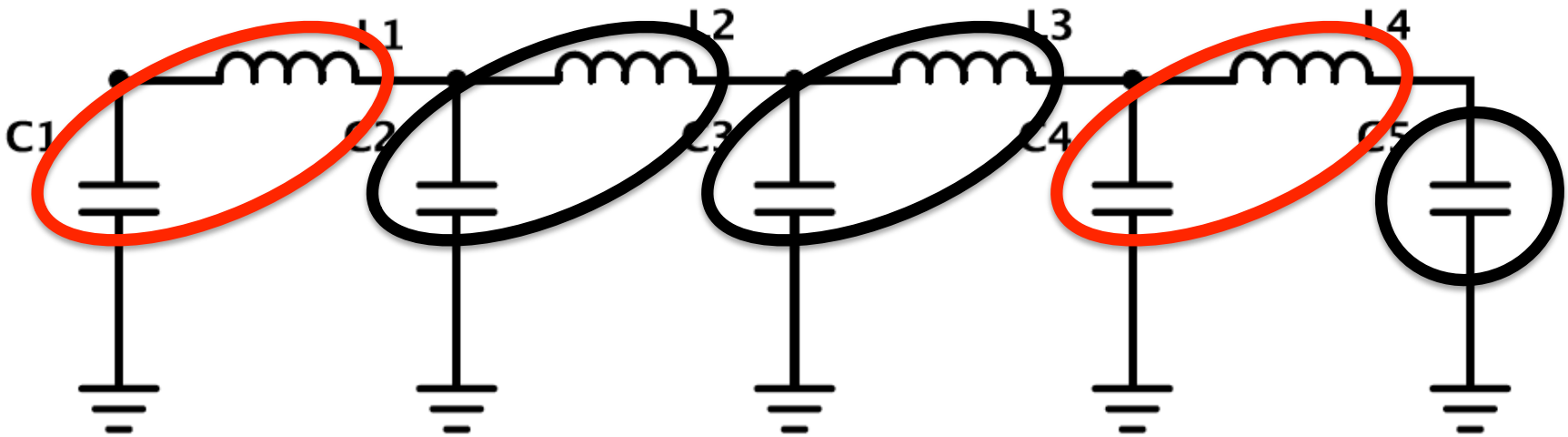
Actuated and sensed | **Only sensed** | **Only sensed** | **Actuated and sensed** | **Only sensed**

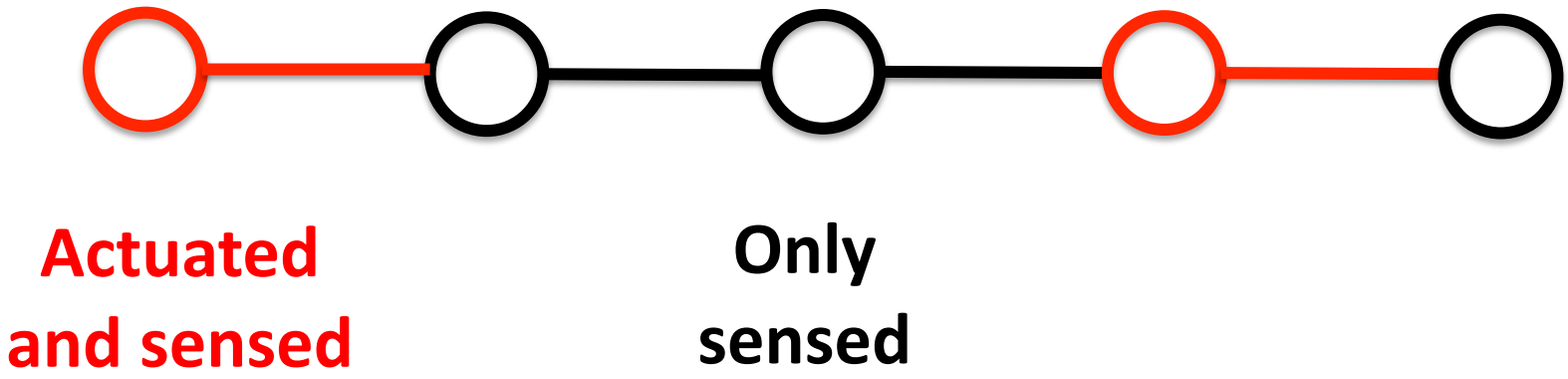


Simplified diagram (2 states per node)

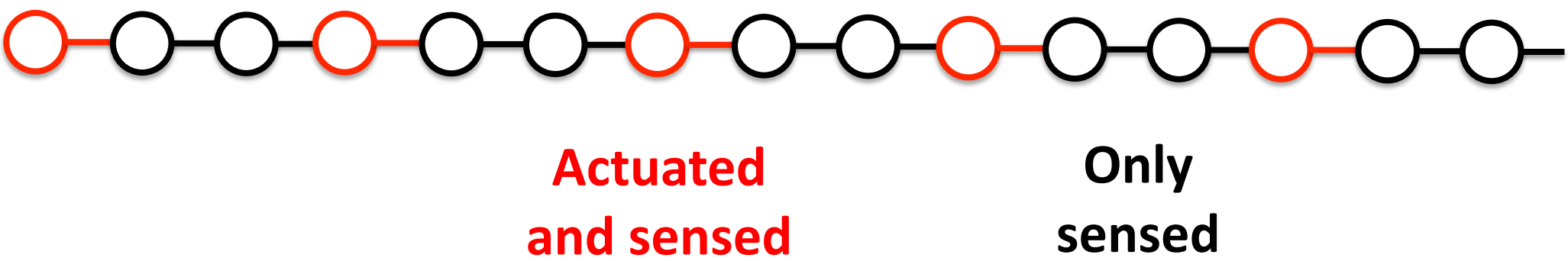


Actuated and sensed | **Only sensed** | **Only sensed** | **Actuated and sensed** | **Only sensed**





Simplified diagram (2 states per node)

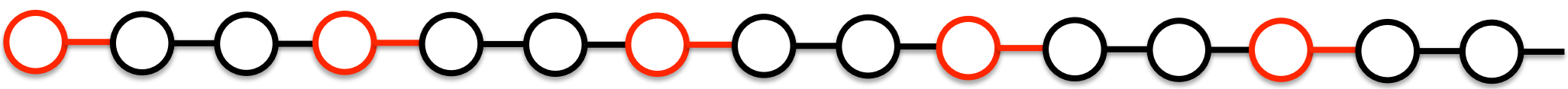


Expensive?

0. Physical
1. Actuation

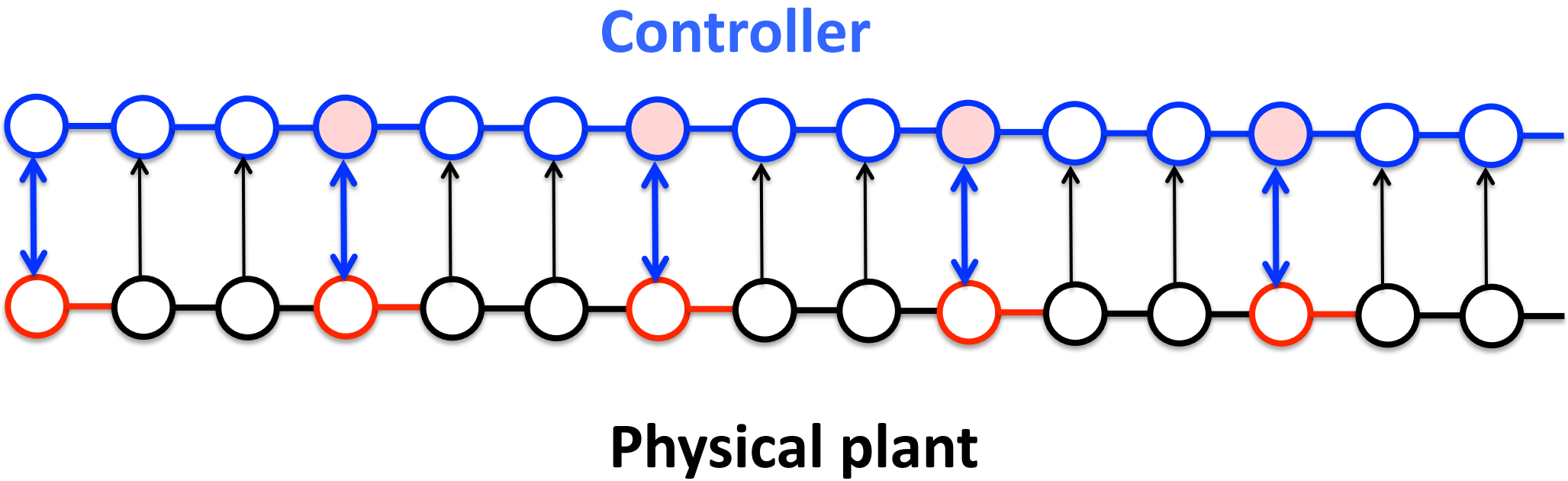
Nominally each has delay 1.

Simplified diagram (2 states per node)



Actuated and sensed

Only sensed

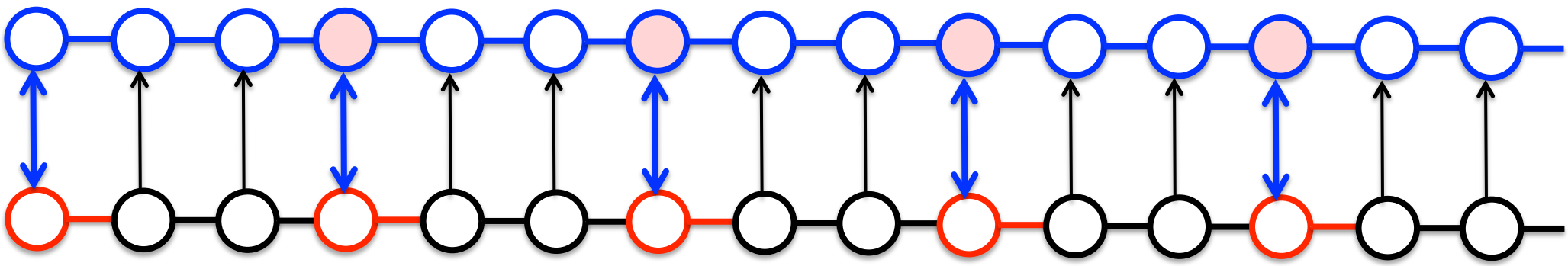

Controller
Physical plant

Expensive?

0. Physical
1. Actuation
2. Comms speed
3. Comp speed
4. Sensing

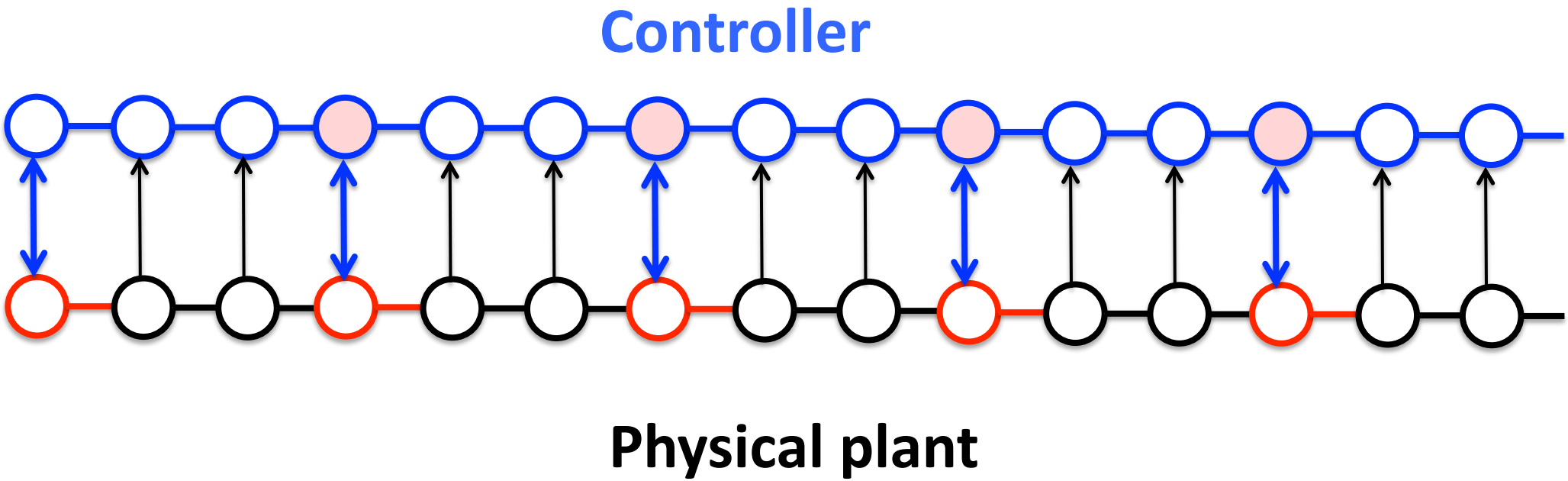
…

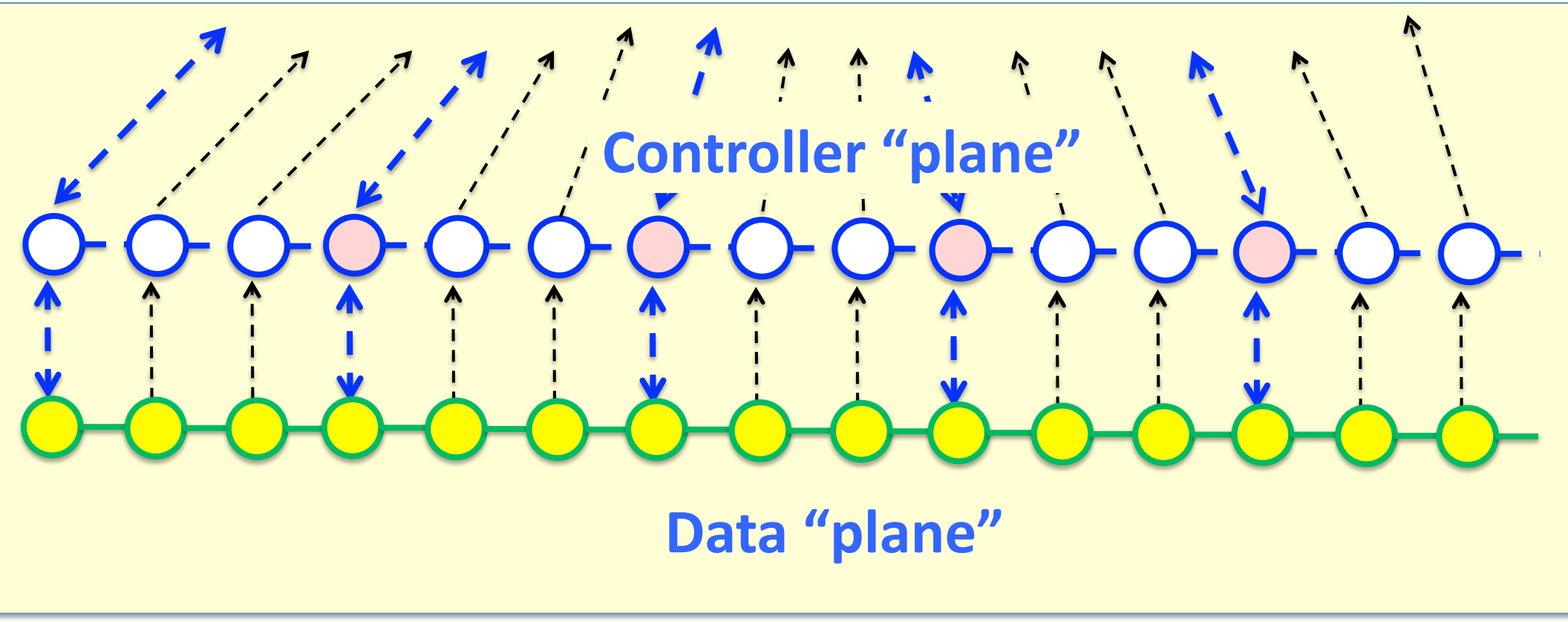
Sense, comm/comp, ***act***.



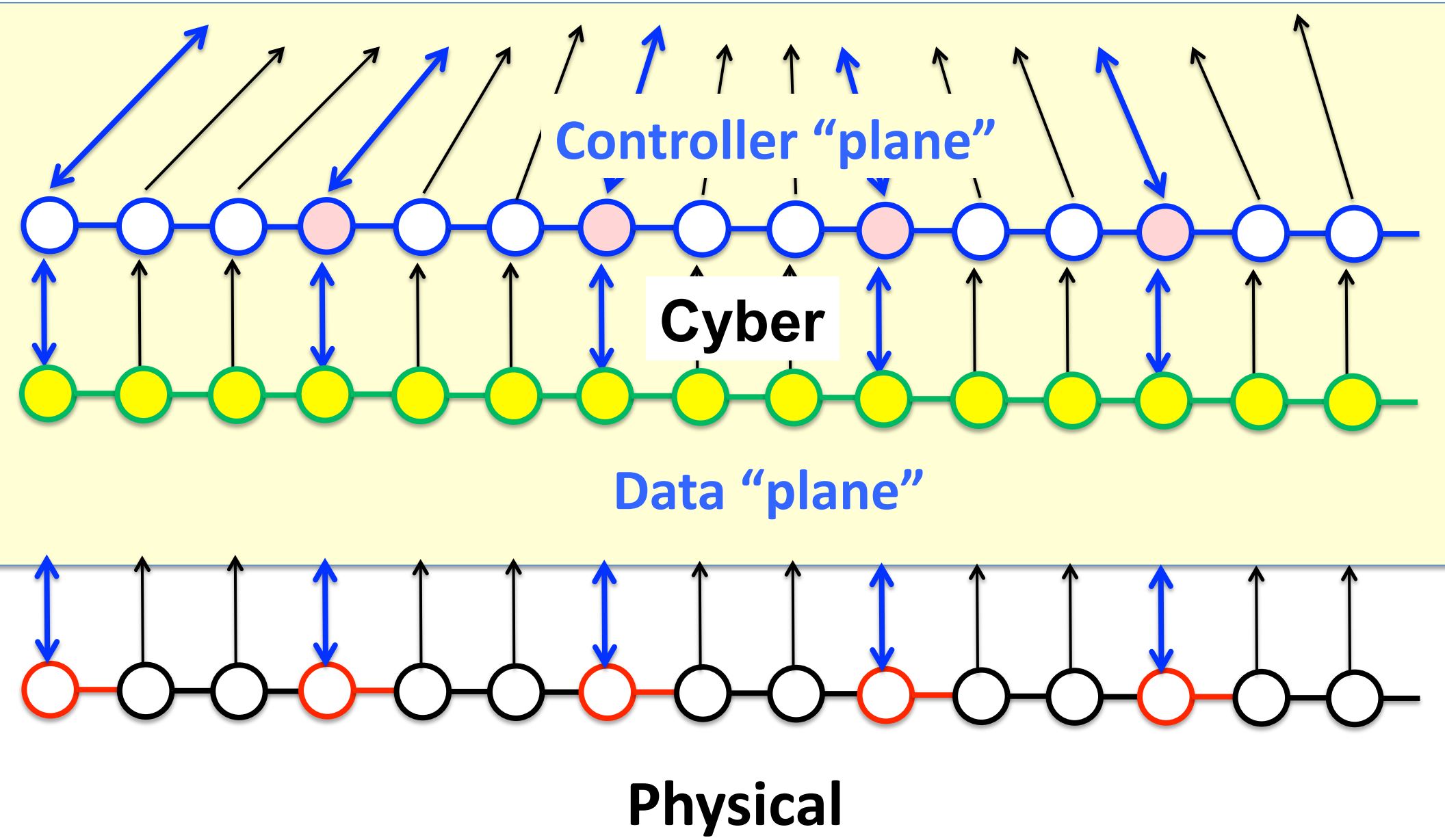
Actuated and sensed

Only sensed


Controller
Physical plant

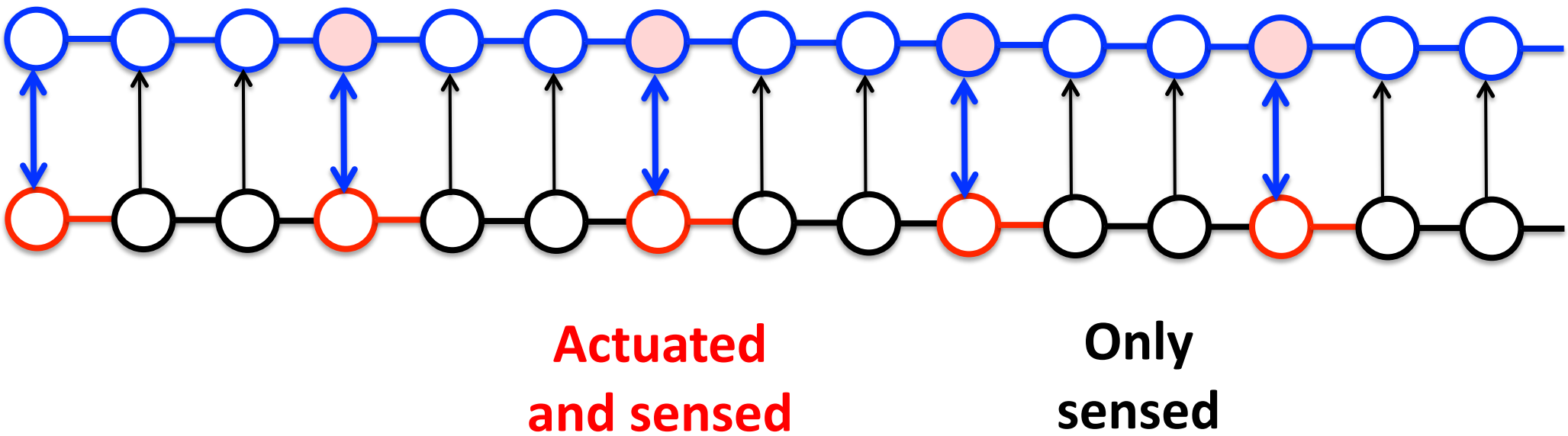


SDN/ODP


Controller "plane"
Cyber
Data "plane"
Physical

Sense, comm/comp, ***act***.

Nominally each has delay 1.

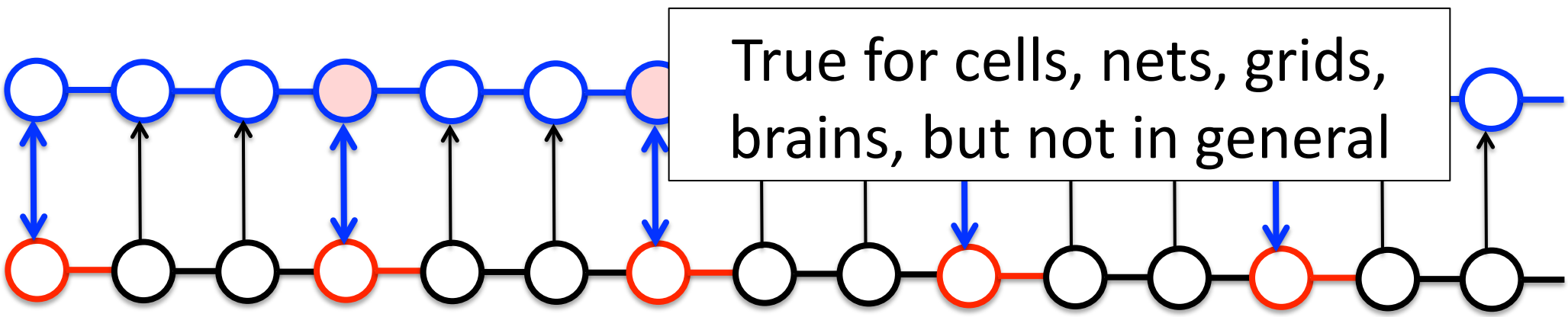


Expensive:

- physical plant
- passive stability
- actuation
- low delay (comms and comp)

Cheap:

- comms bandwidth
- compute memory
- sensing



Actuated and sensed

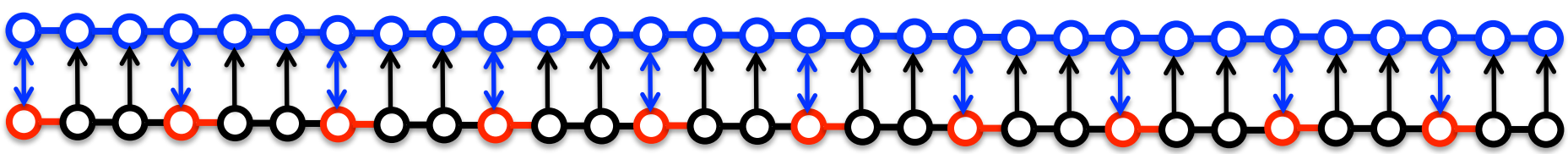
Only sensed

System Model

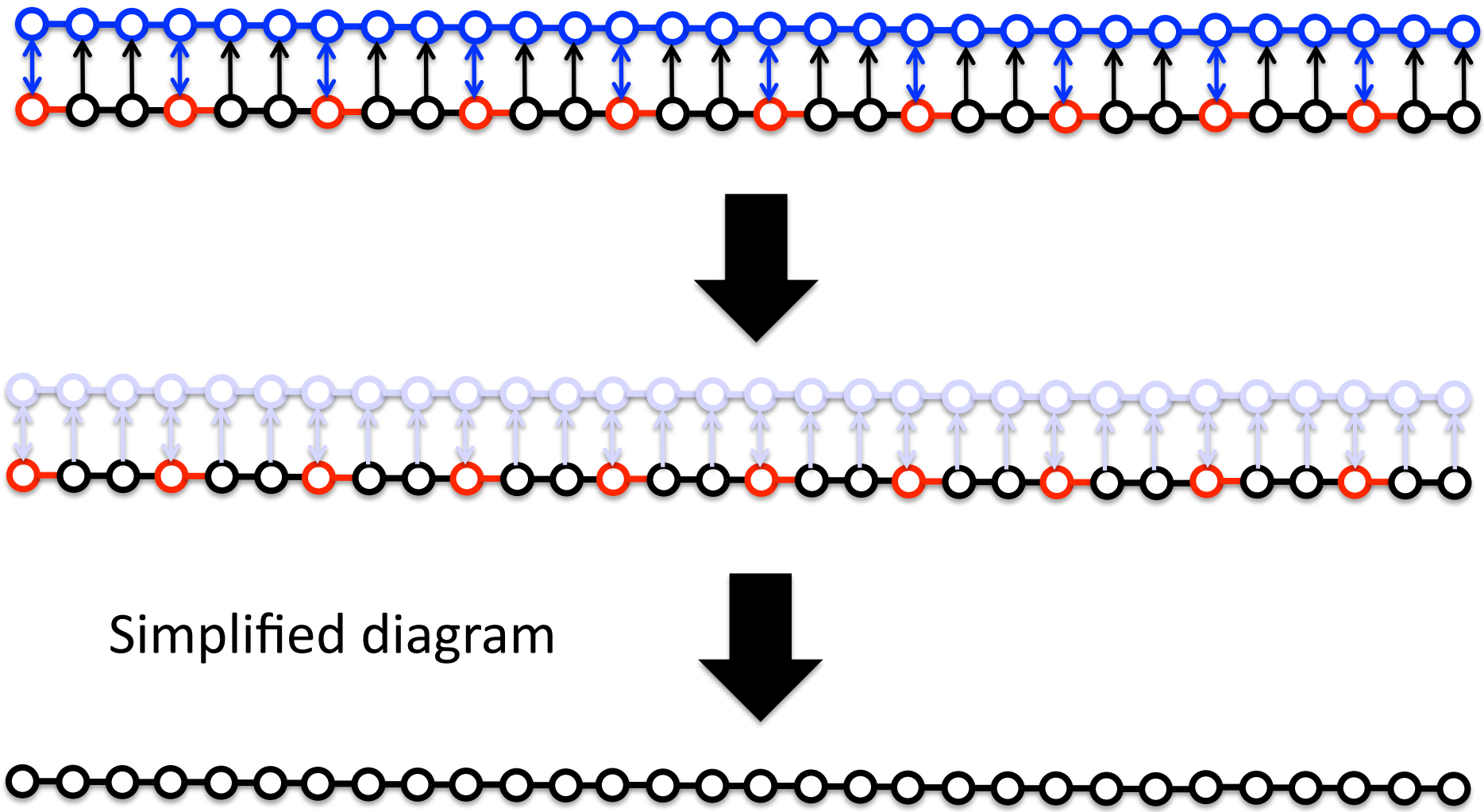
- The discrete time system equation is

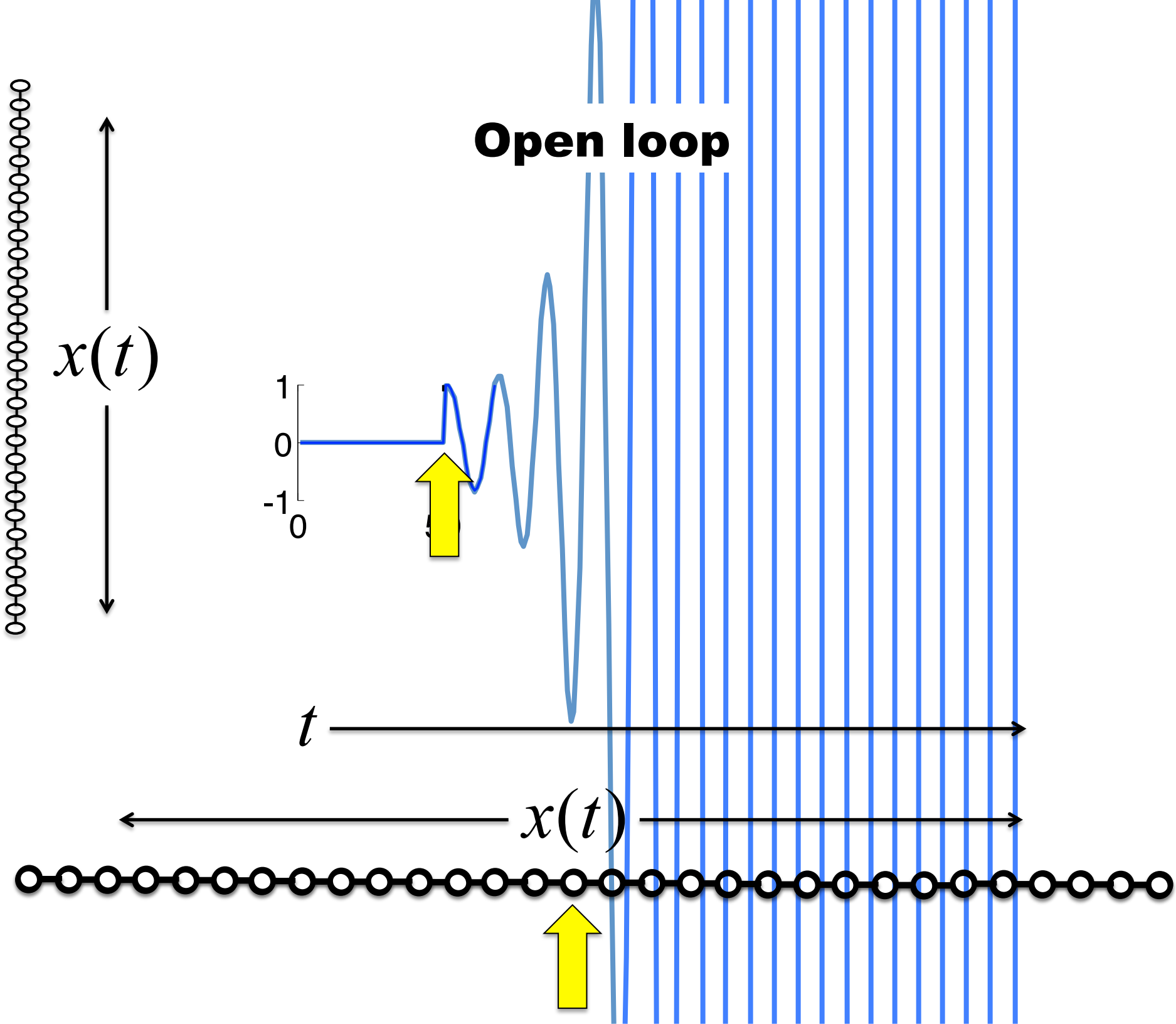
$$x[k+1] = A_d x[k] + Bu[k] + w[k]$$

- Example: 30 C, 29 L

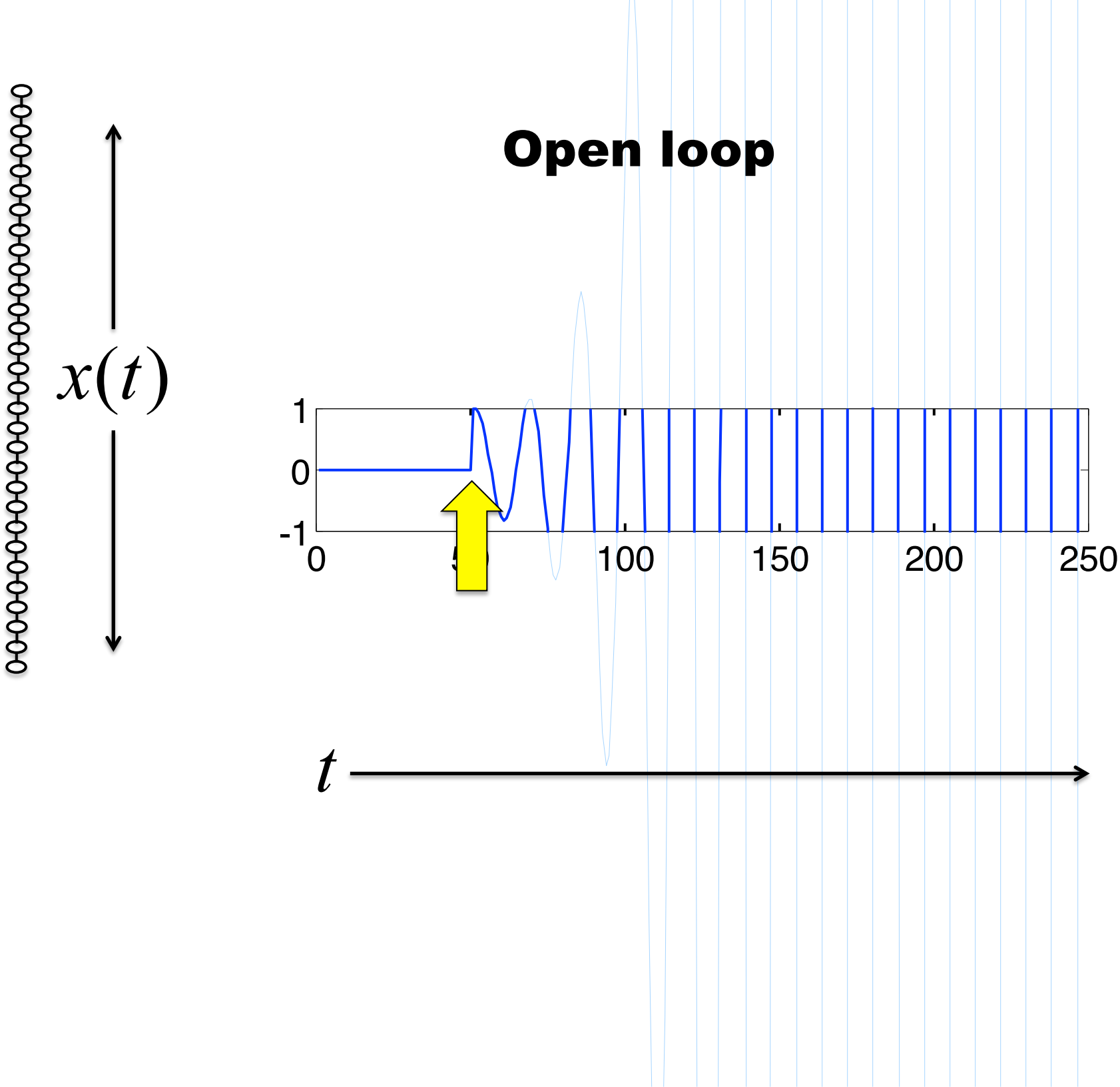


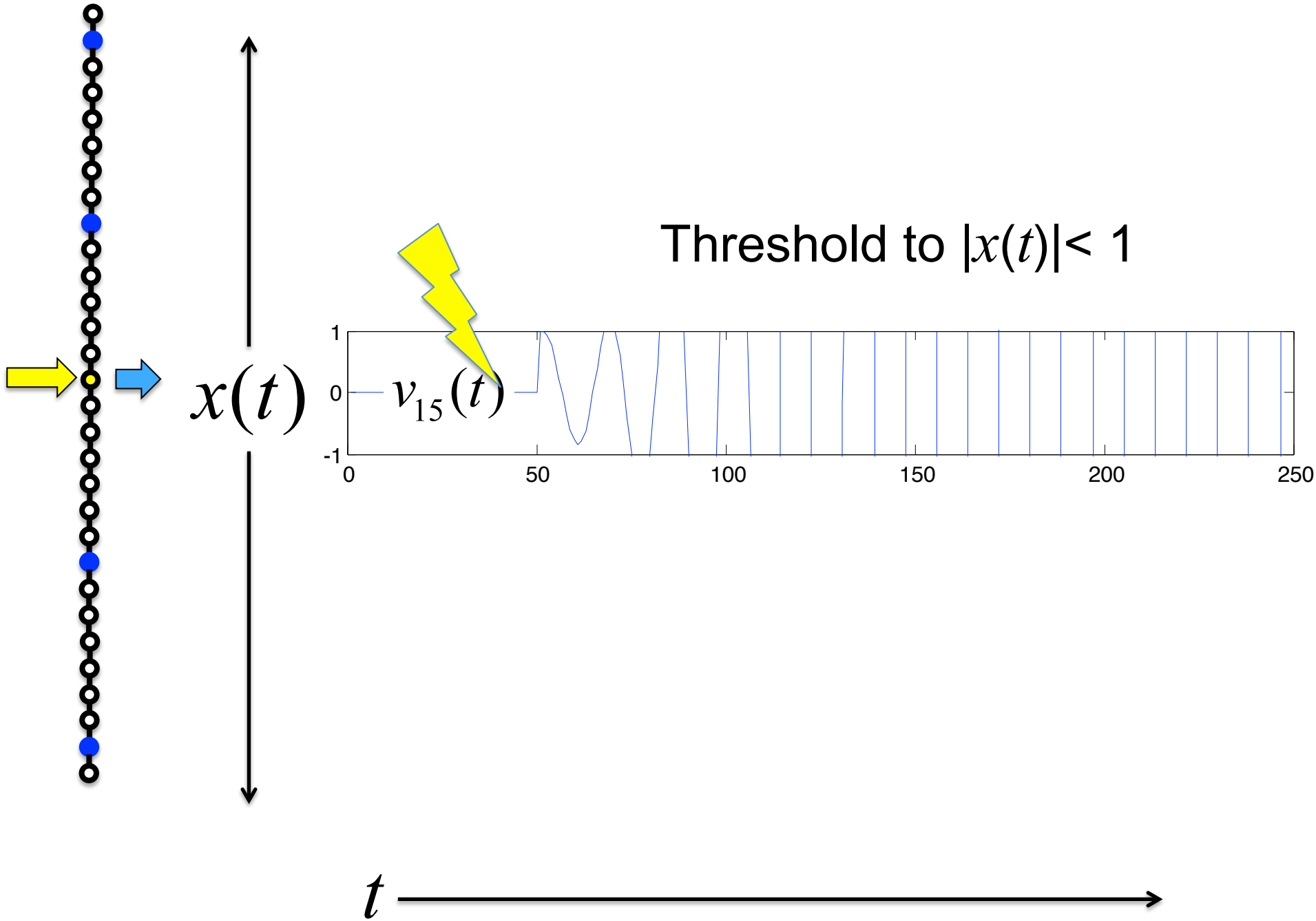
Open loop dynamics

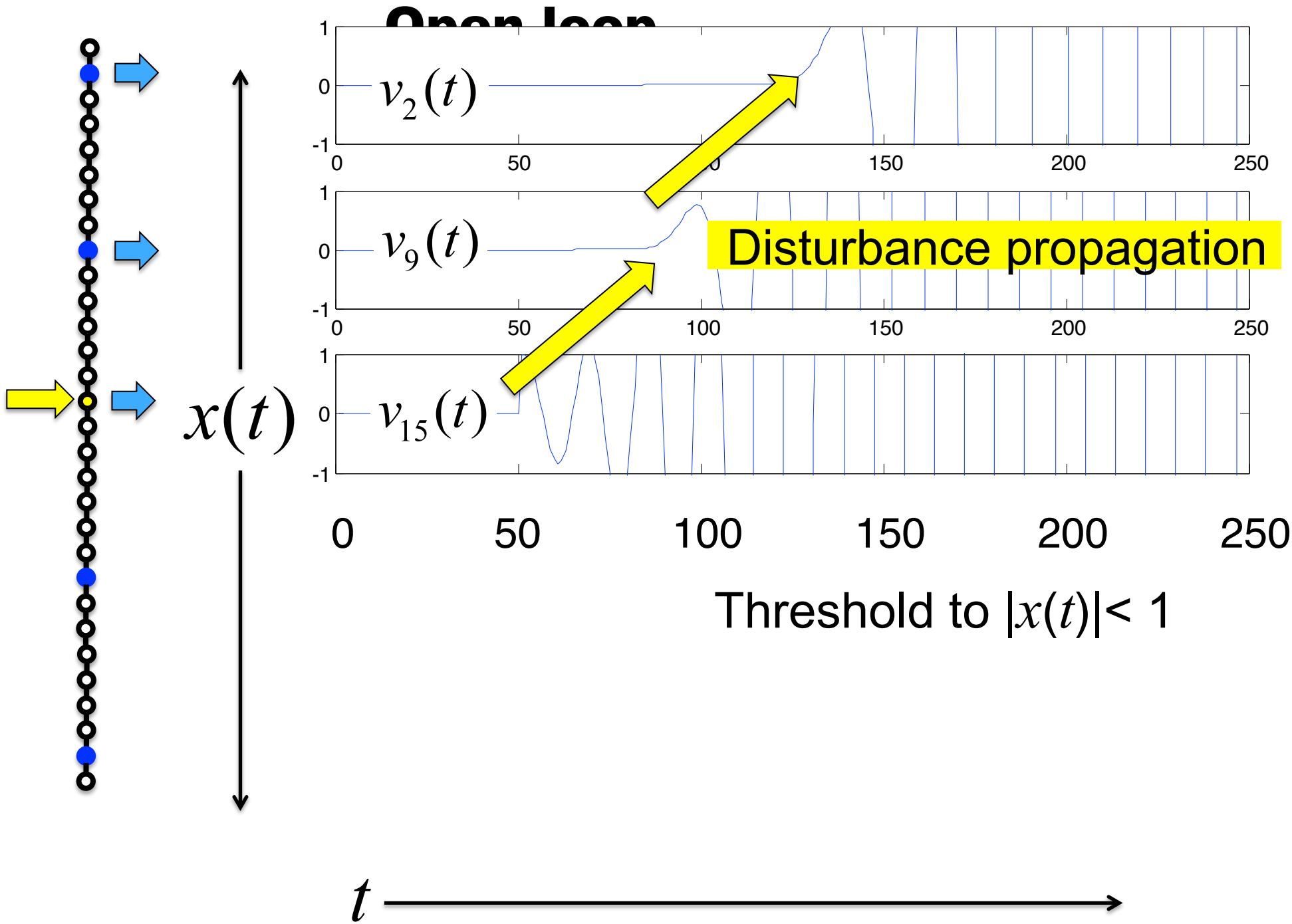


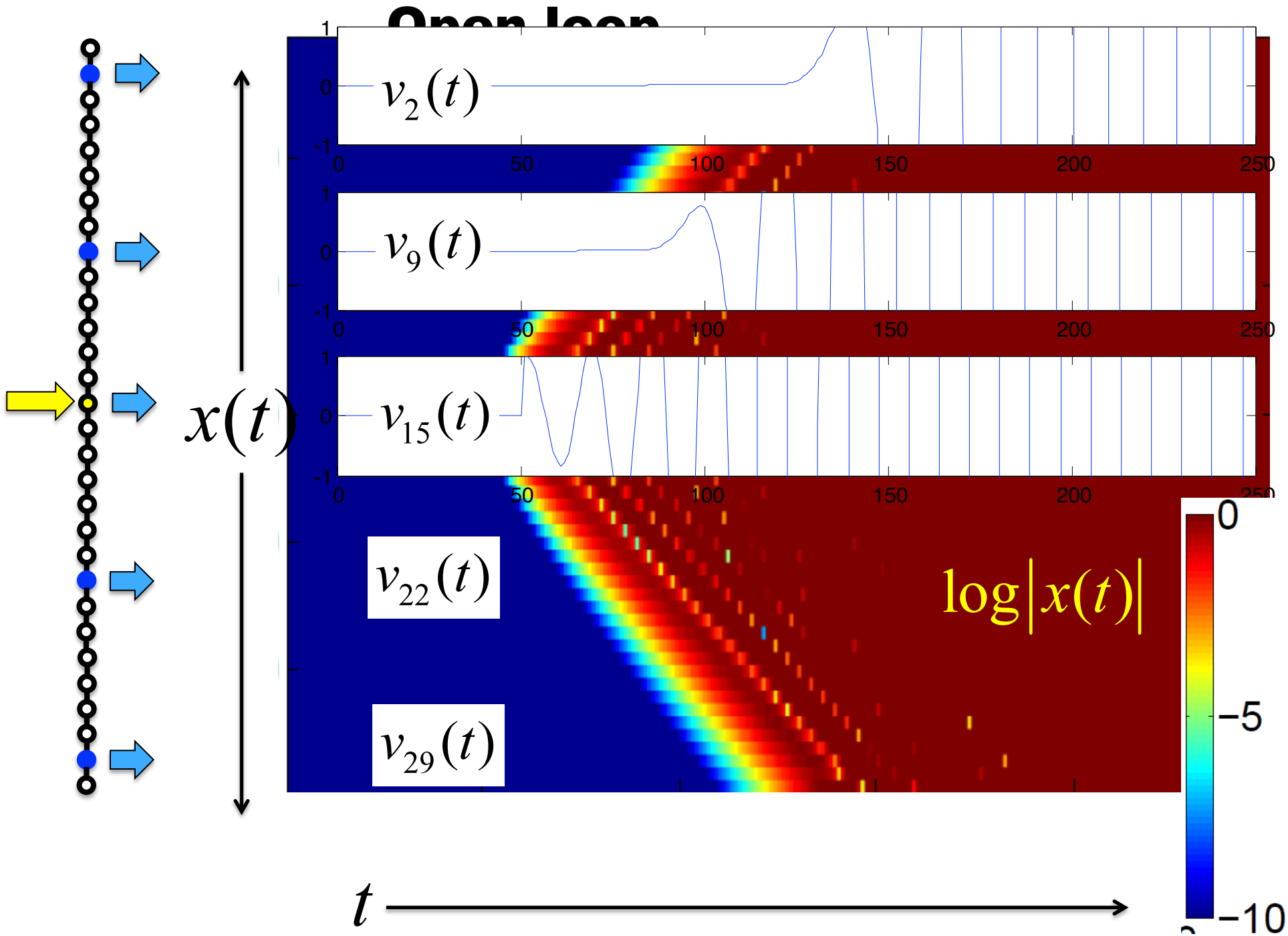

Open loop
$x(t)$
1
0
-1
0
t
$x(t)$

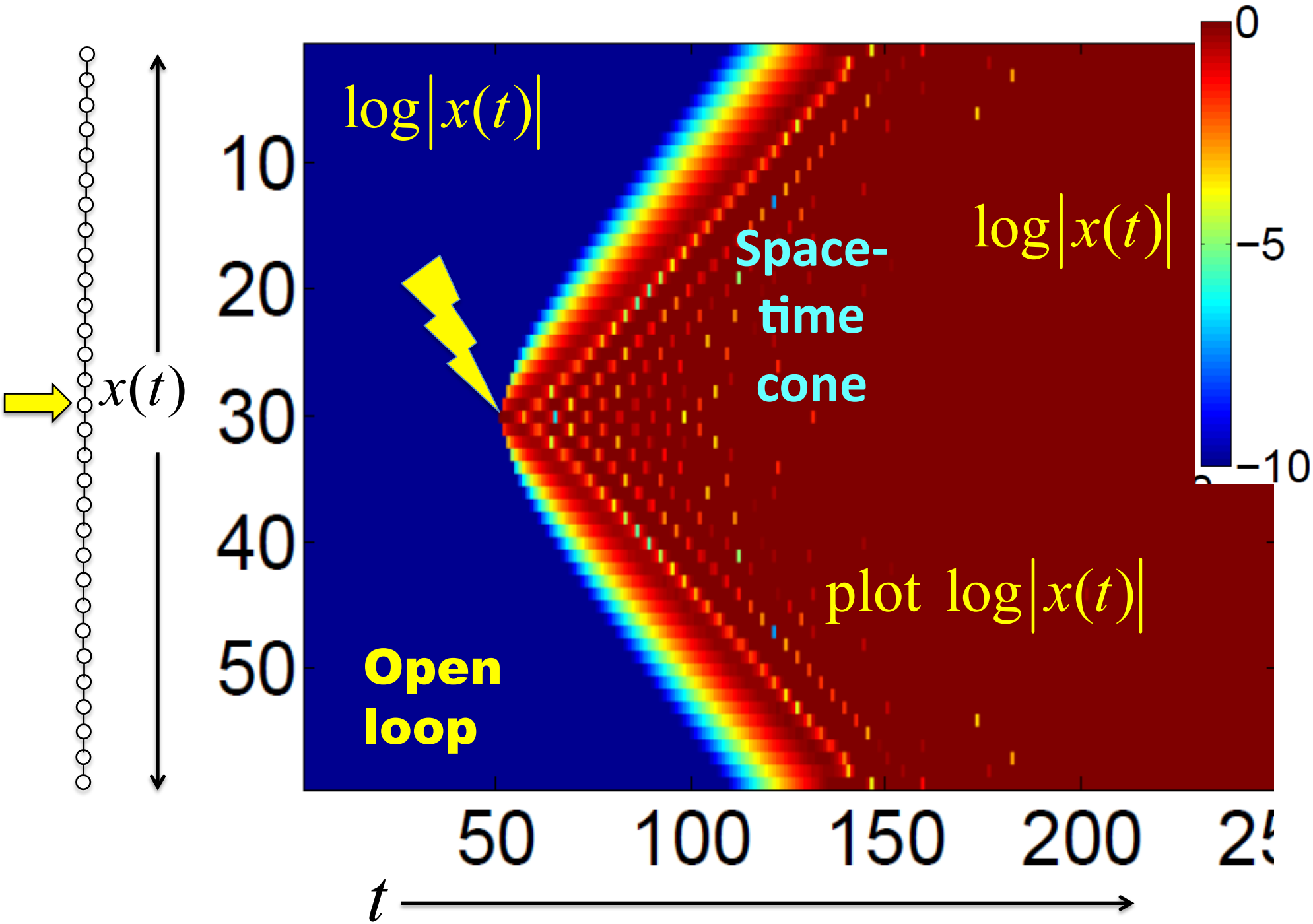
Open loop

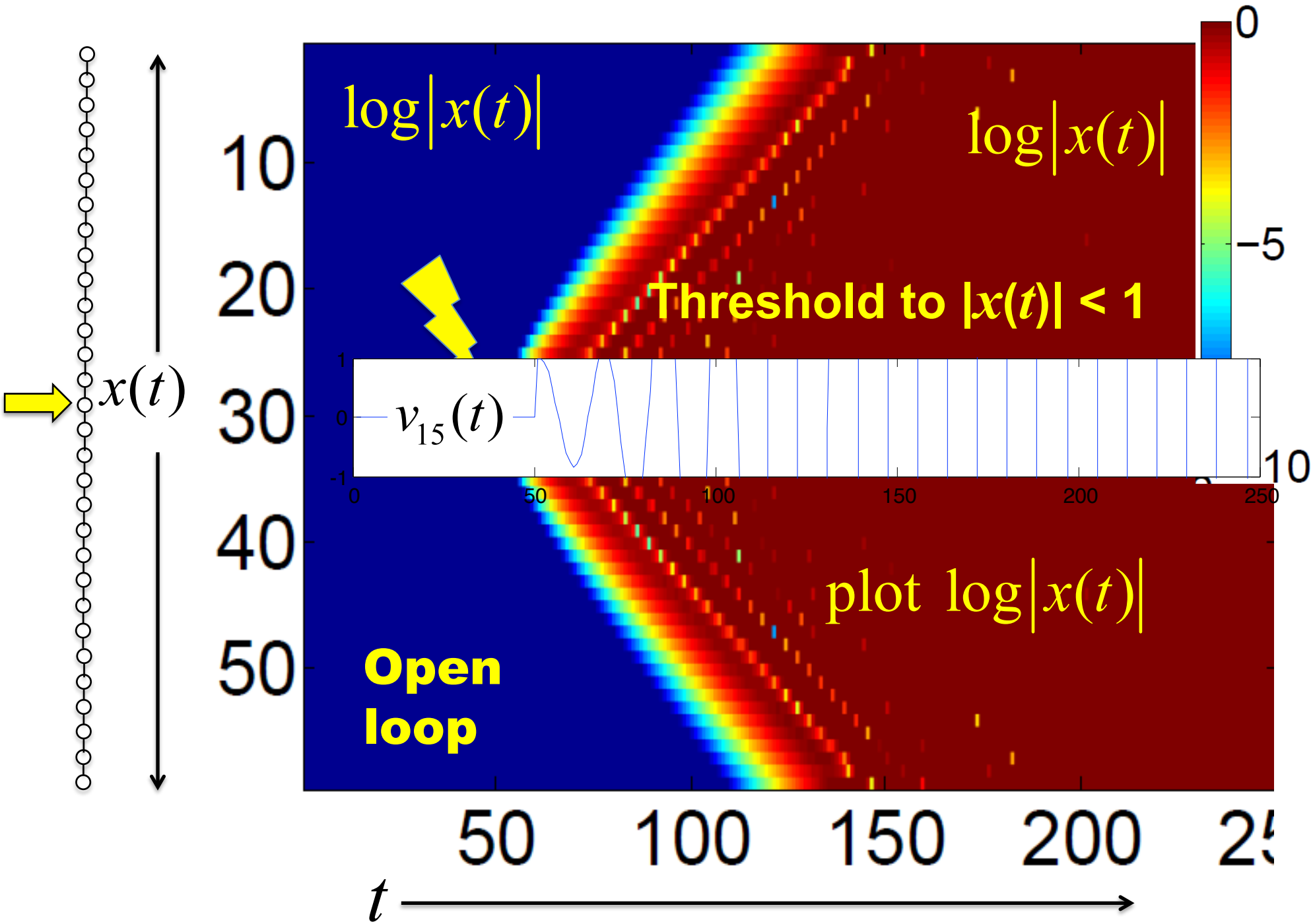


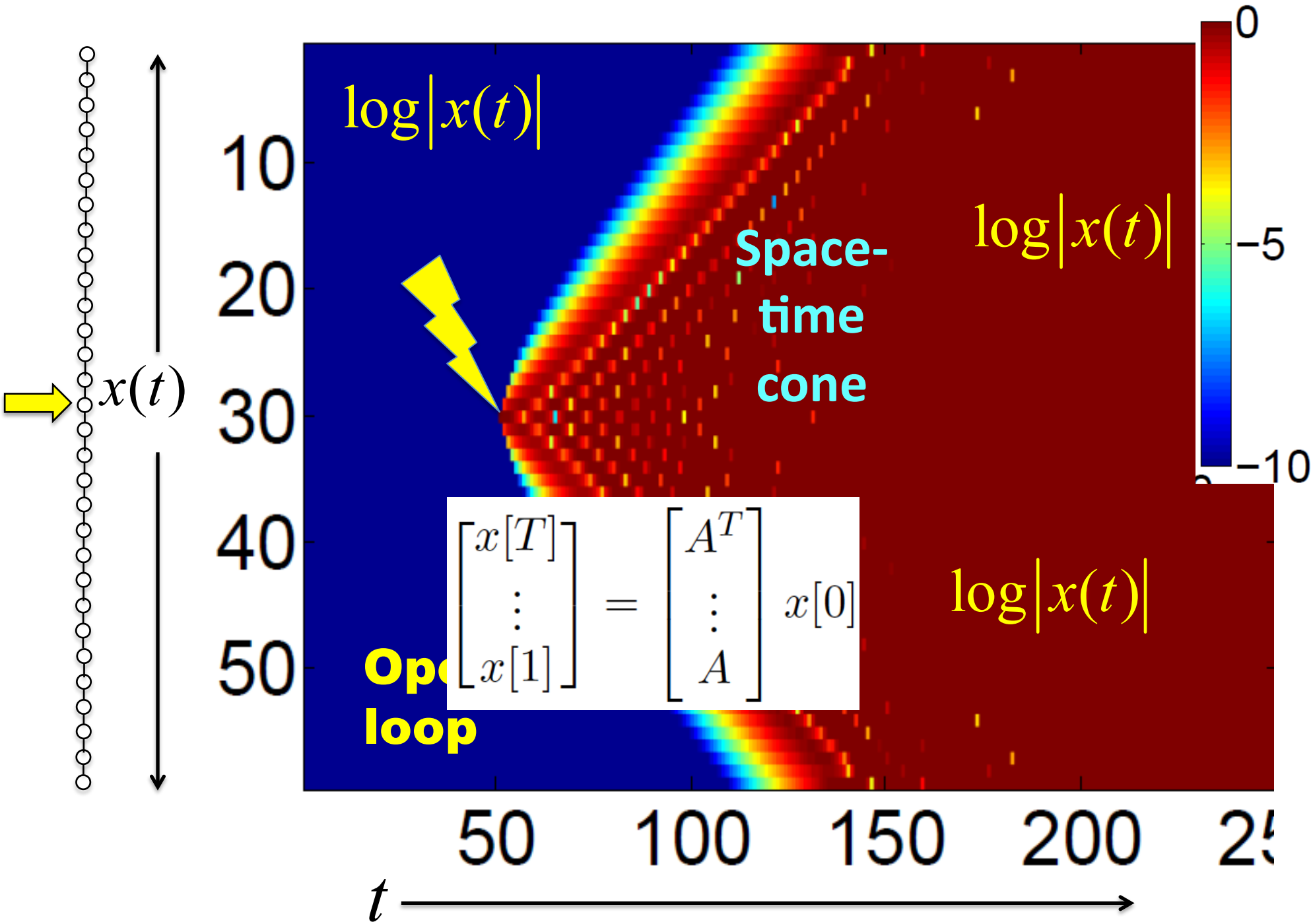

Threshold to |x(t)|< 1
x(t)
v15(t)
1
0
-1
0
50
100
150
200
250
t

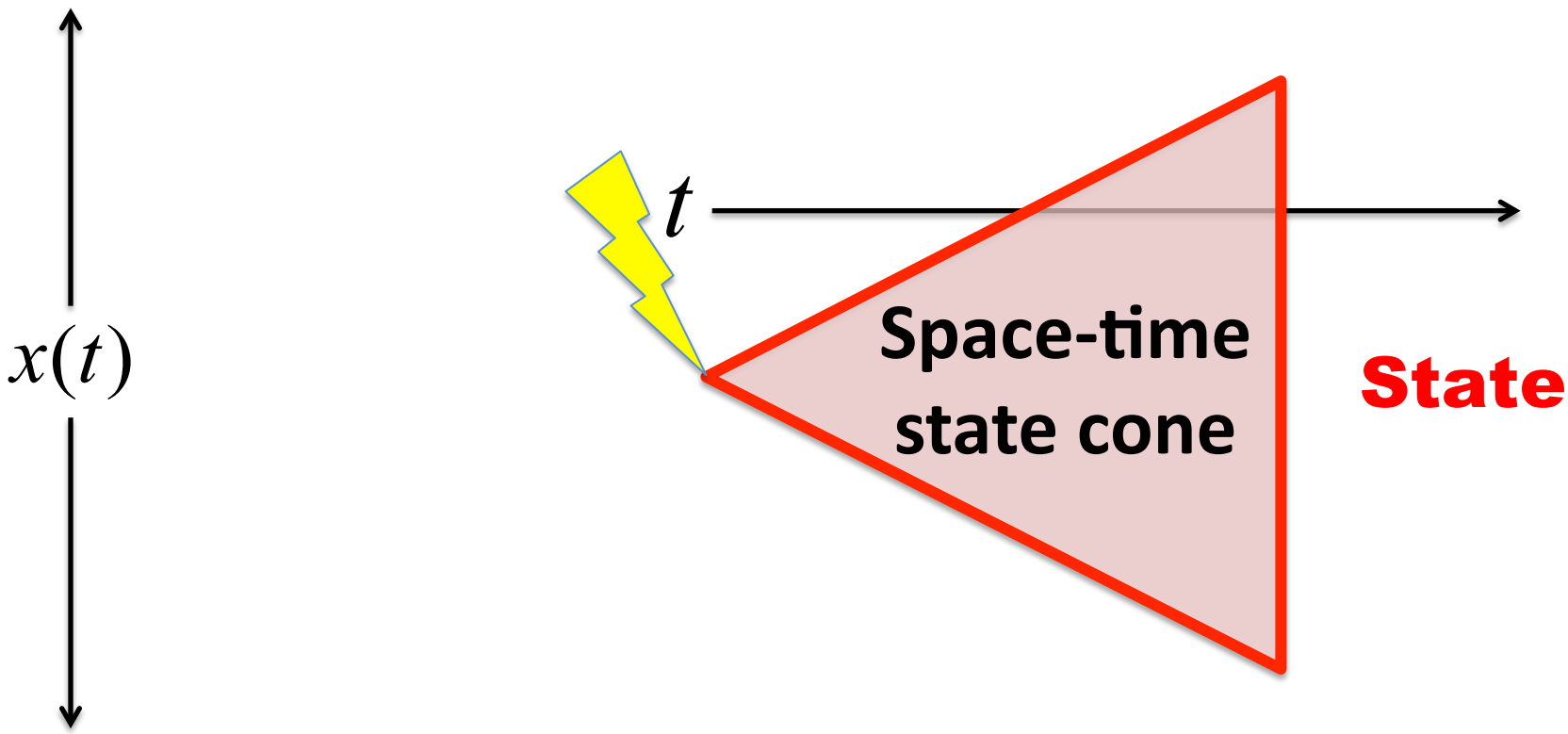

Open loop
$v_2(t)$
$v_9(t)$
$v_{15}(t)$
Disturbance propagation
$x(t)$
1
0
-1
0
50
100
150
200
250
Threshold to $|x(t)|< 1$
t


$v_2(t)$
$v_9(t)$
$v_{15}(t)$
$v_{22}(t)$
$v_{29}(t)$
$x(t)$
t
$\log|x(t)|$
0
−5
−10
1
0
-1
0
50
100
150
200
250


$\log|x(t)|$
Space-time cone
$\log|x(t)|$
plot $\log|x(t)|$
Open loop
$x(t)$
10
20
30
40
50
50
100
150
200
25
t
0
−5
−10


$\log|x(t)|$
$\log|x(t)|$
Threshold to $|x(t)| < 1$
$v_{15}(t)$
plot $\log|x(t)|$
Open loop
$x(t)$
t
10
20
30
40
50
50
100
150
200
0
−5
10


$x(t)$
$\log|x(t)|$
Space-time cone
$\log|x(t)|$
$\log|x(t)|$
$\begin{bmatrix} x[T] \\ \vdots \\ x[1] \end{bmatrix} = \begin{bmatrix} A^T \\ \vdots \\ A \end{bmatrix} x[0]$
Ope
loop
10
20
30
40
50
50
100
150
200
25
t
0
−5
−10



$$\begin{bmatrix} x[T] \\ \vdots \\ x[1] \end{bmatrix} = \begin{bmatrix} A^T \\ \vdots \\ A \end{bmatrix} x[0]$$

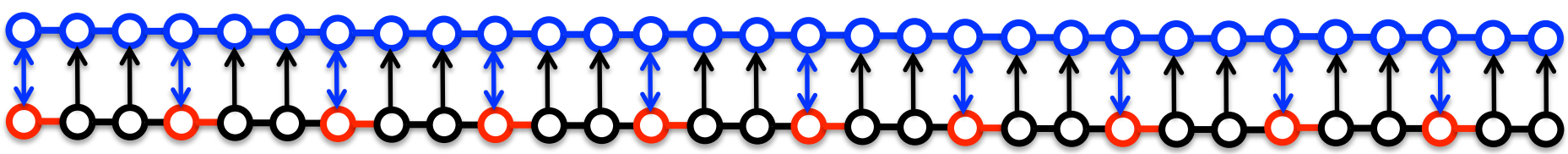
Controller Design

Critical Issues

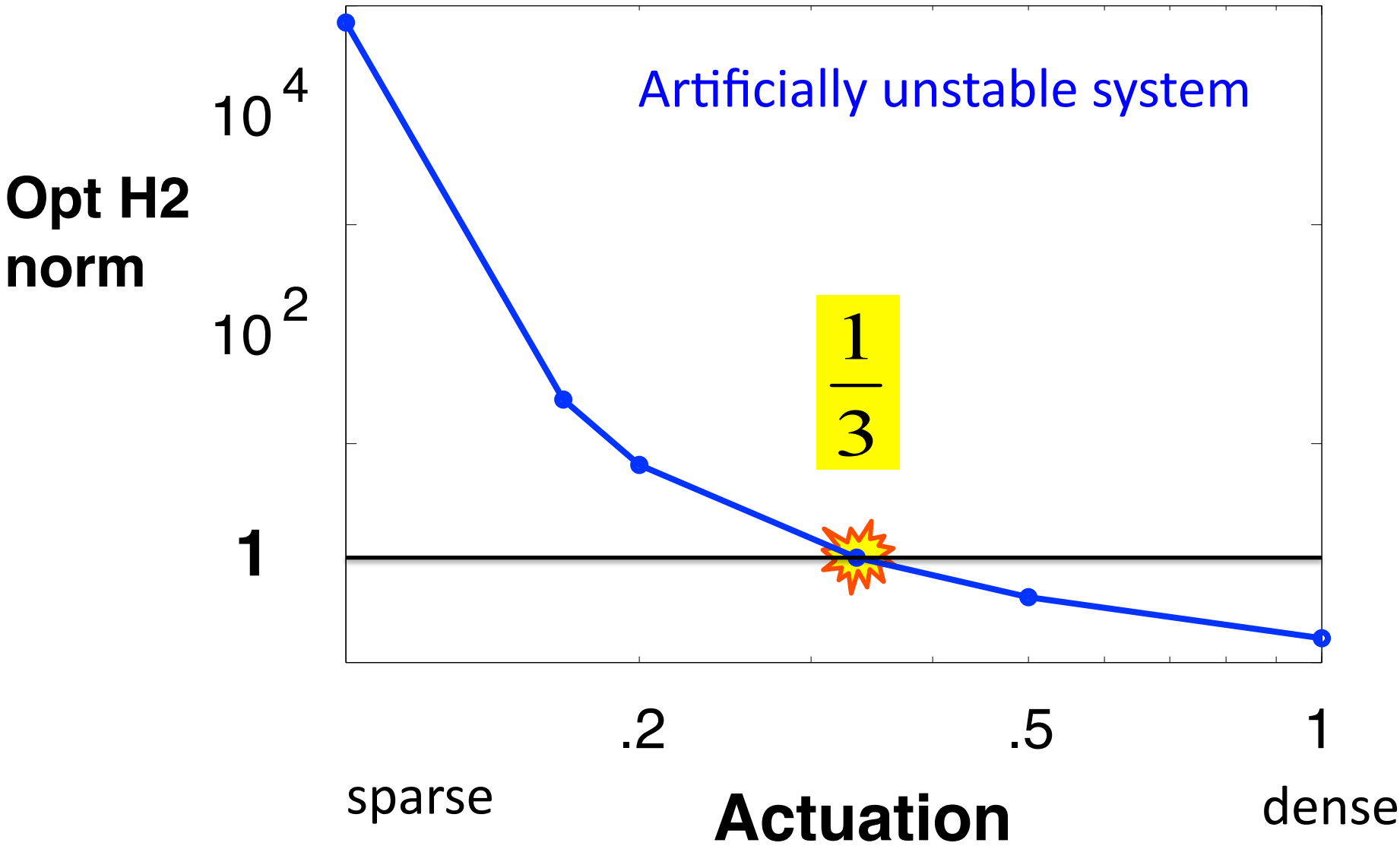
1. Transient LQ (H2) cost: $\Sigma(x'x+u'u)$
2. Actuator Density
3. Communication (vs plant) Speed
4. Locality/Scalability (Computation)
5. Time/space horizon

Actuator Density

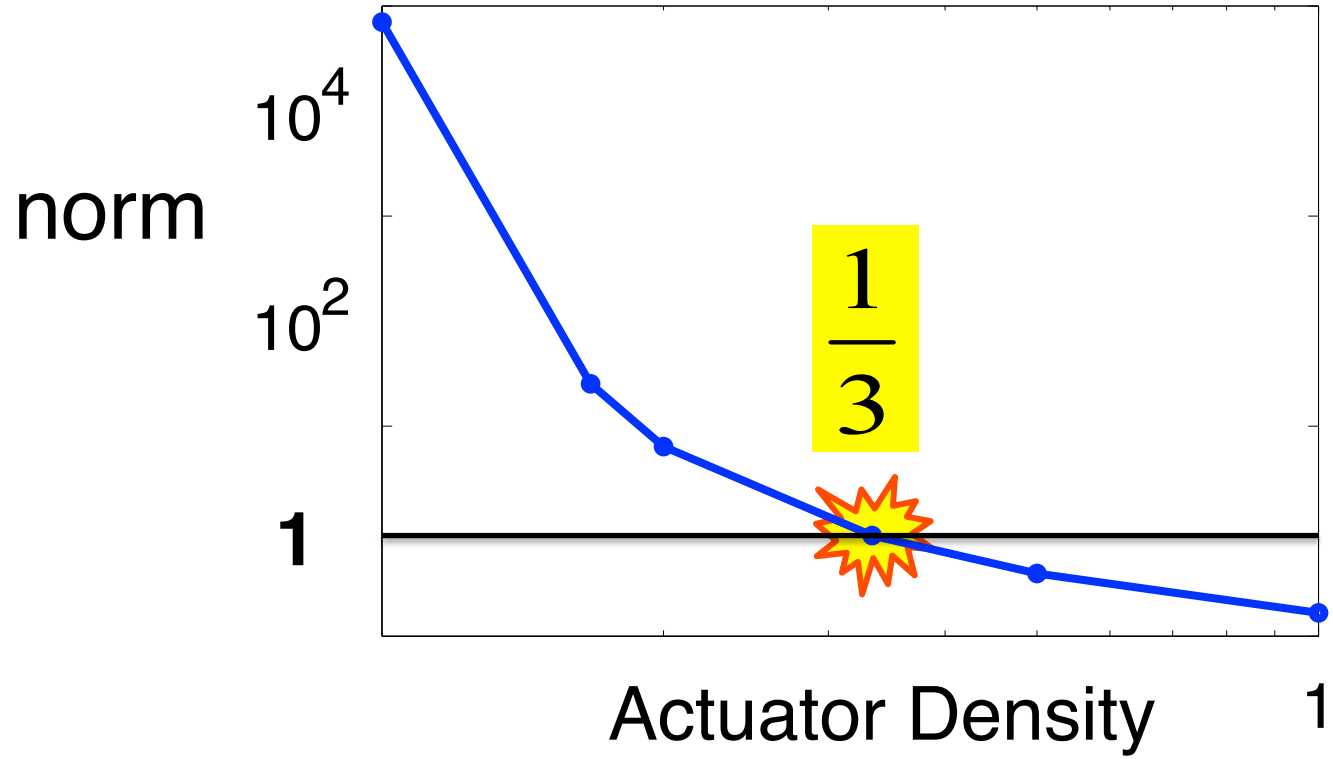
- ***Standard*** (centralized) optimal H2 control
- No delay (initially)
- Defer other issues (∞ comm, comp, sense)
- Objective: min sum (x'x+u'u)
- Actuator density = # actuators / # states
- Trade-off: actuator density vs norm
- Example: 30 C, 29 L



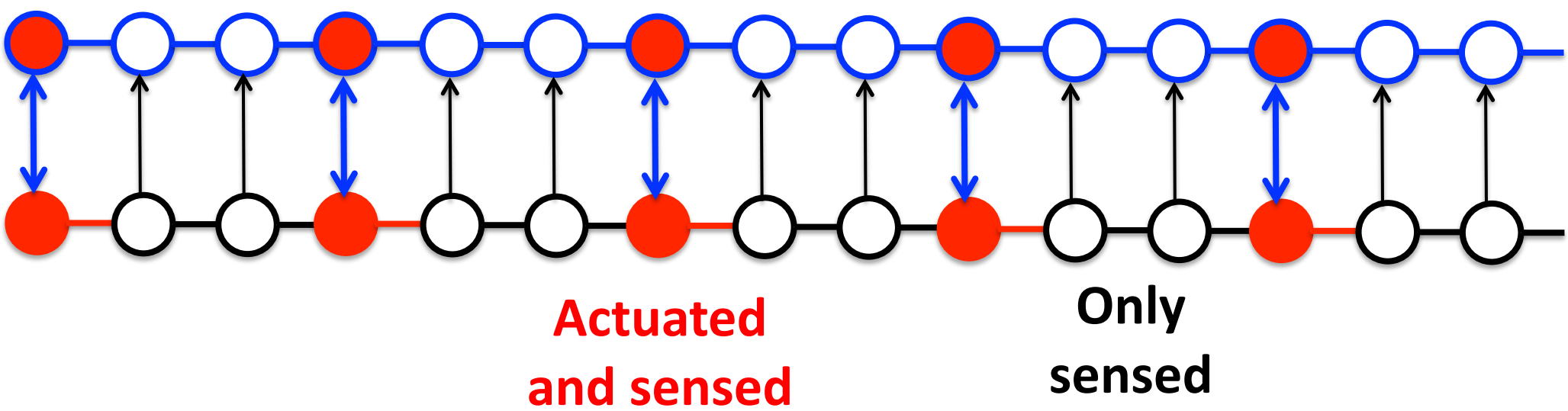
Norm - Actuator Density (normalized)

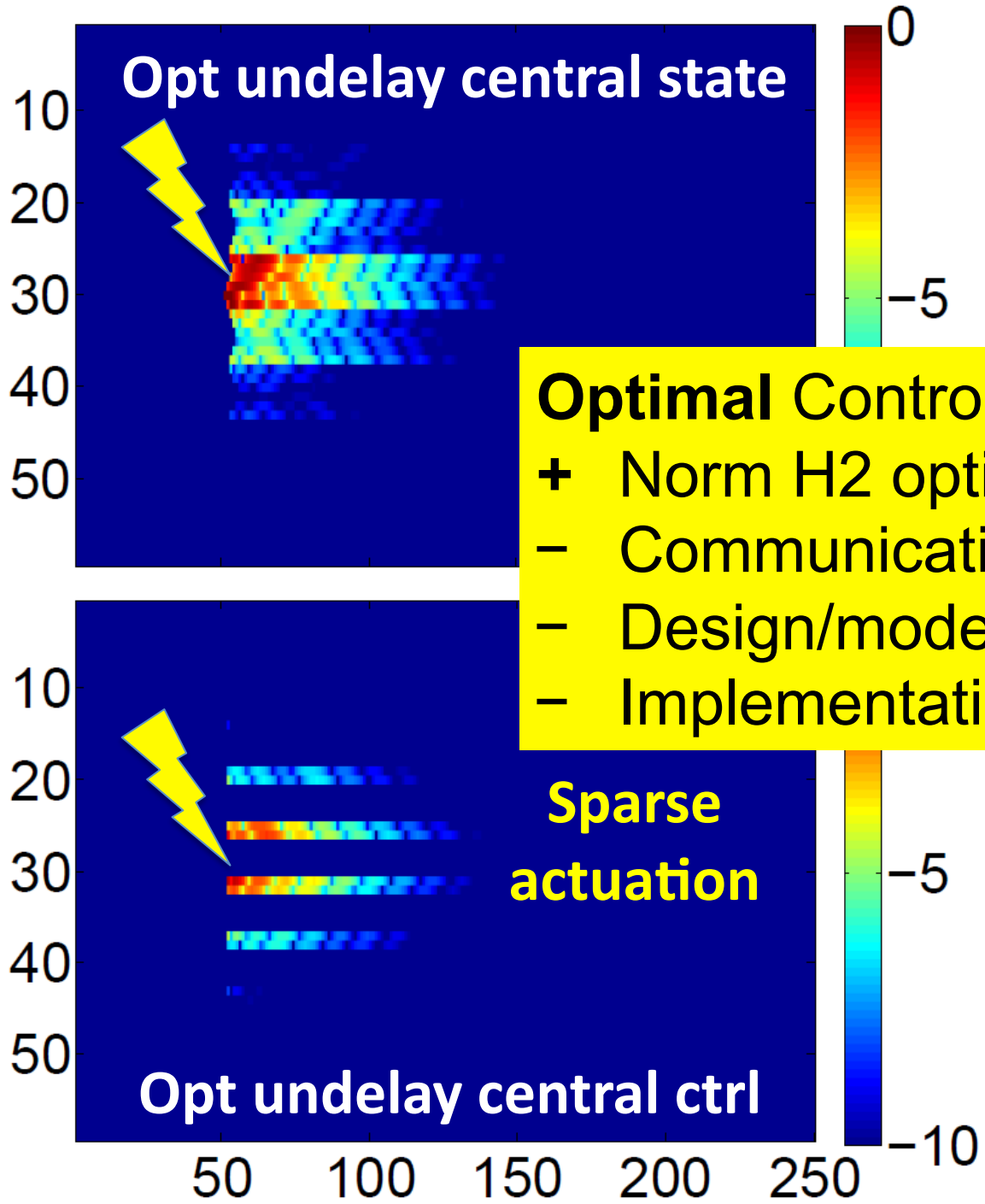


Standard control (circa 1970)



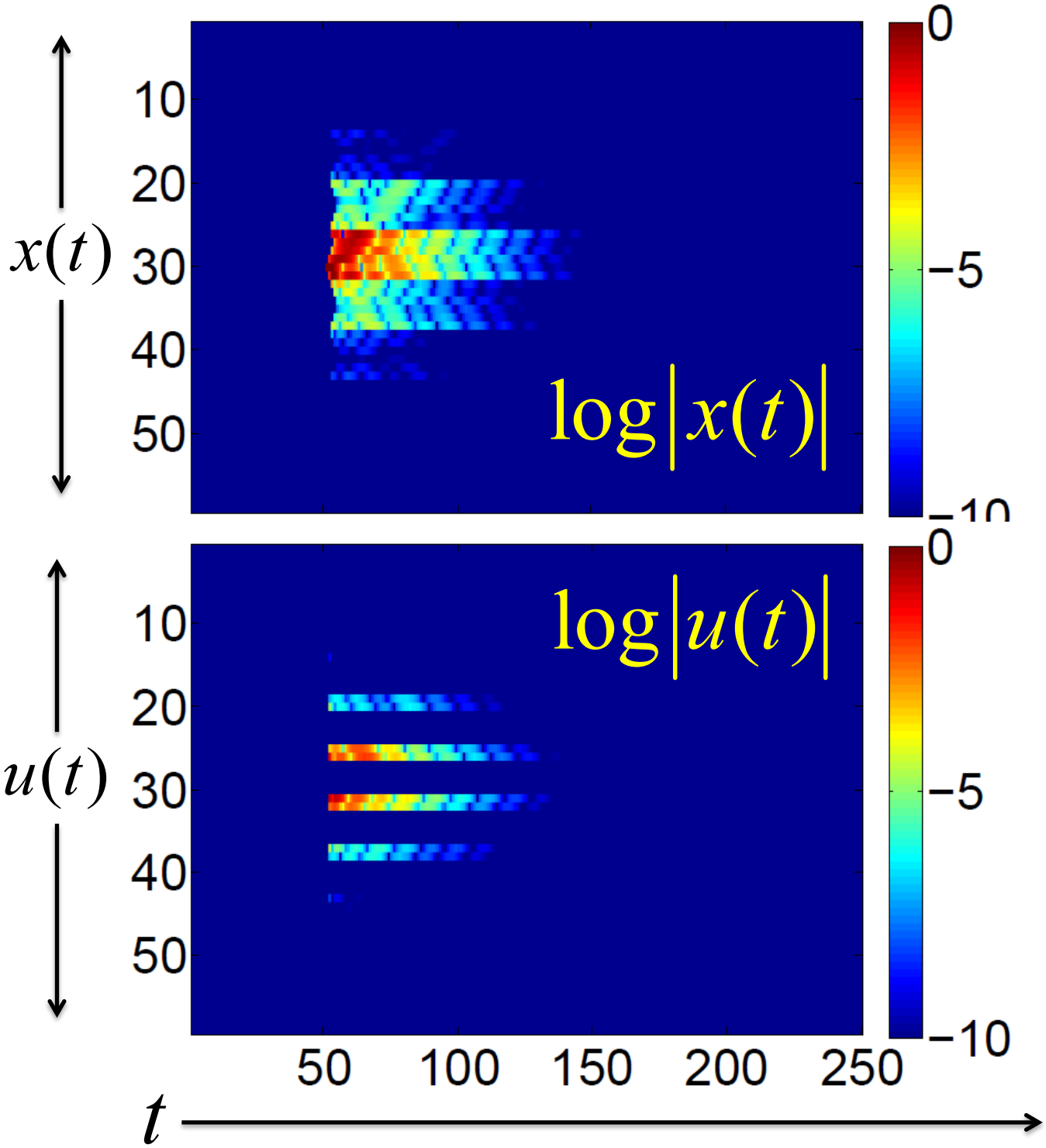
∞ Comm speed = 0 delay





Optimal Controller

+ Norm H2 optimal
− Communication undelayed
− Design/model global/huge P
− Implementation local/huge P

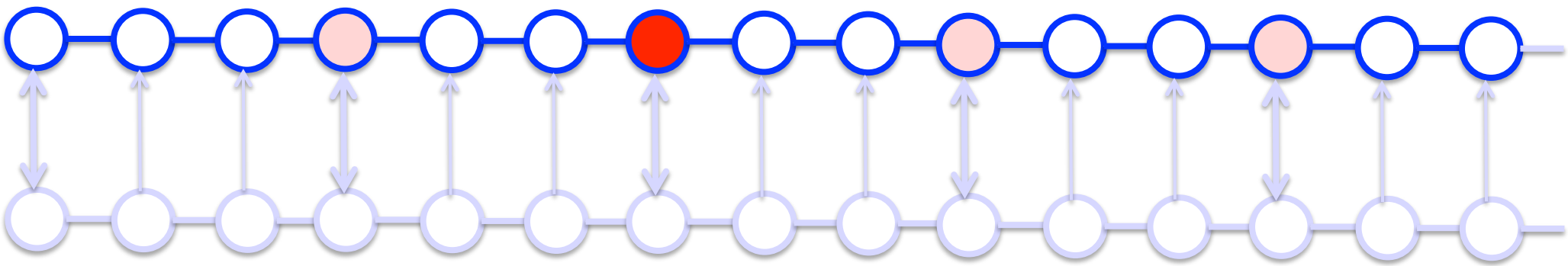

$x(t)$
10
20
30
40
50
$\log|x(t)|$
0
−5
−10
$u(t)$
10
20
30
40
50
$\log|u(t)|$
0
−5
−10
50
100
150
200
250
t

Color code?

Expensive?

0. Physical
1. Actuation
2. **Comms speed**
3. Comp speed
4. Sensing

…



Actuated and sensed

Only sensed

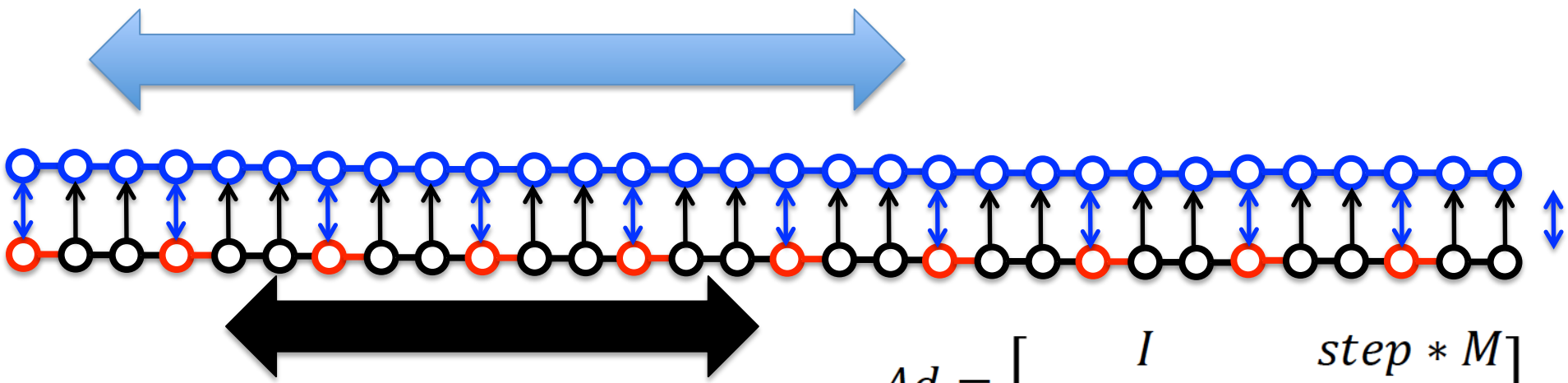
Nominally delay 1.

Expensive?

0. Physical
1. Actuation
2. **Comms speed**
3. Comp speed
4. Sensing

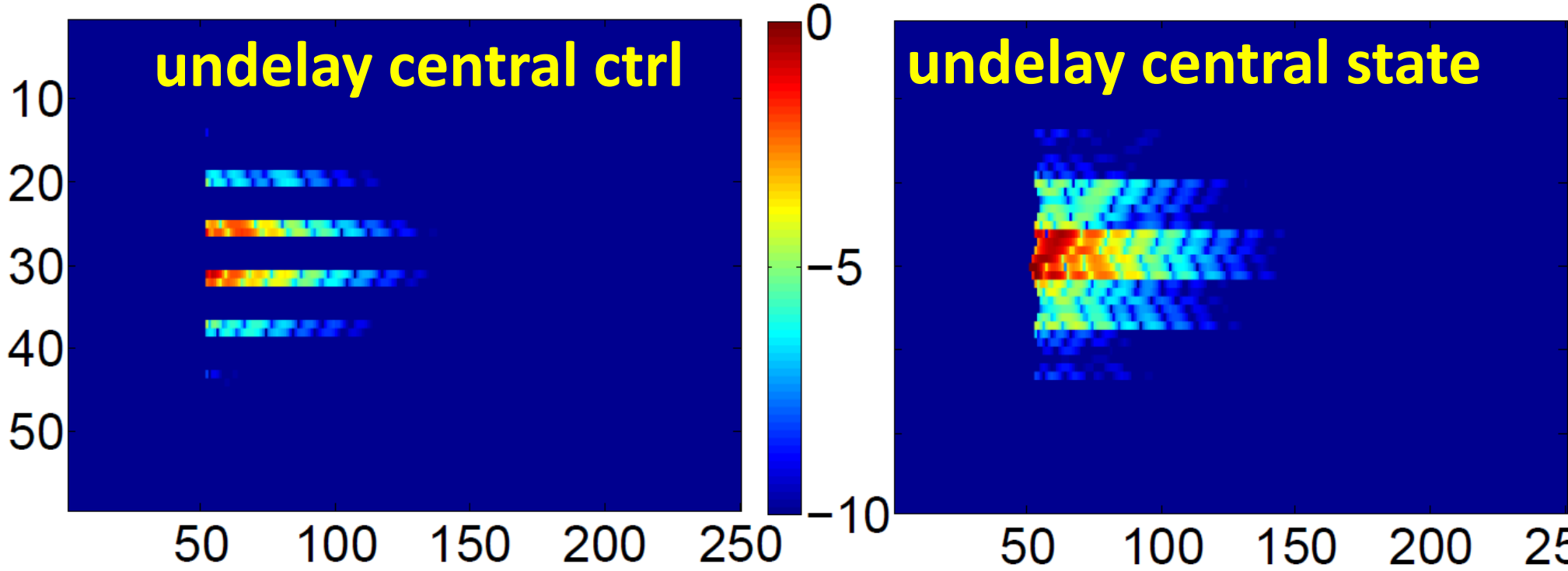
…

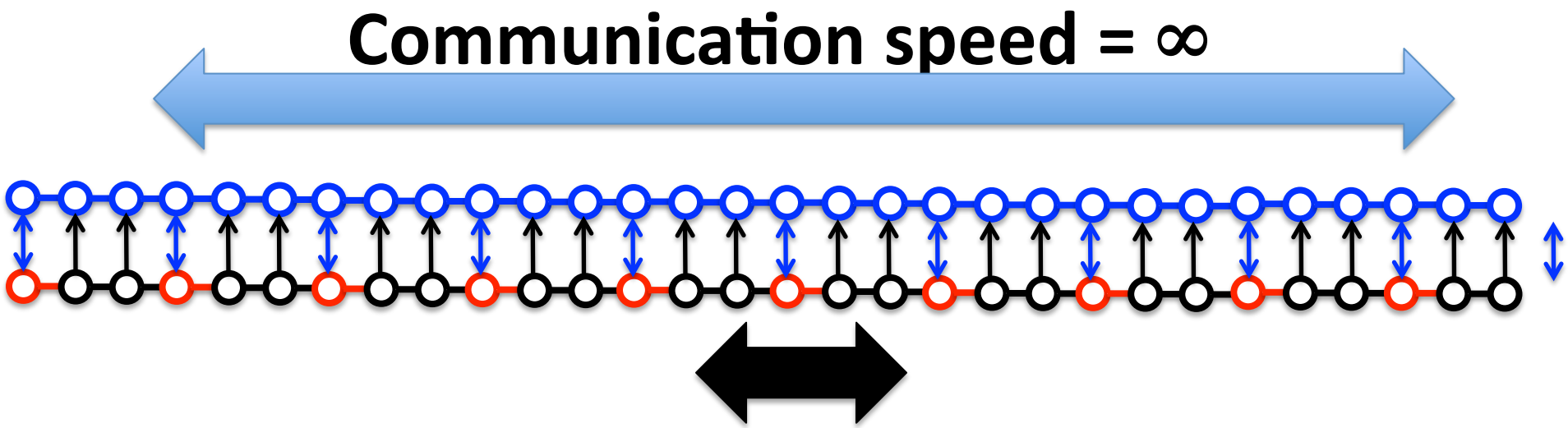
Communication speed

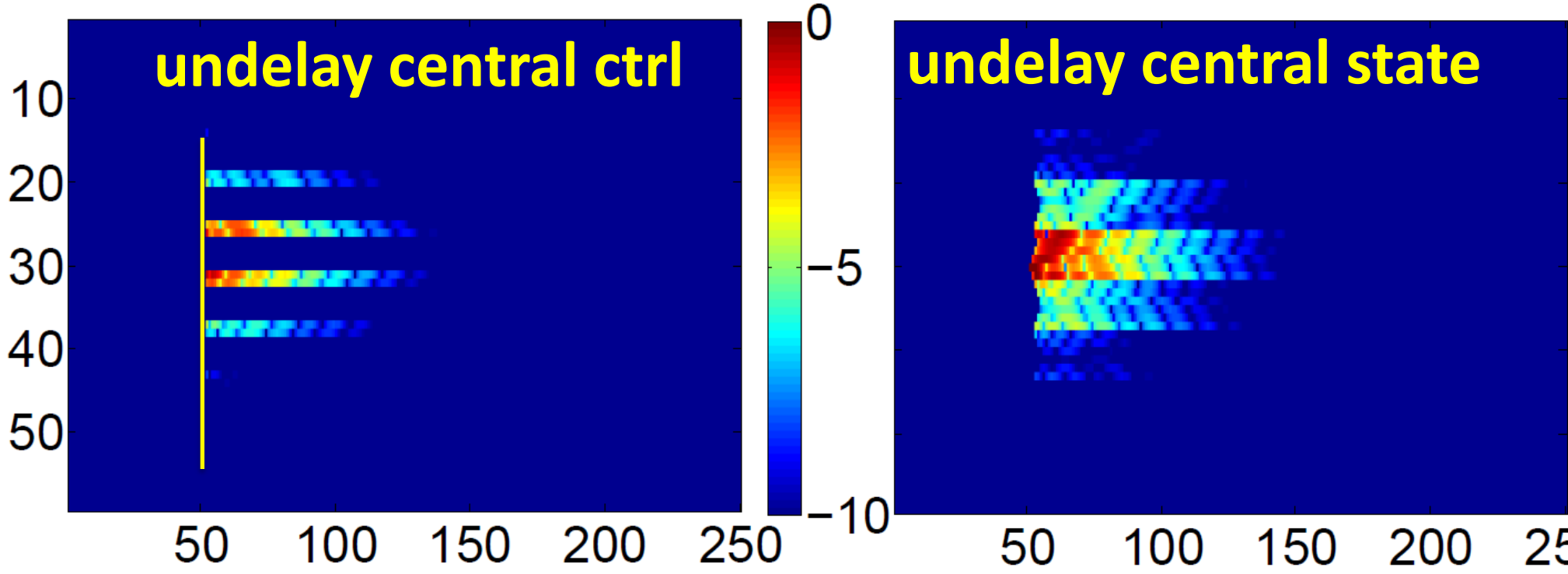


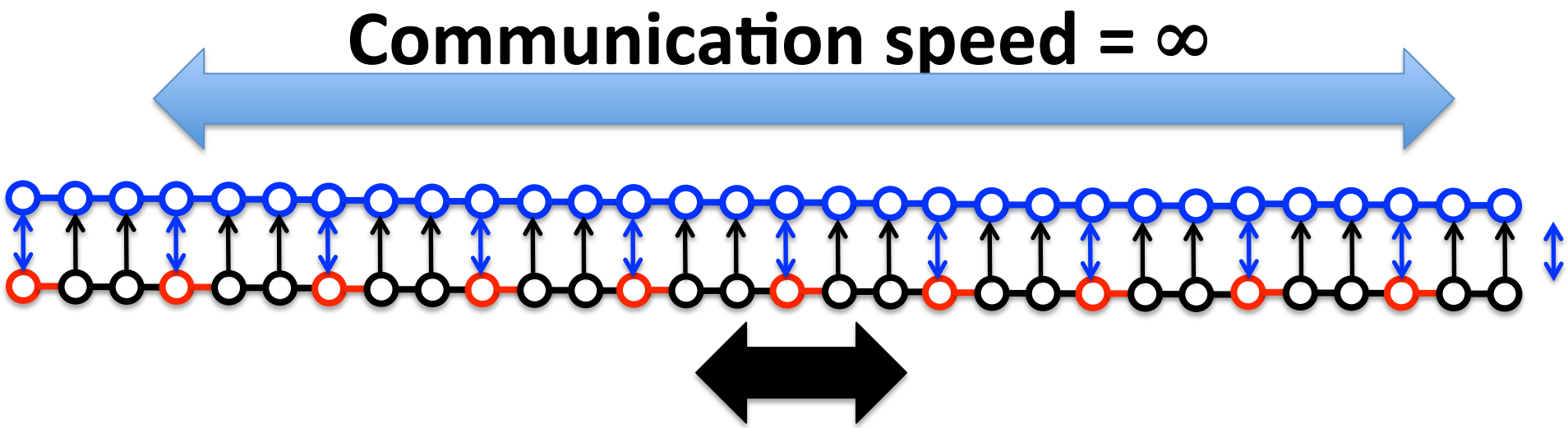
Versus plant speed

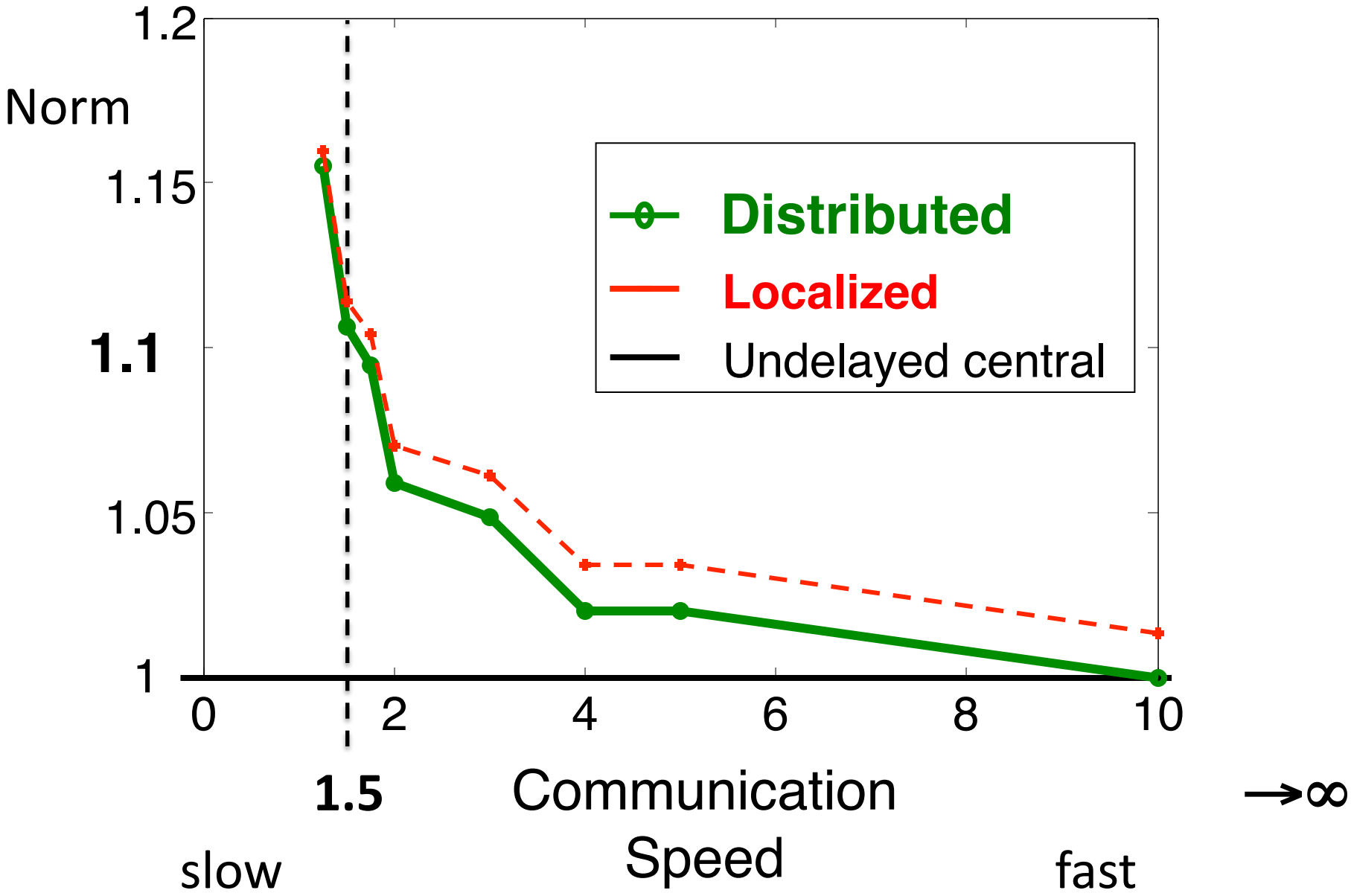
$$Ad = \begin{bmatrix} I & step * M \\ -step * M' & I \end{bmatrix}$$

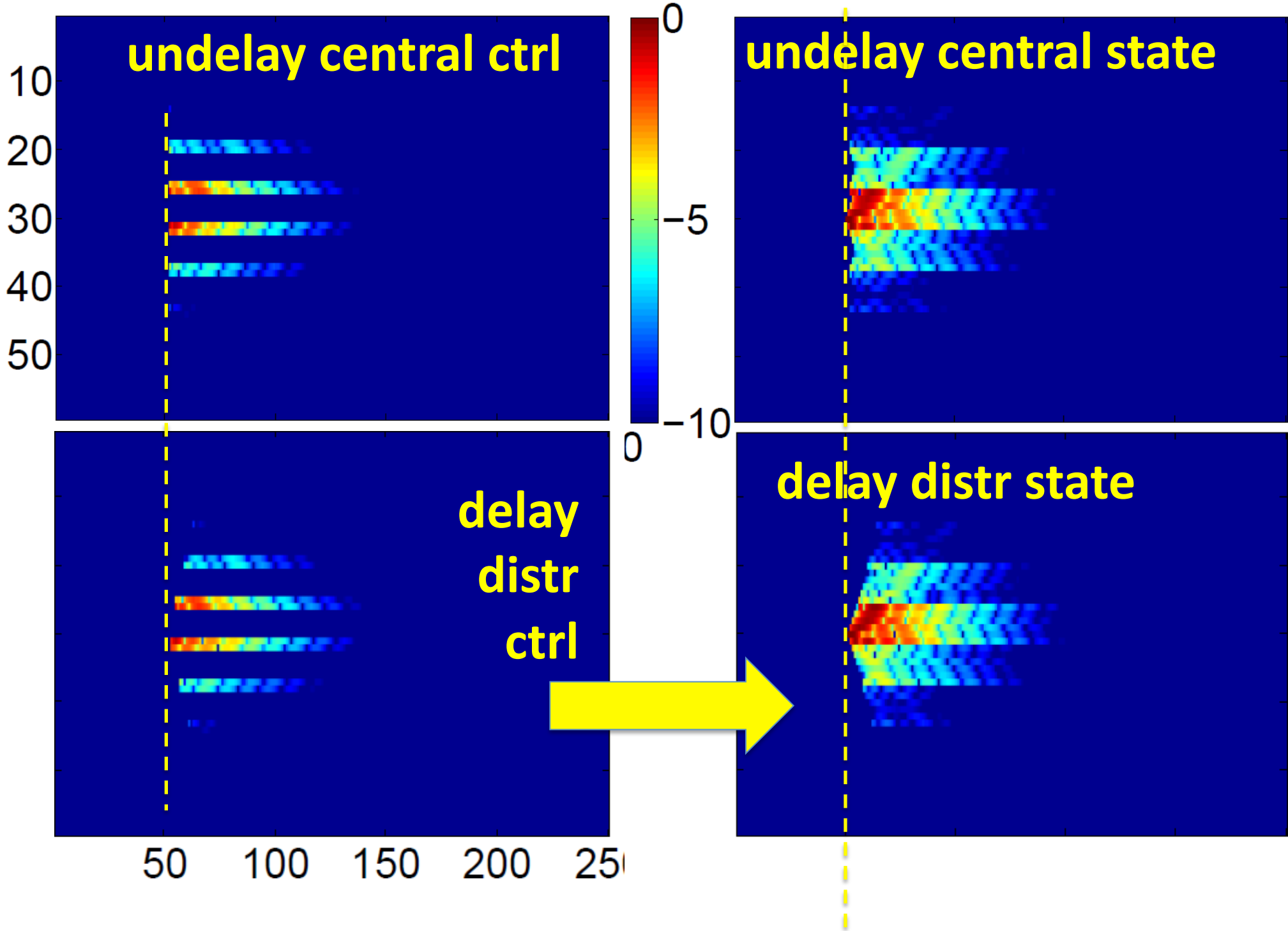

undelay central ctrl
undelay central state
10
20
30
40
50
50
100
150
200
250
0
−5
−10

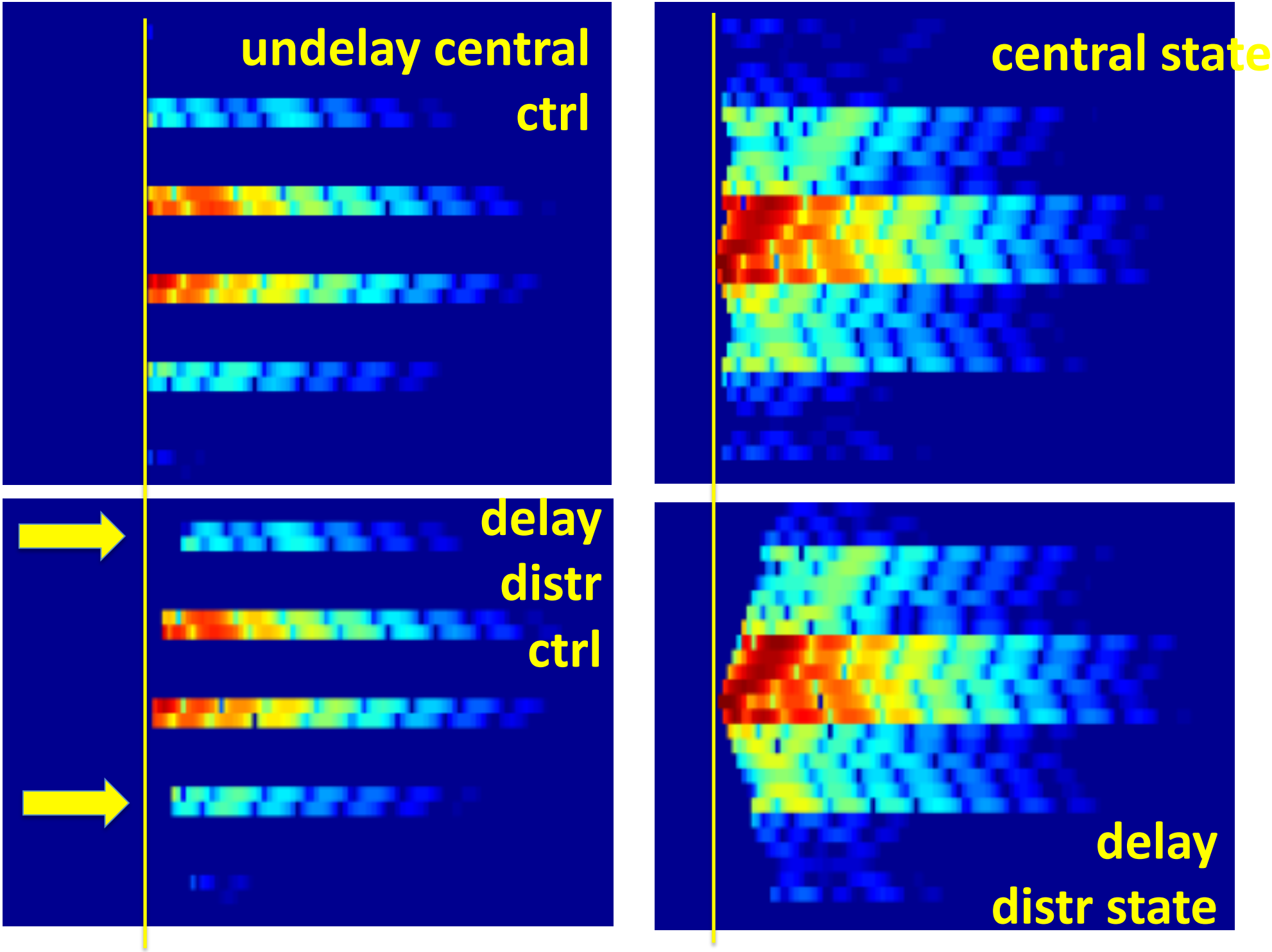

Communication speed = ∞


undelay central ctrl
undelay central state
10
20
30
40
50
50
100
150
200
250
0
−5
−10


Communication speed = ∞

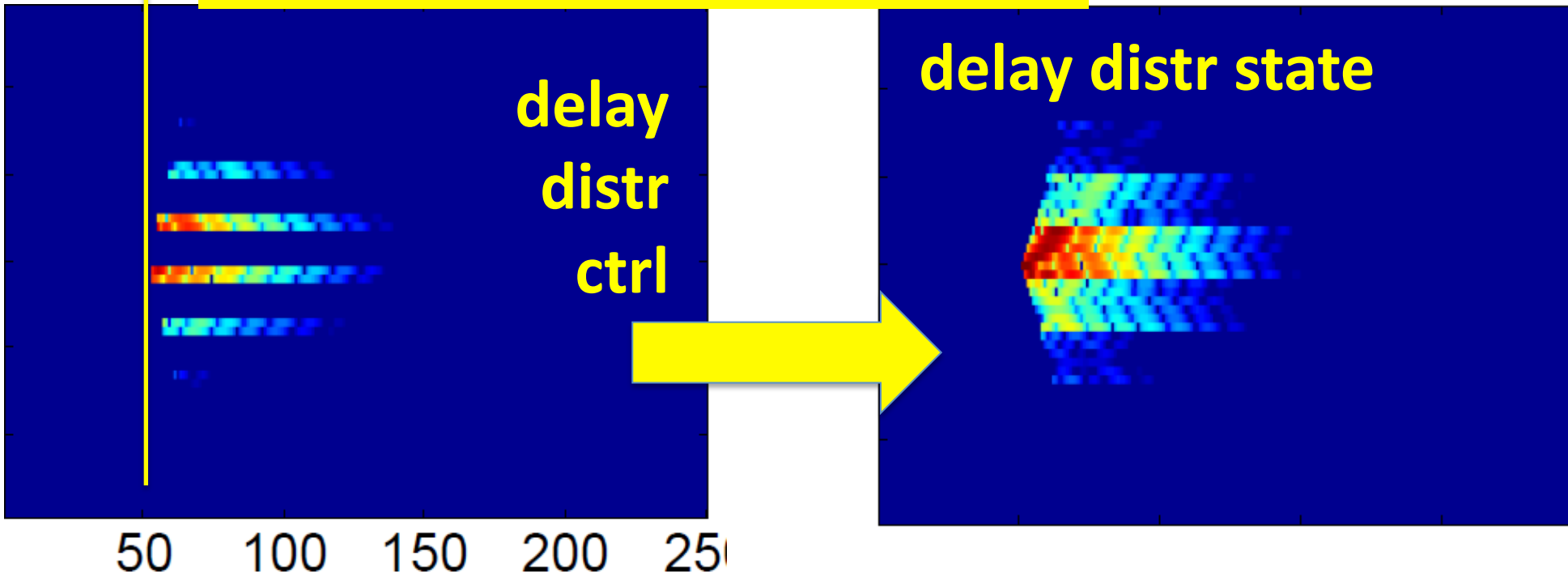

1.2
Norm
1.15
1.1
1.05
1
0
2
4
6
8
10
Distributed
Localized
Undelayed central
1.5
Communication
Speed
→∞
slow
fast

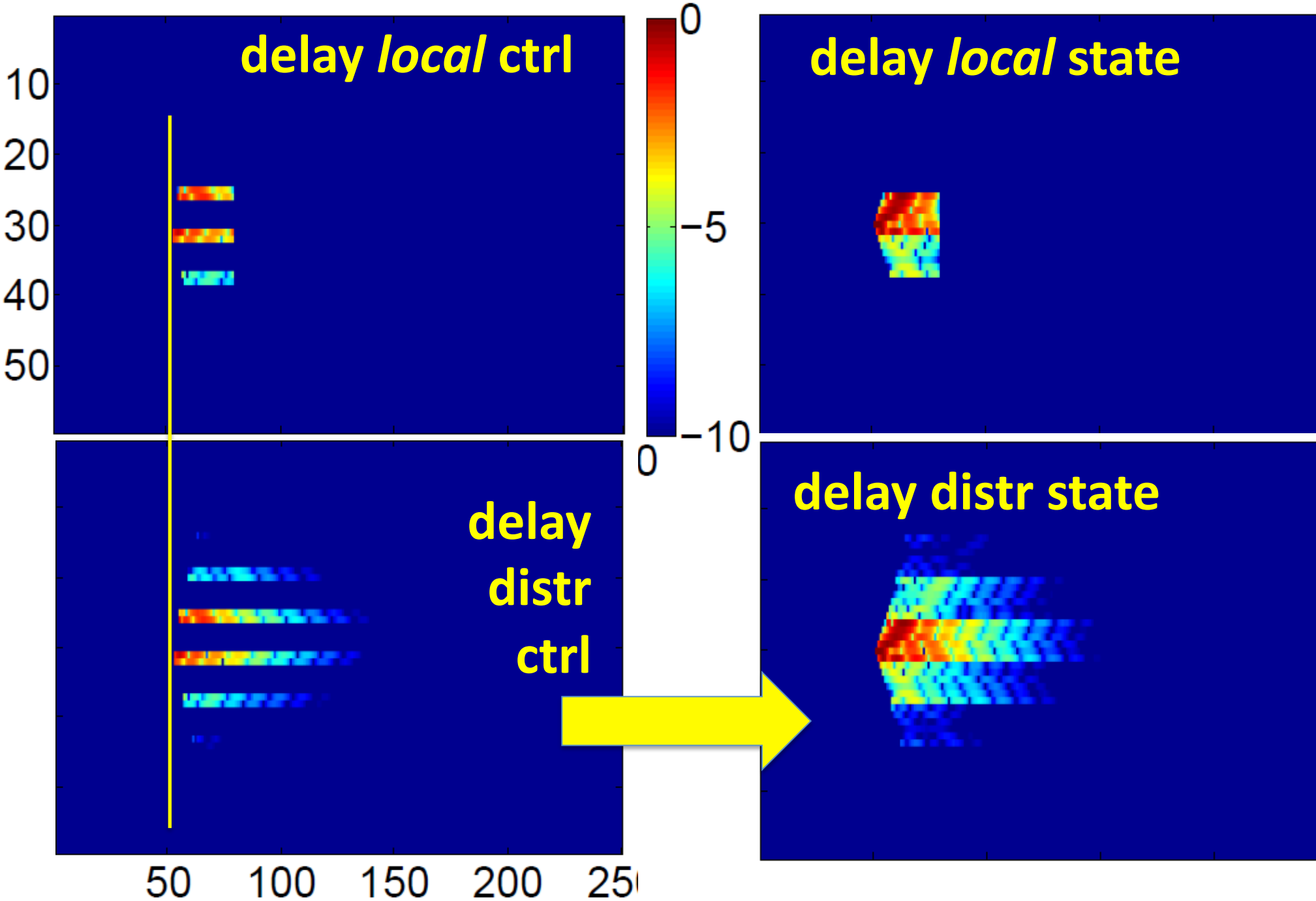

undelay central ctrl
undelay central state
delay distr ctrl
delay distr state
10
20
30
40
50
50
100
150
200
0
−5
−10

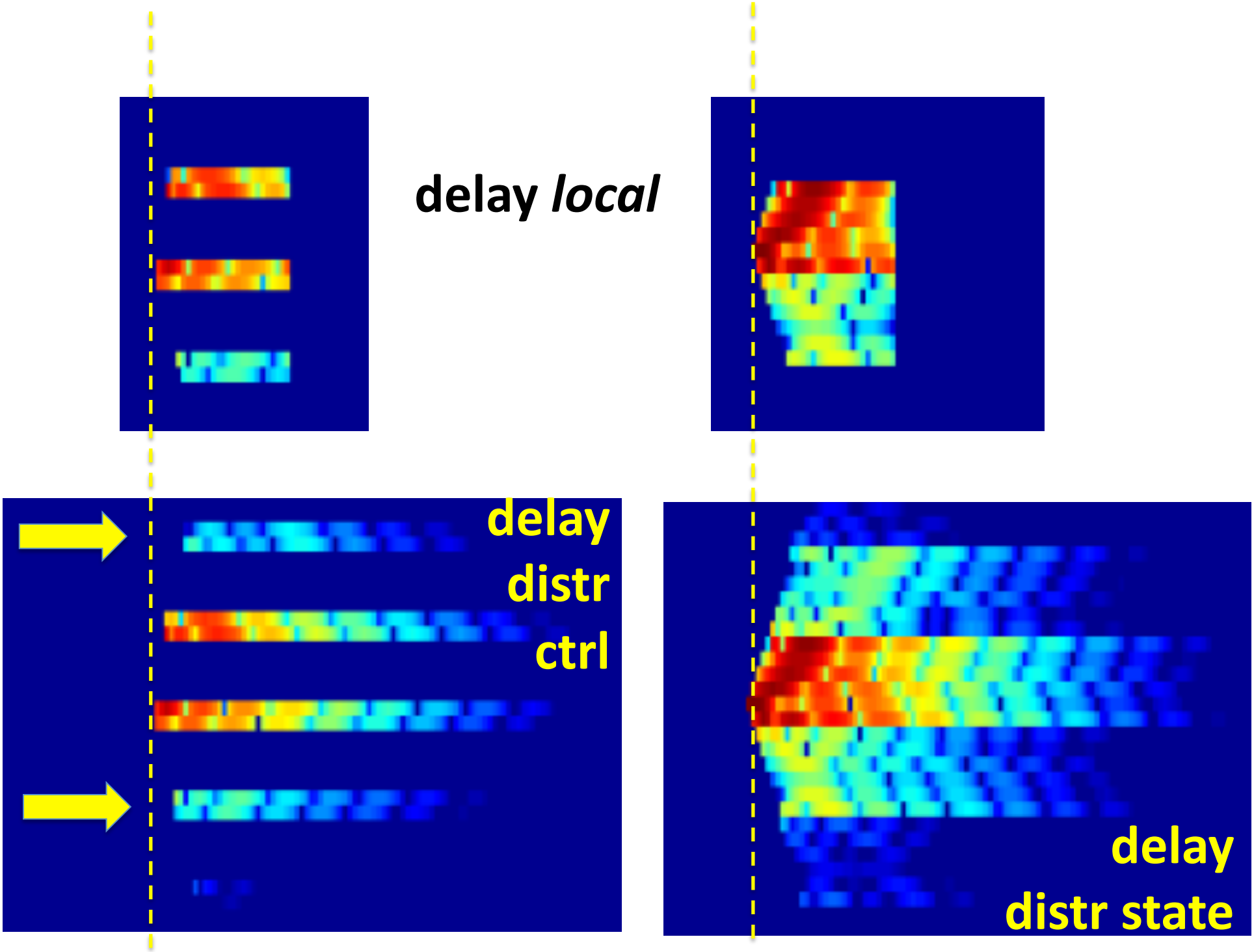

undelay central
ctrl
central state
delay
distr
ctrl
delay
distr state

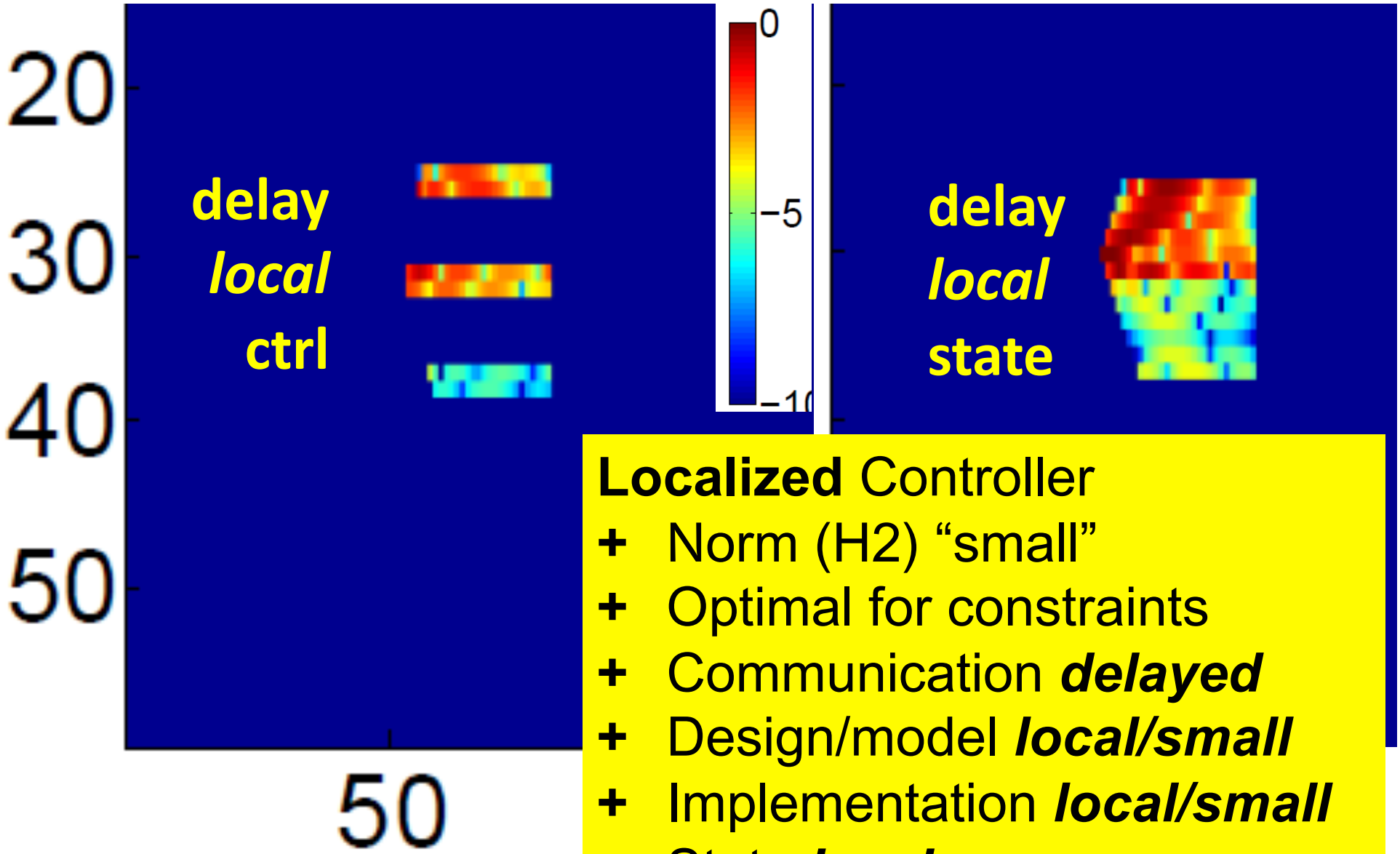
Distributed (QI) Controller

+ Norm (H2) “small”
+ Optimal for constraints
+ Communication delayed
− Design/model global/huge P
− Implementation local/huge P




delay *local* ctrl
delay *local* state
delay distr ctrl
delay distr state
10
20
30
40
50
50
100
150
200
0
−5
−10
0


delay *local*
delay
distr
ctrl
delay
distr state

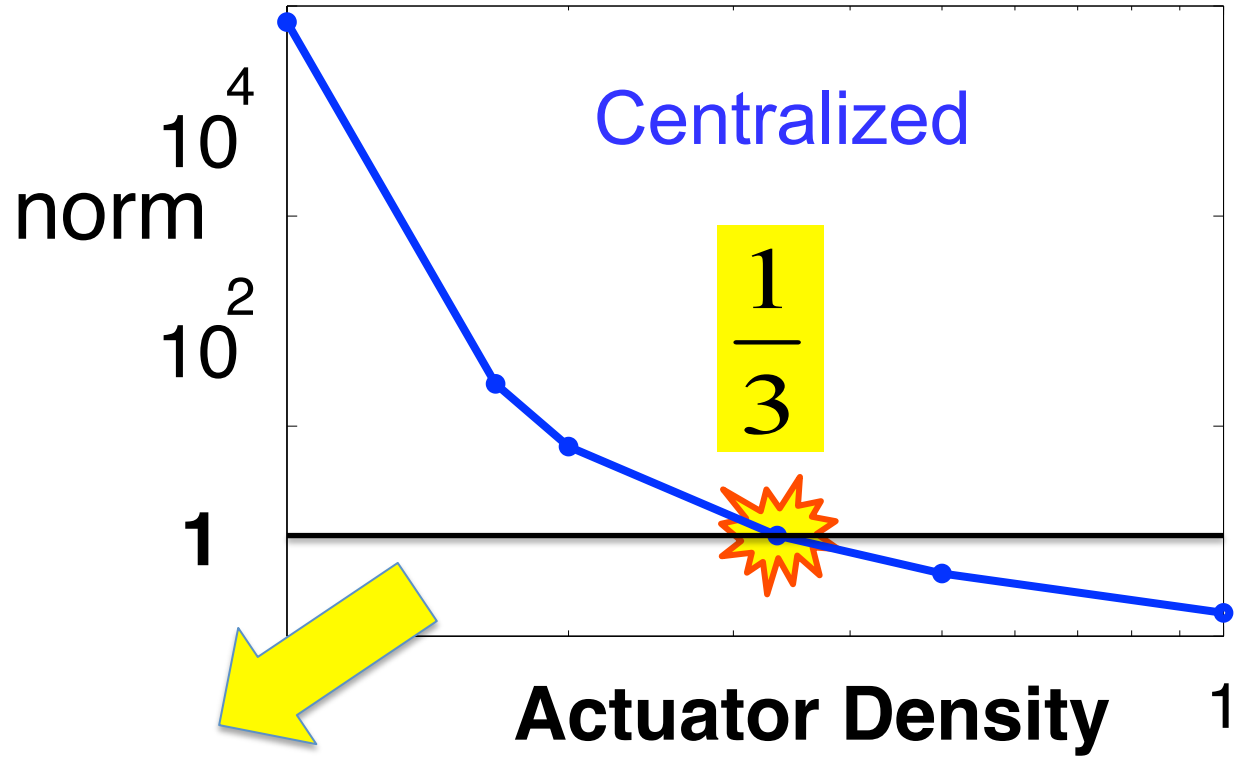


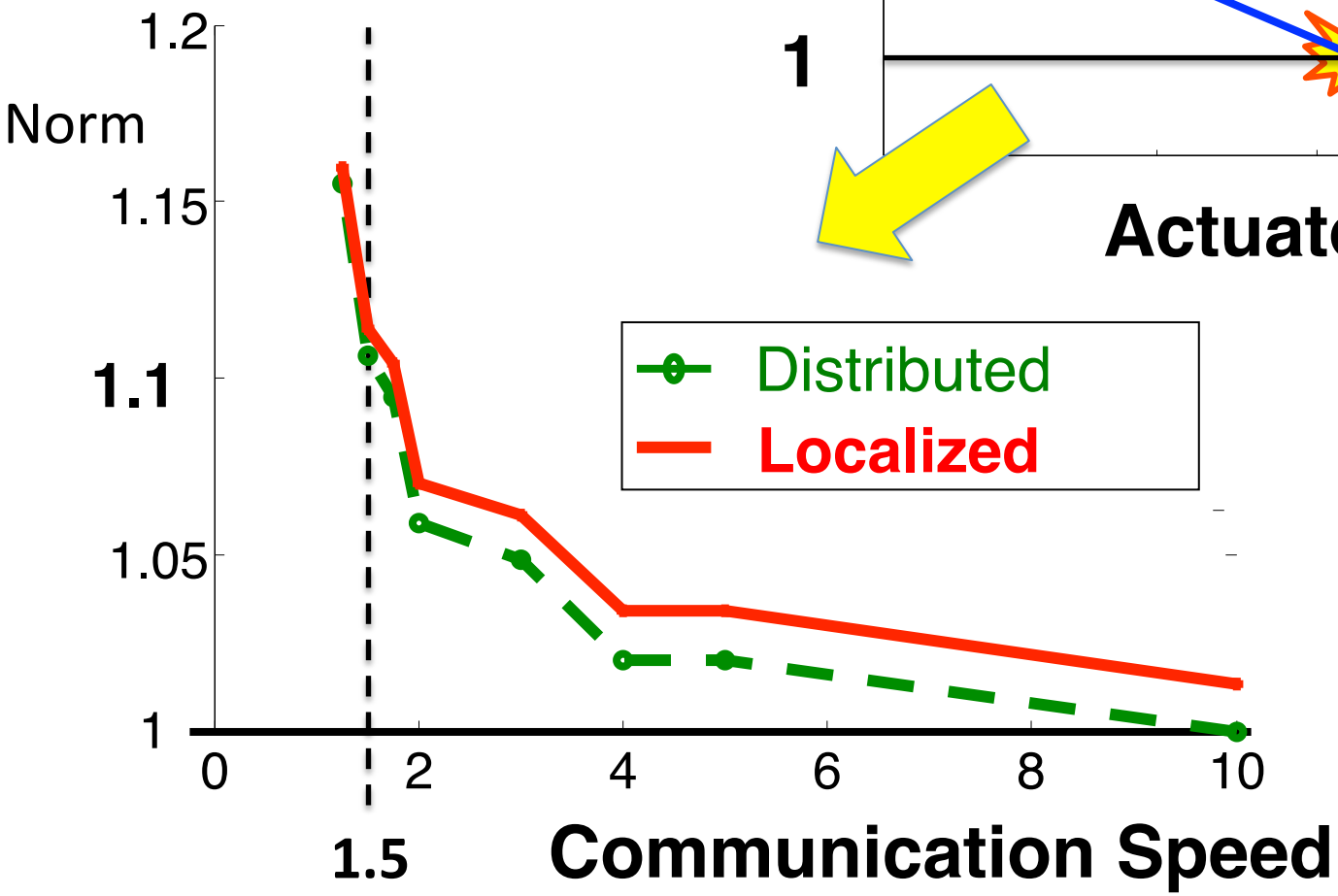
Localized Controller

- **+** Norm (H2) “small”
- **+** Optimal for constraints
- **+** Communication ***delayed***
- **+** Design/model ***local/small***
- **+** Implementation ***local/small***
- **+** State ***local***

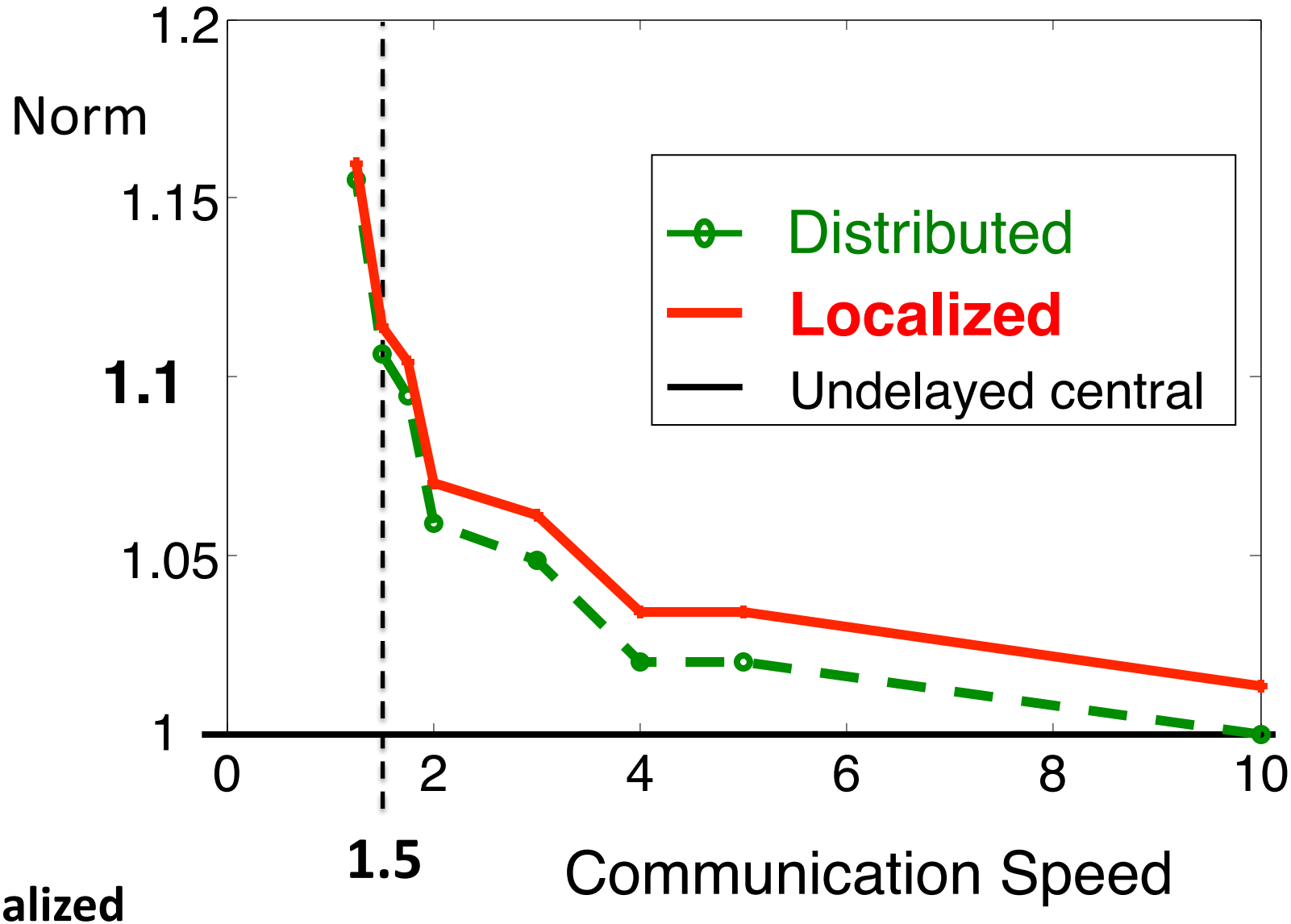
Everything is scalable.

Conjecture:
Norm bad before method breaks



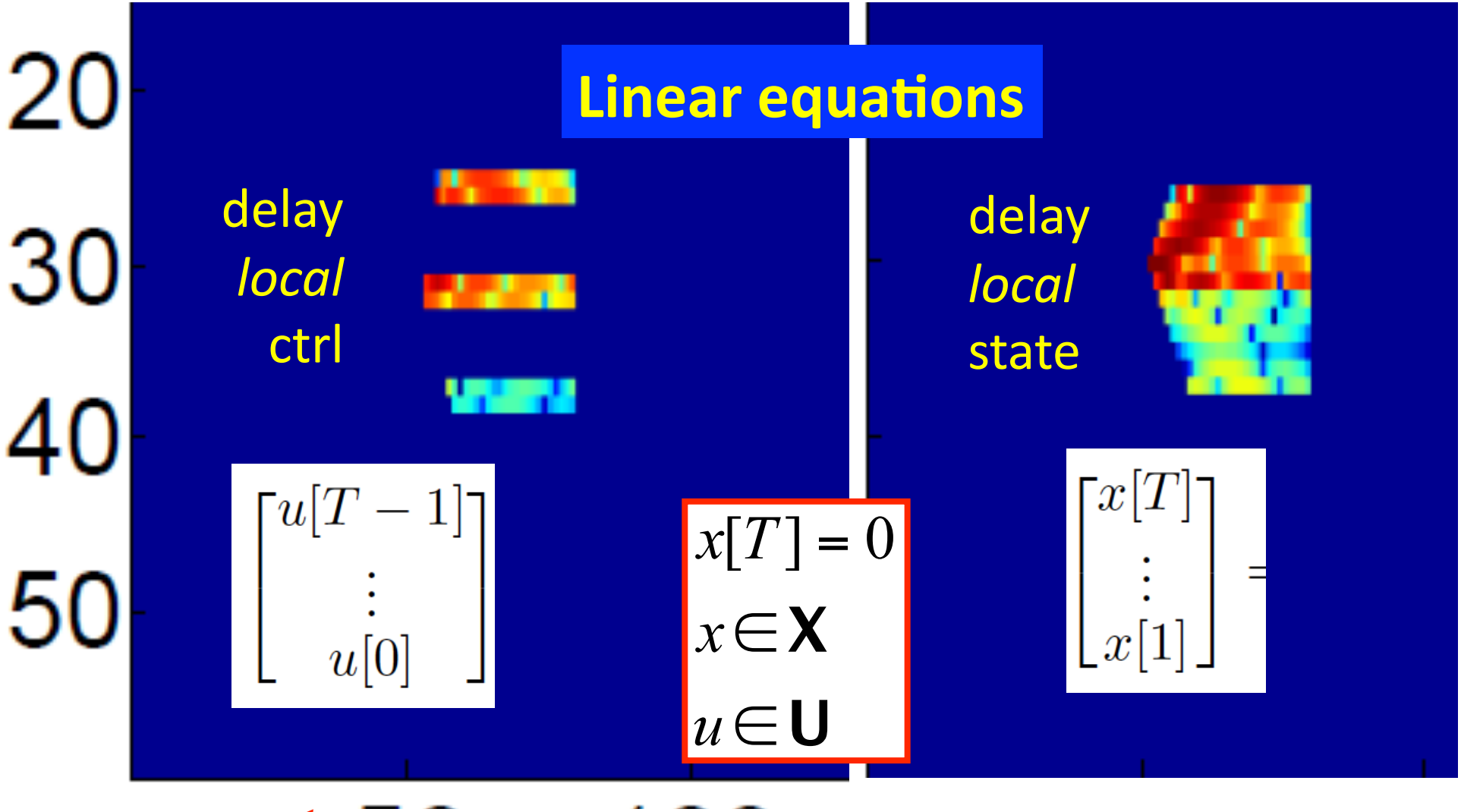


Tradeoffs

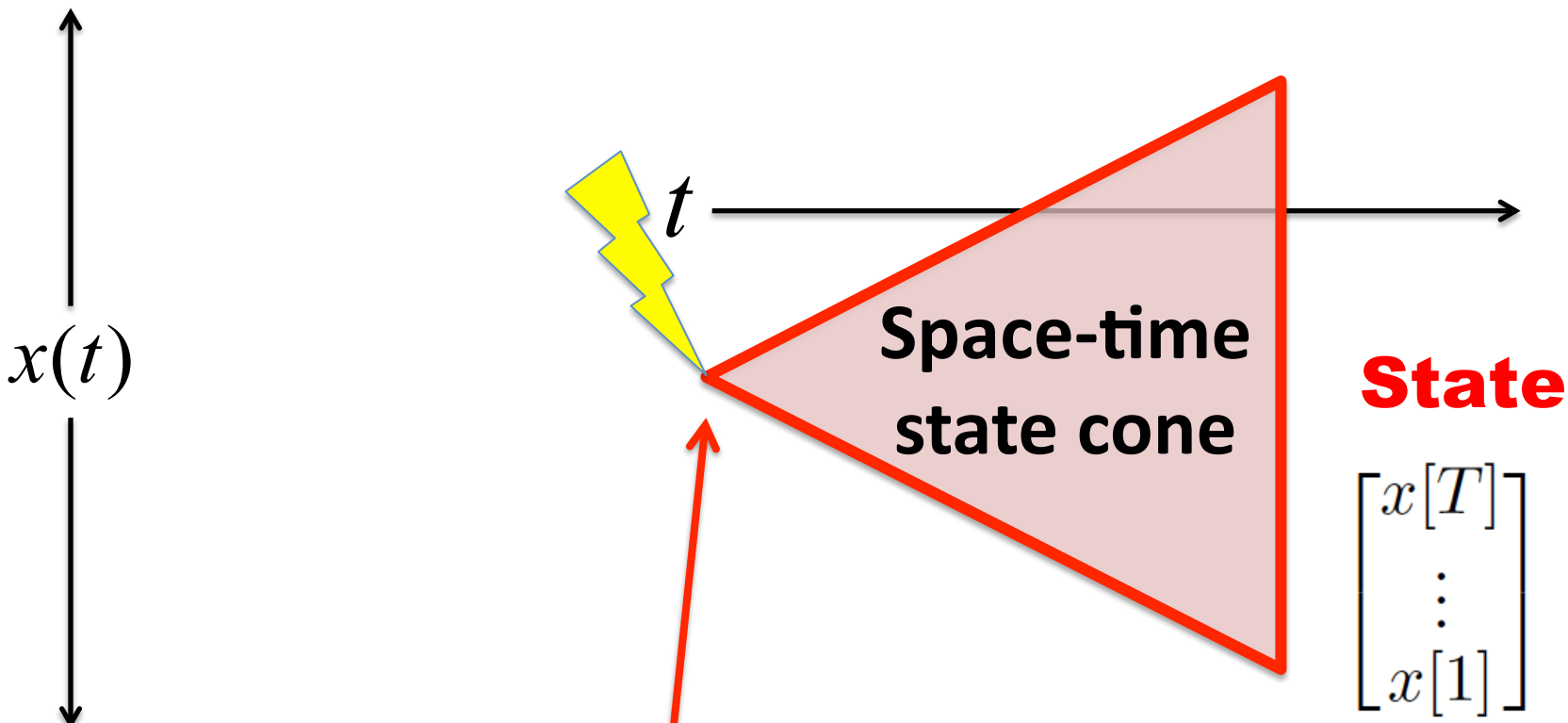


Normalized by undelayed centralized

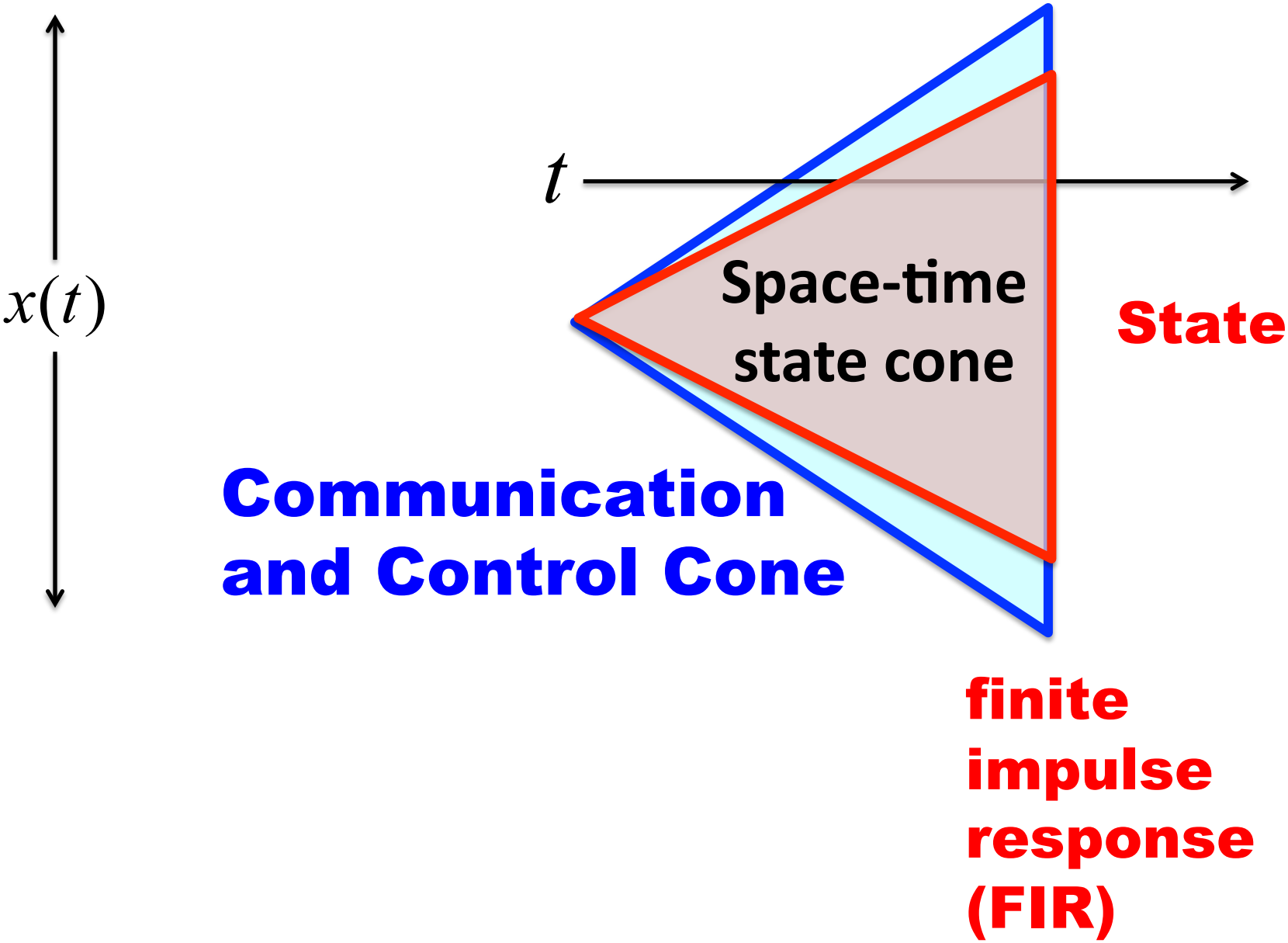
Linear equations

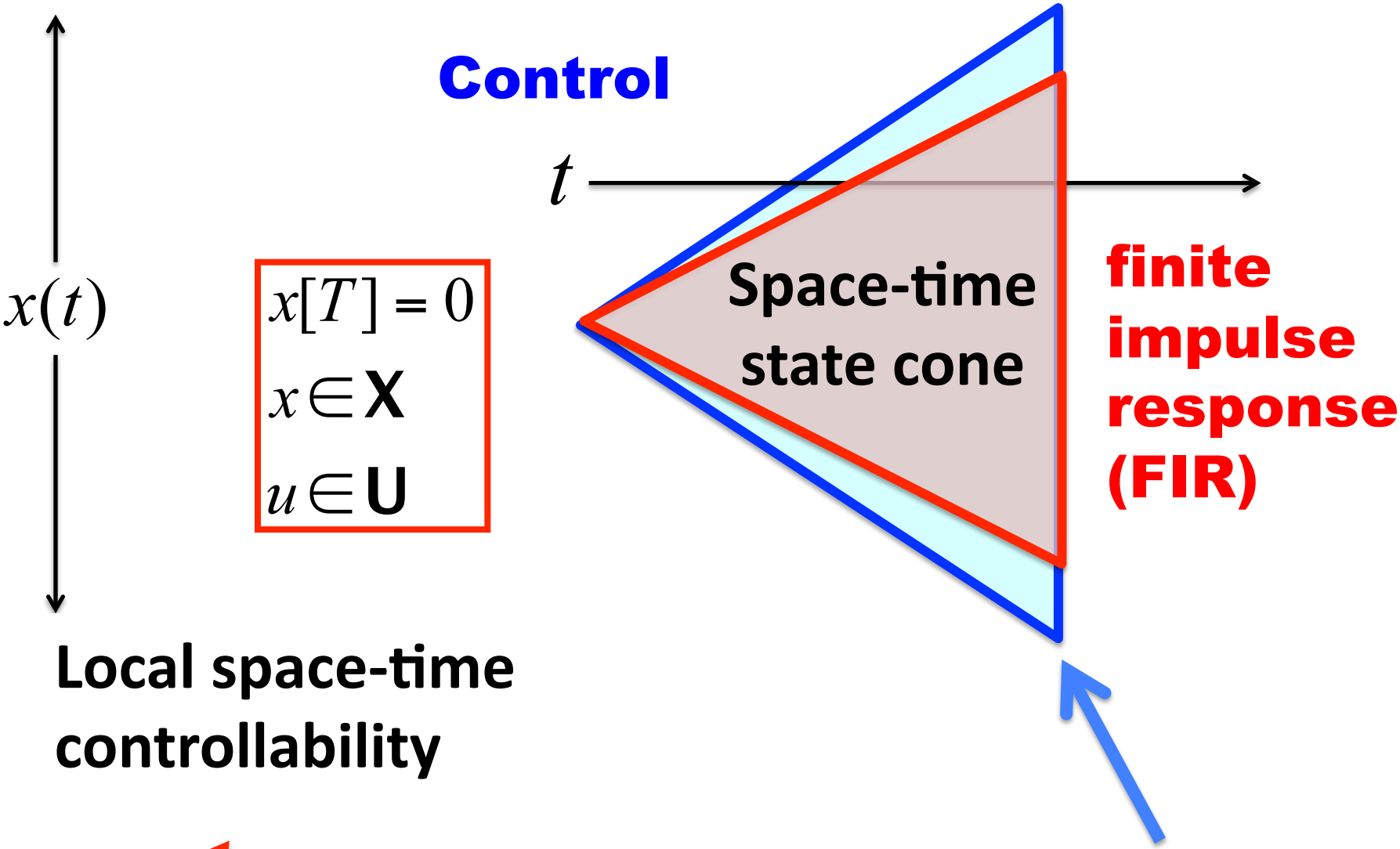


$$\begin{bmatrix} x[T] \\ \vdots \\ x[1] \end{bmatrix} = \begin{bmatrix} A^T \\ \vdots \\ A \end{bmatrix} x[0] + \begin{bmatrix} B & \cdots & A^{T-1}B \\ & \ddots & \vdots \\ 0 & & B \end{bmatrix} \begin{bmatrix} u[T-1] \\ \vdots \\ u[0] \end{bmatrix}$$



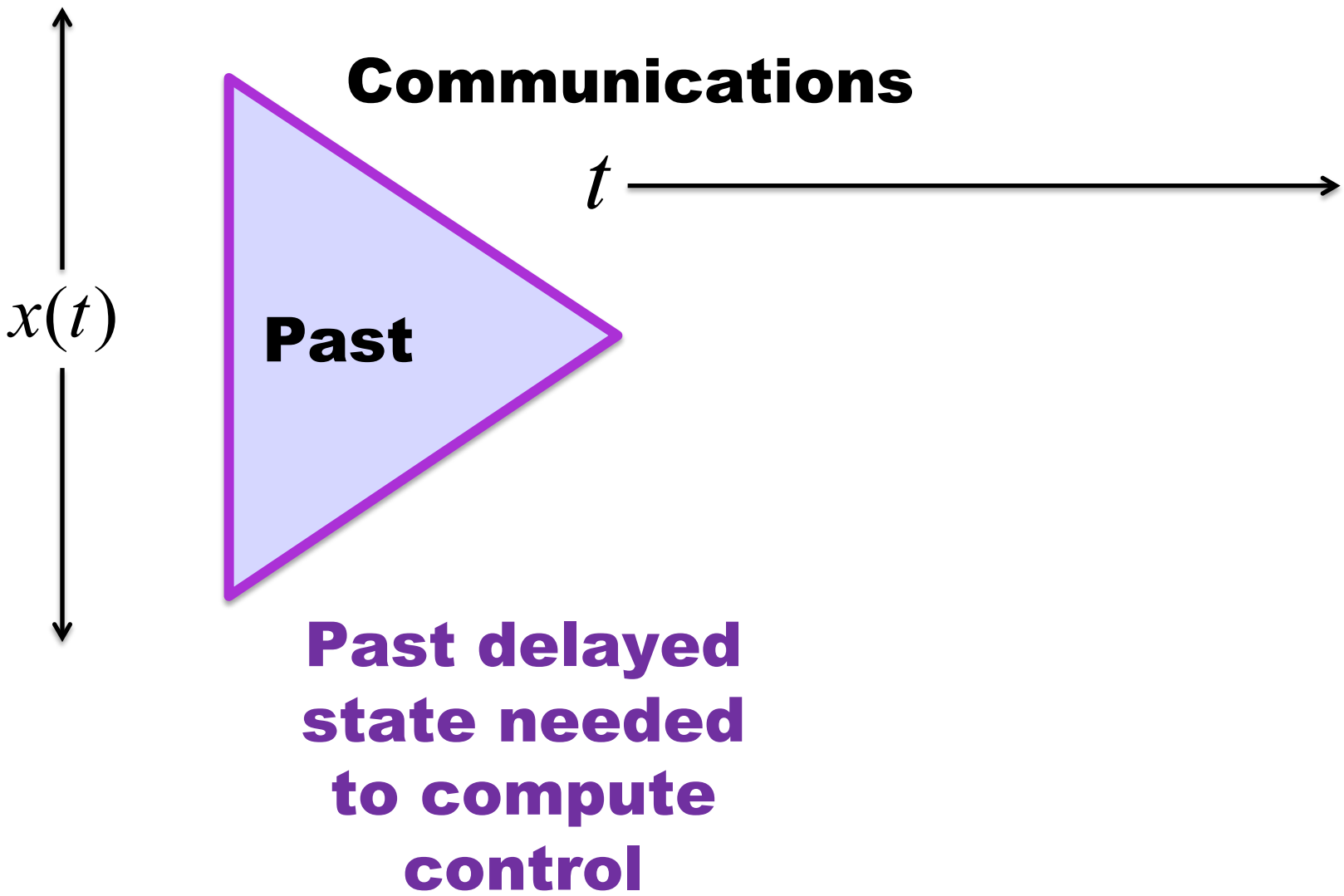
$$\begin{bmatrix} x[T] \\ \vdots \\ x[1] \end{bmatrix} = \begin{bmatrix} A^T \\ \vdots \\ A \end{bmatrix} x[0] + \begin{bmatrix} B & \cdots & A^{T-1}B \\ & \ddots & \vdots \\ 0 & & B \end{bmatrix} \begin{bmatrix} u[T-1] \\ \vdots \\ u[0] \end{bmatrix}$$

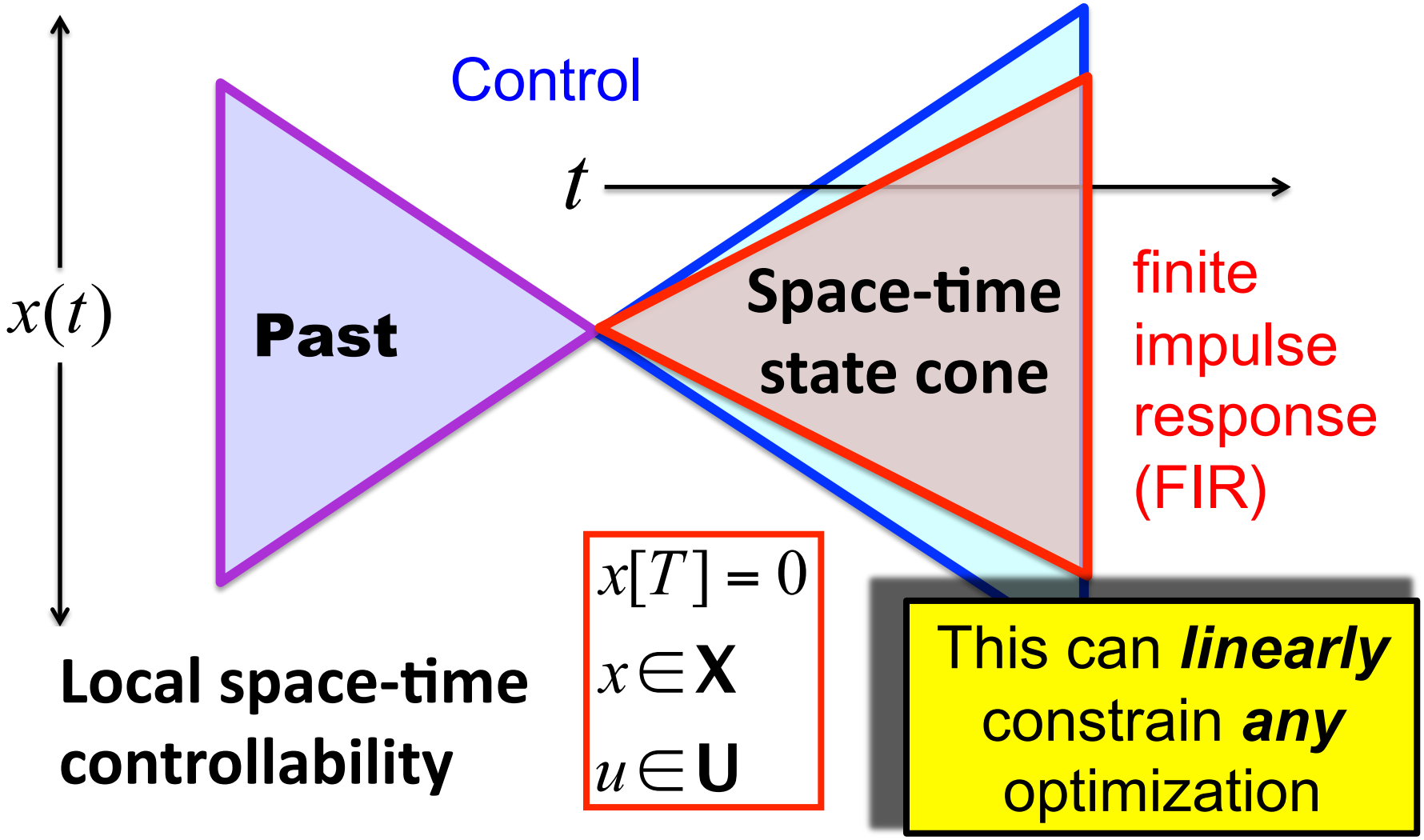

$x(t)$
t
Space-time
state cone
State
Communication
and Control Cone
finite
impulse
response
(FIR)



Local space-time controllability

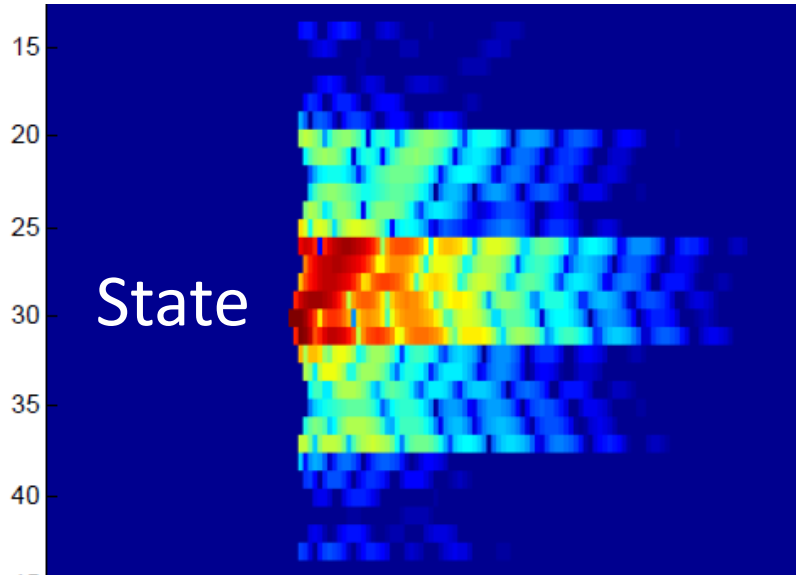
$$\begin{bmatrix} x[T] \\ \vdots \\ x[1] \end{bmatrix} = \begin{bmatrix} A^T \\ \vdots \\ A \end{bmatrix} x[0] + \begin{bmatrix} B & \cdots & A^{T-1}B \\ & \ddots & \vdots \\ 0 & & B \end{bmatrix} \begin{bmatrix} u[T-1] \\ \vdots \\ u[0] \end{bmatrix}$$


Communications
t
$x(t)$
Past
Past delayed state needed to compute control

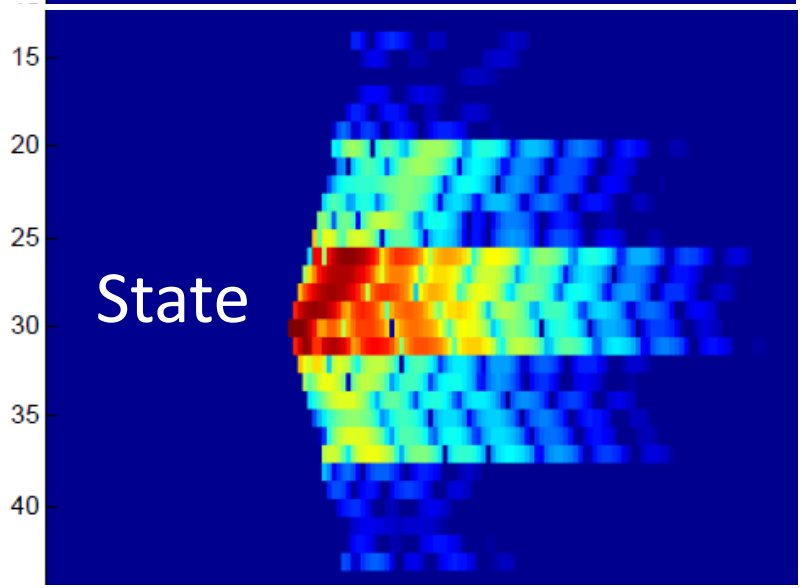


Local space-time controllability

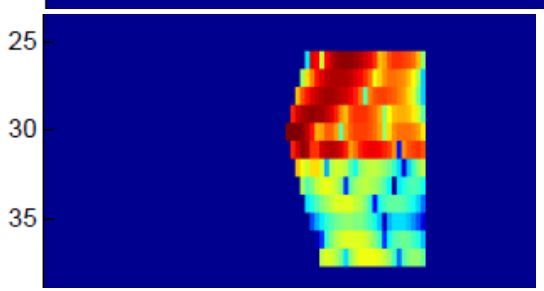
$$\begin{bmatrix} x[T] \\ \vdots \\ x[1] \end{bmatrix} = \begin{bmatrix} A^T \\ \vdots \\ A \end{bmatrix} x[0] + \begin{bmatrix} B & \cdots & A^{T-1}B \\ & \ddots & \vdots \\ 0 & & B \end{bmatrix} \begin{bmatrix} u[T-1] \\ \vdots \\ u[0] \end{bmatrix}$$



Optimal undelayed centralized state (old)

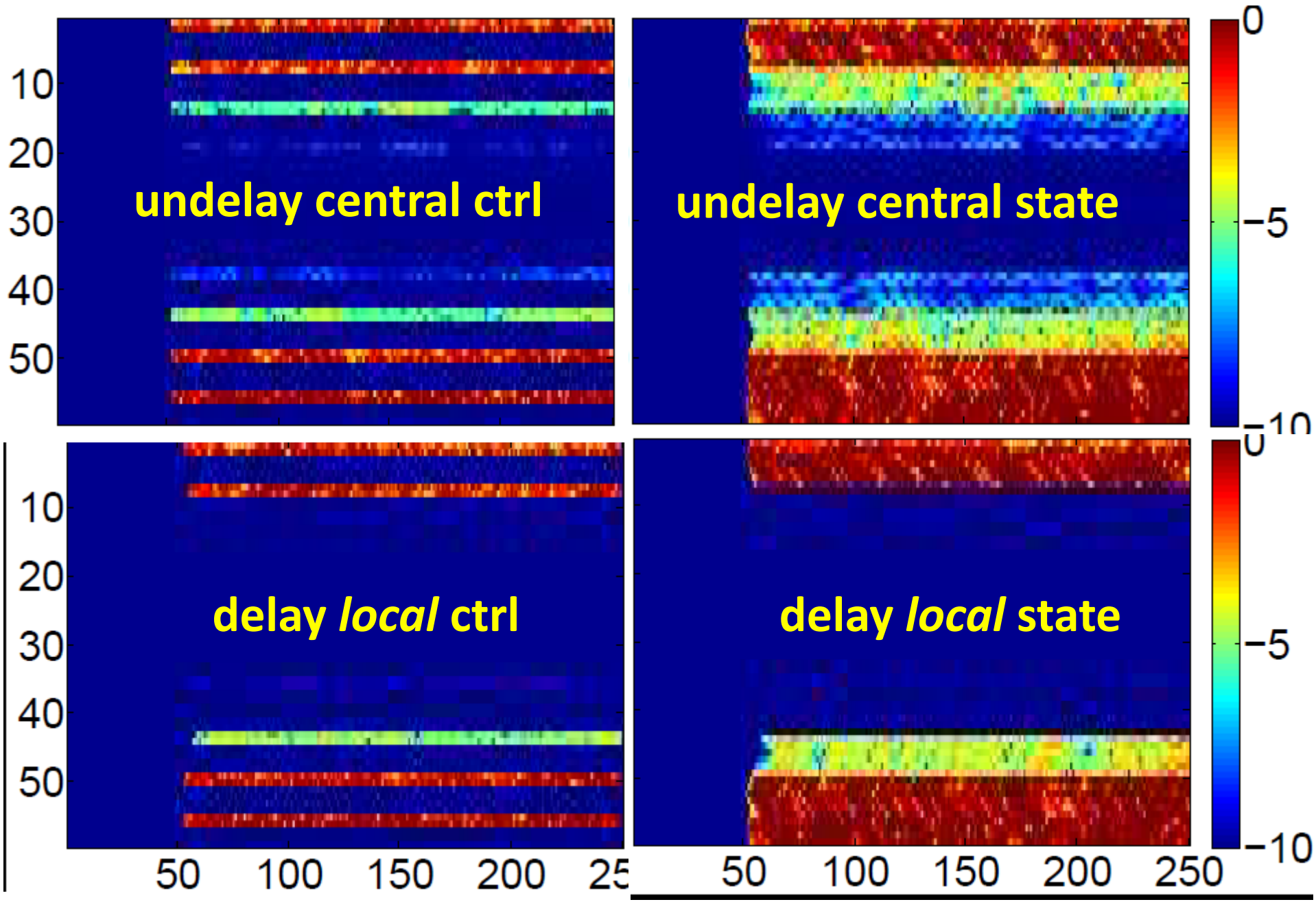


Optimal delayed distributed (newish) (but not scalable)



Optimal delayed *localized* (very new, scalable)

AWGN in C2, L26, C29



Control

$x(t)$

Localized Controller

+ Norm (H2) ***small***
+ Optimal for constraints
+ Communication is ***delayed***
+ Design/model ***local/small***
+ Implementation ***local/small***
+ State ***local***

finite impulse response (FIR)

Local space-time controllability

$x \in \mathbf{X}$

$u \in \mathbf{U}$

$$\begin{bmatrix} x[T] \\ \vdots \\ x[1] \end{bmatrix} = \begin{bmatrix} A^T \\ \vdots \\ A \end{bmatrix} x[0] + \begin{bmatrix} B & \cdots & \\ & \ddots & \\ 0 & & \end{bmatrix}$$

This can ***linearly*** constrain any optimization

Localized Controller

+ Norm (H2) small
+ Optimal for constraints
+ Design/model is local
+ Implementation is local
+ State stays local

- Bandwidth is ∞

? **Output feedback?**
? **Approximately local?**
? **Layering?**
? **Nonlinear, MPC, etc?**
? **Comms codesign?**

See also Javad's new relaxations

Mostly good news, but incomplete

Extensions

- Scalable optimal control
  - Localizable control: Y.-S. Wang, N. Matni, S. You and J. C. Doyle ACC ’14
  - Localized LQR control: Y.-S. Wang, N. Matni, and J. C. Doyle CDC’14
  - Output feedback progress
- Dealing with varying-delays (jitter)
  - Two player LQR with varying delays: N. Matni and J. C. Doyle CDC’ 13, N. Matni, A. Lamperski and J C. Doyle IFAC ‘14

More Nikolai Matni

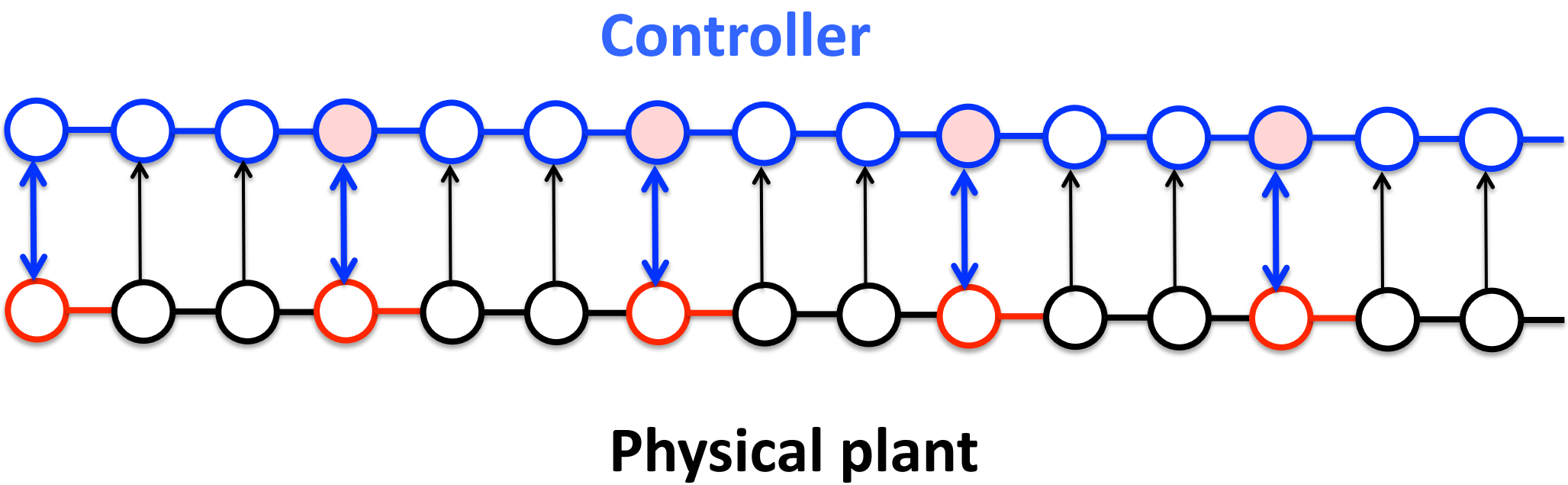
- Next
- Stay tuned

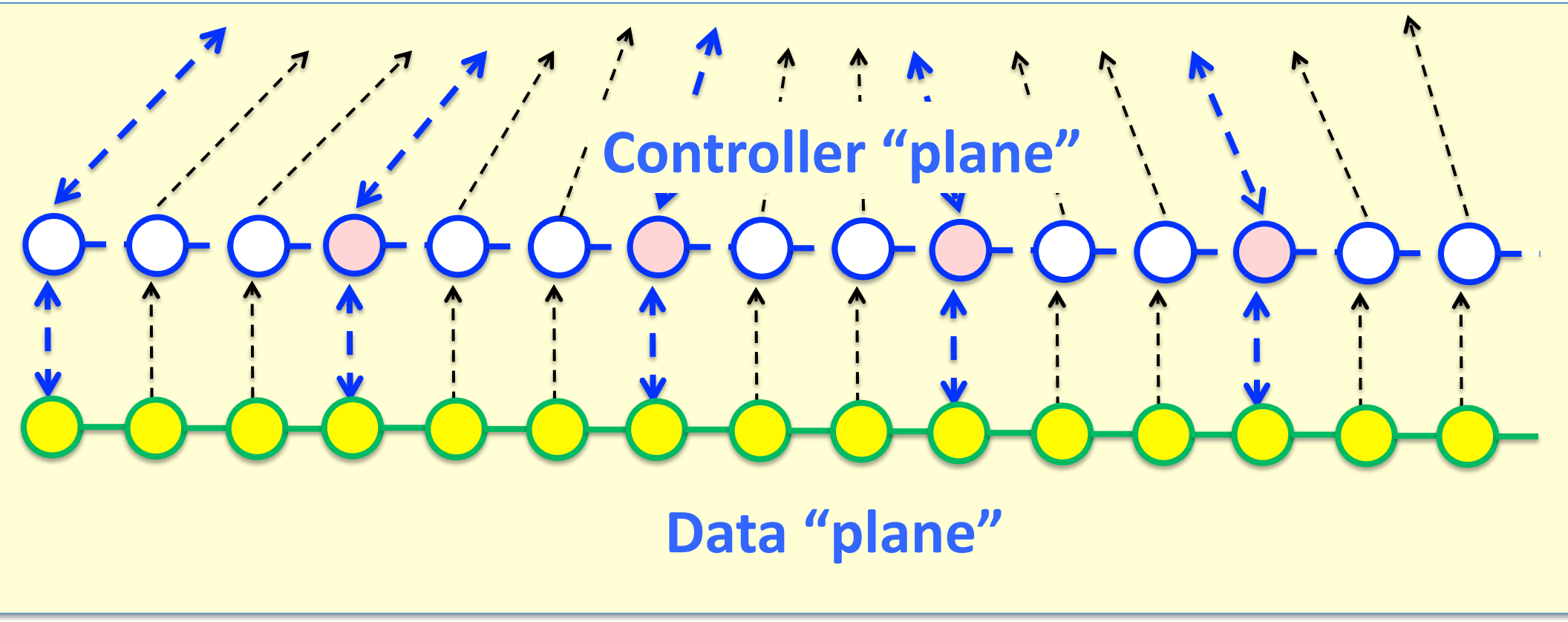
More Extensions/Apps

- Apps: neuro, smartgrid, CPS, cells
- IMC/RHC, etc (all of centralized control theory)
- Cyber theory: Delay jitter (uncertainty)
- Cyber: Comms co-design (CDC student prize paper)
- Physical: Robustness (unmodeled dynamics, noise)
- Cyber-phys: System ID, ML, adaptive
- SDN (Software defined nets, OpenDaylight)

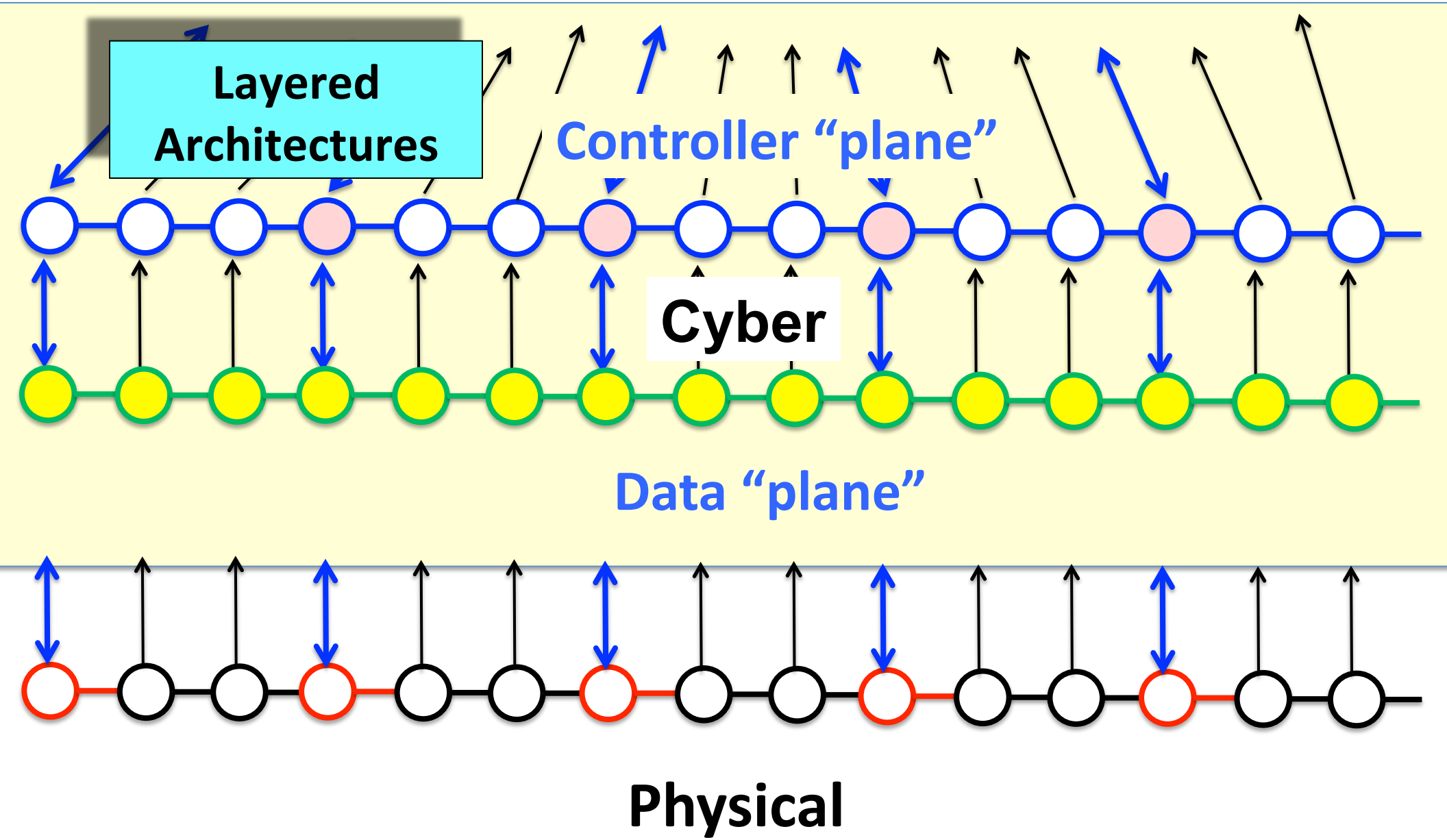
- Revisit “layering as optimization”?
- Poset causality (streamlining)?
- Quantization and network coding?

Revisit layering as optimization decomposition
Chiang, Low, Calderbank, Doyle, 2007

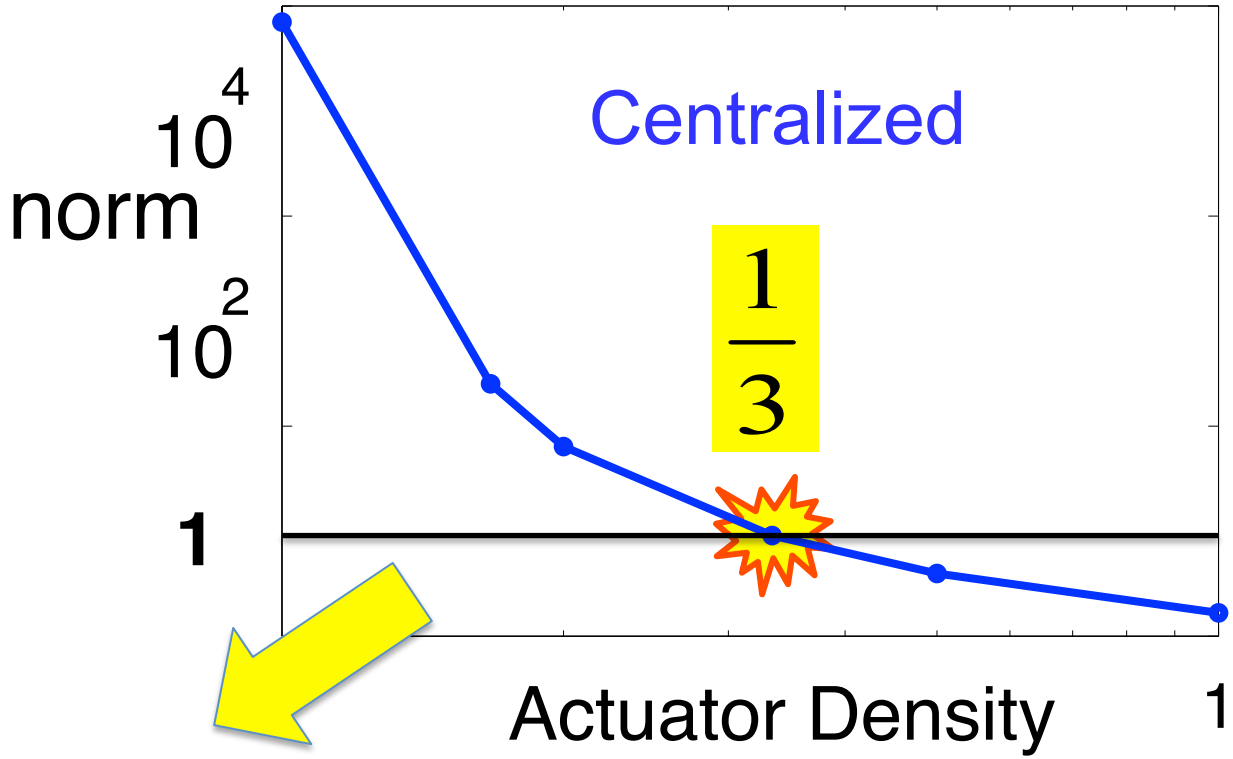


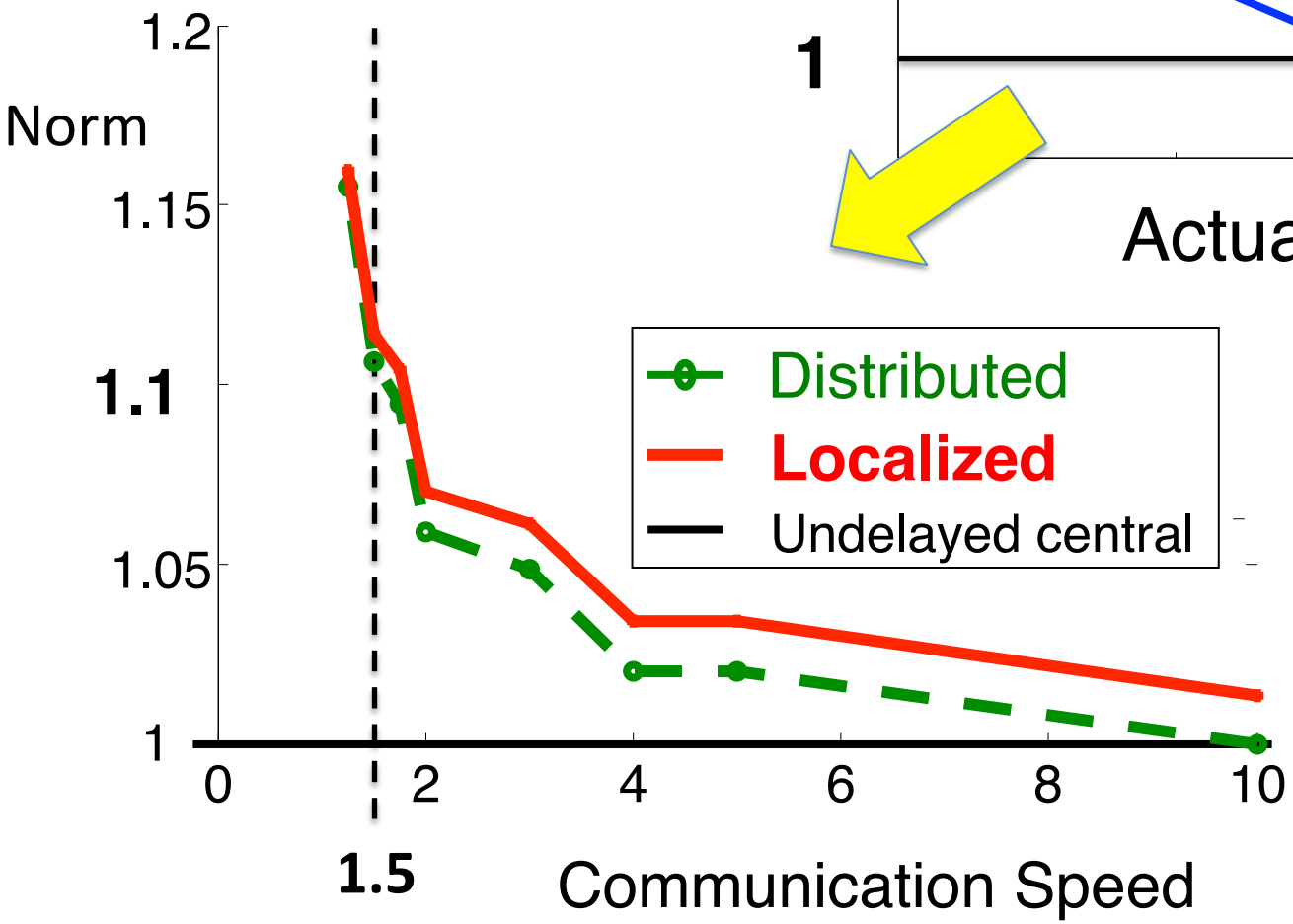


SDN/ODP

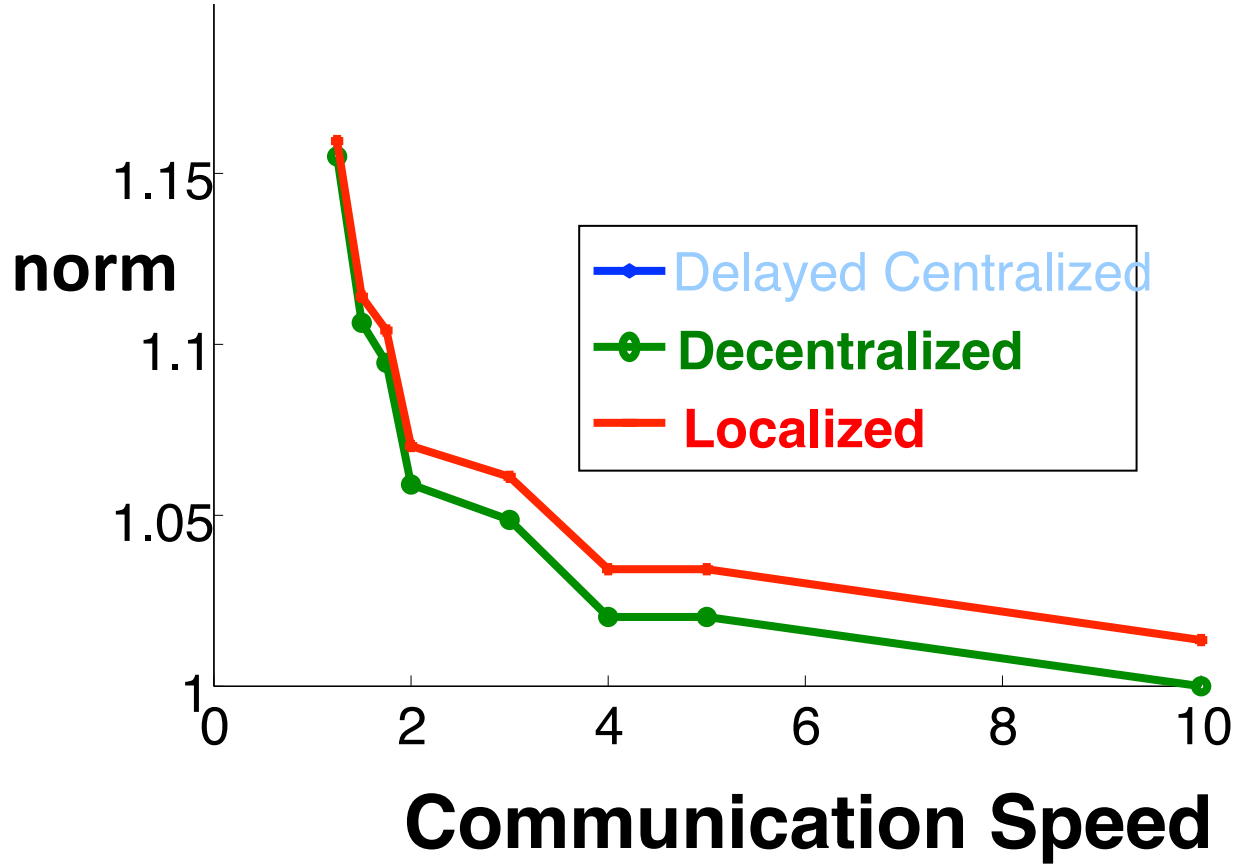

Layered Architectures
Controller "plane"
Cyber
Data "plane"
Physical

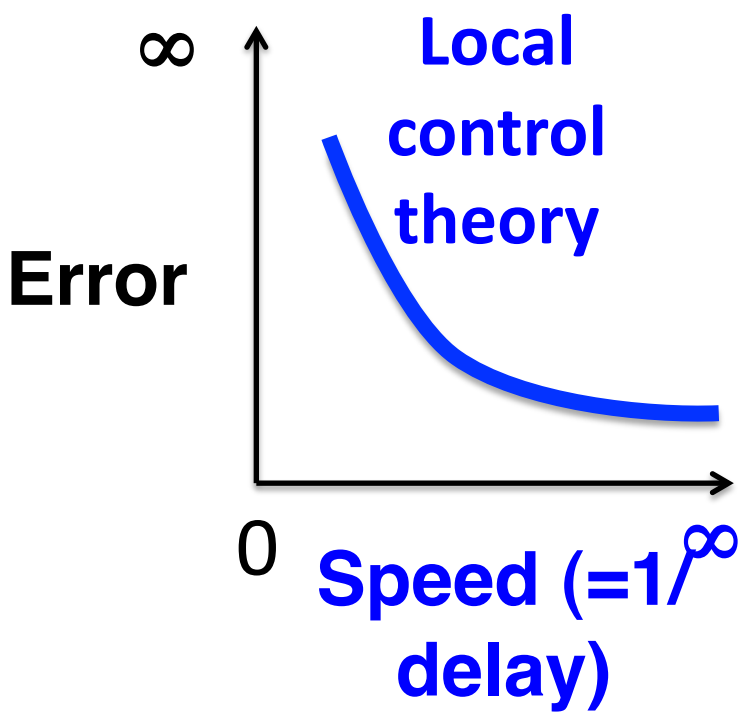
Conjecture:
Norm bad before method breaks

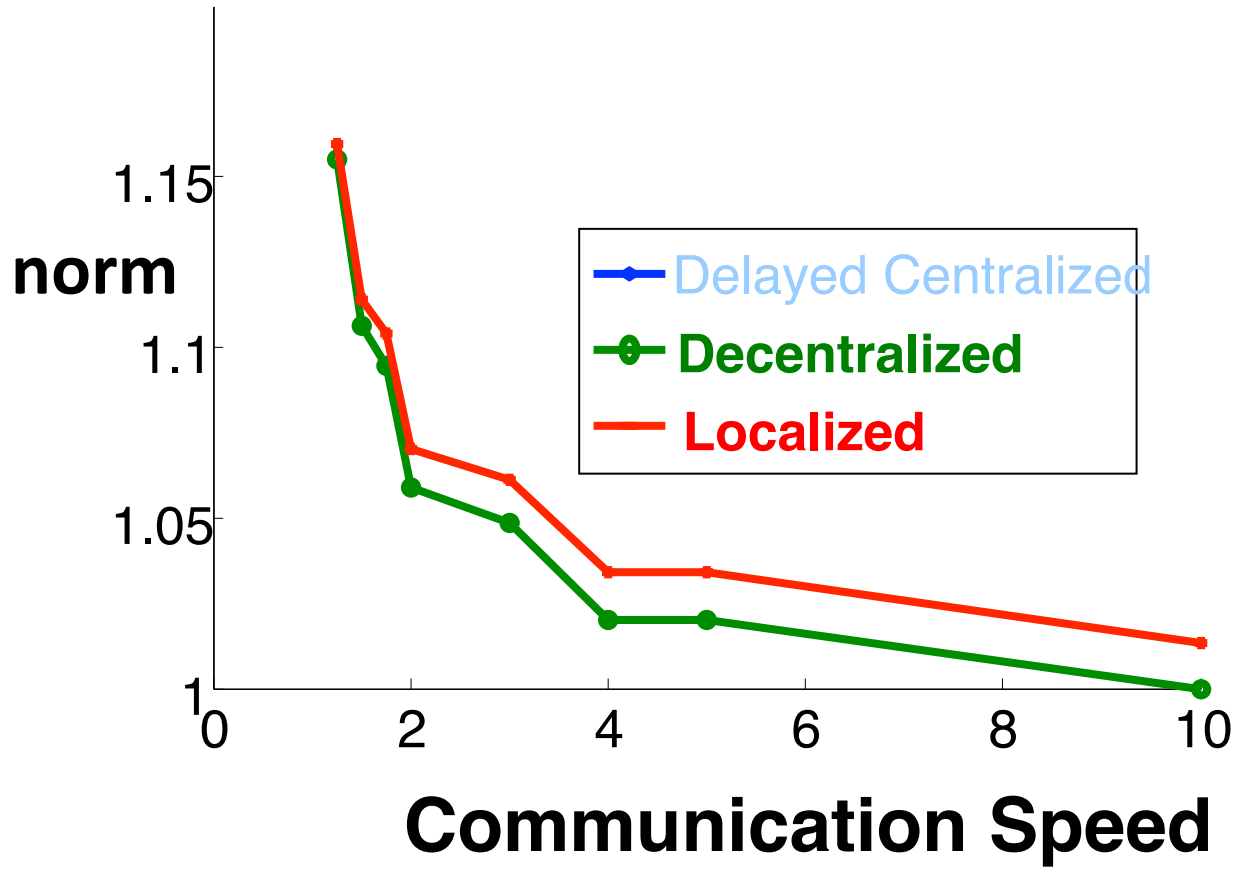


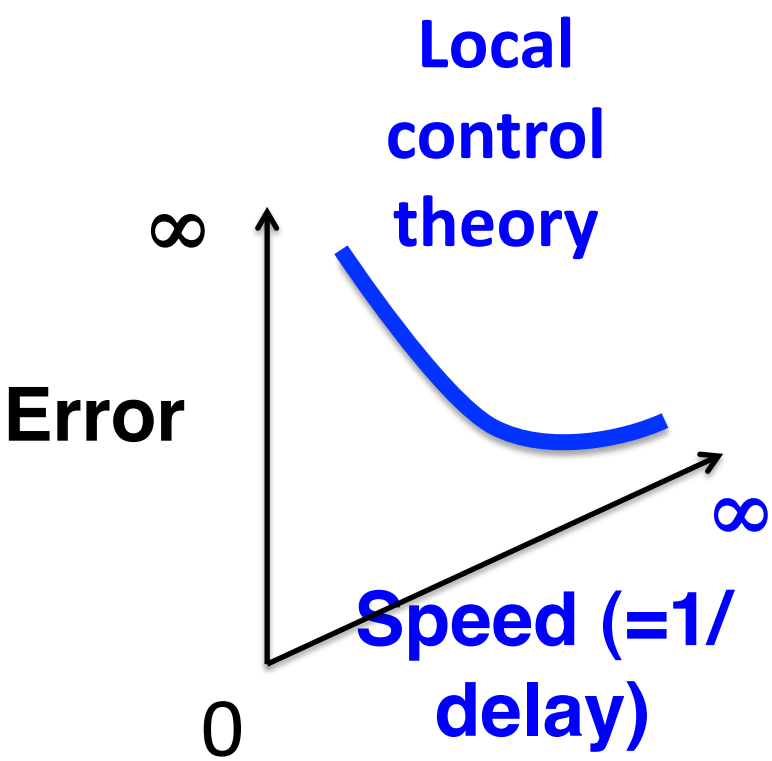


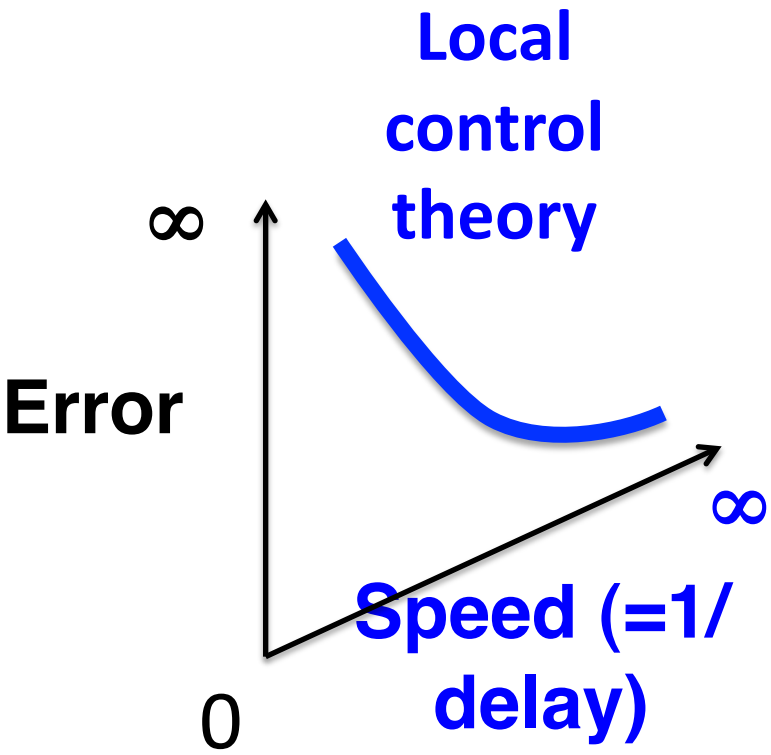
Tradeoffs

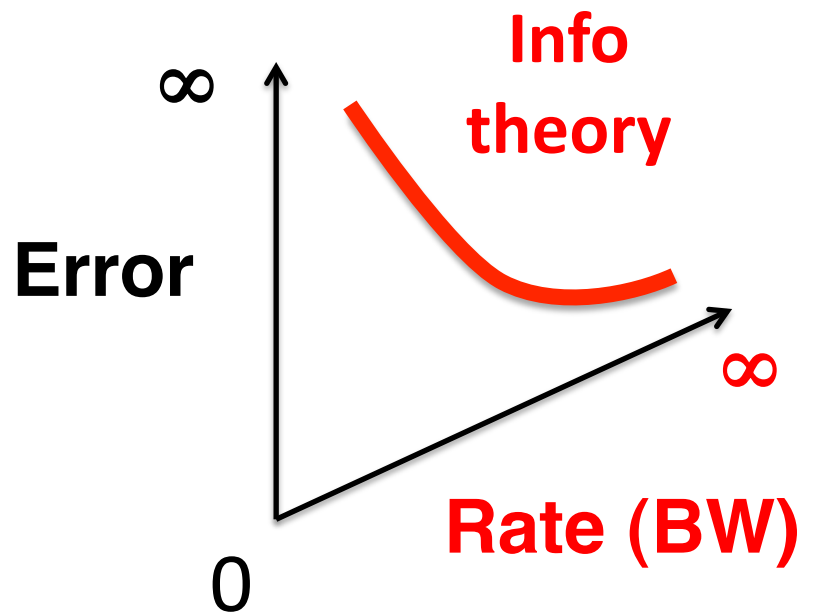

norm
1.15
1.1
1.05
1
0
2
4
6
8
10
Communication Speed
Delayed Centralized
Decentralized
Localized


∞
Error
Local control theory
0
Speed (=1/∞ delay)


norm
1.15
1.1
1.05
1
0
2
4
6
8
10
Communication Speed
Delayed Centralized
Decentralized
Localized

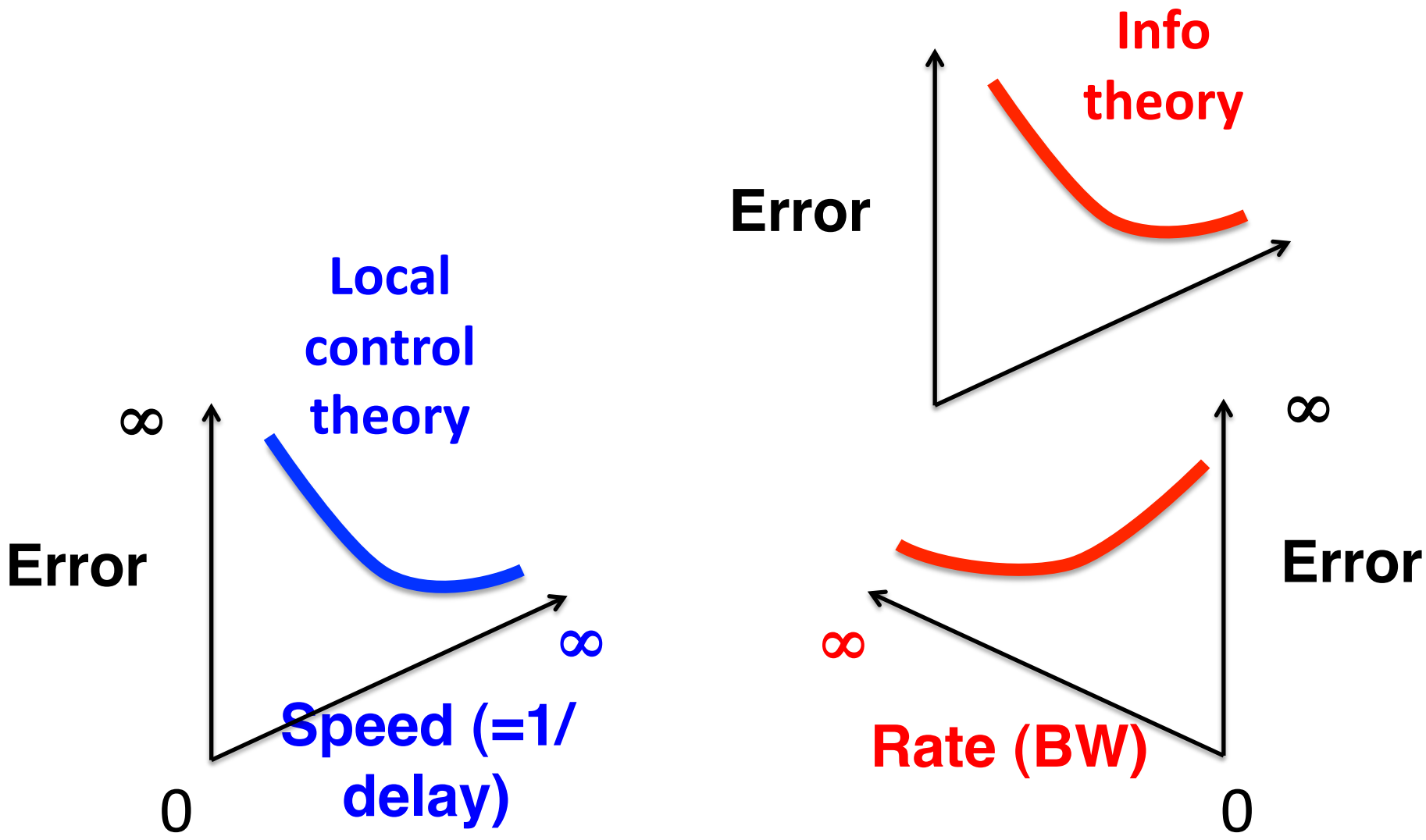

Local control theory
∞
Error
∞
0
Speed (=1/ delay)



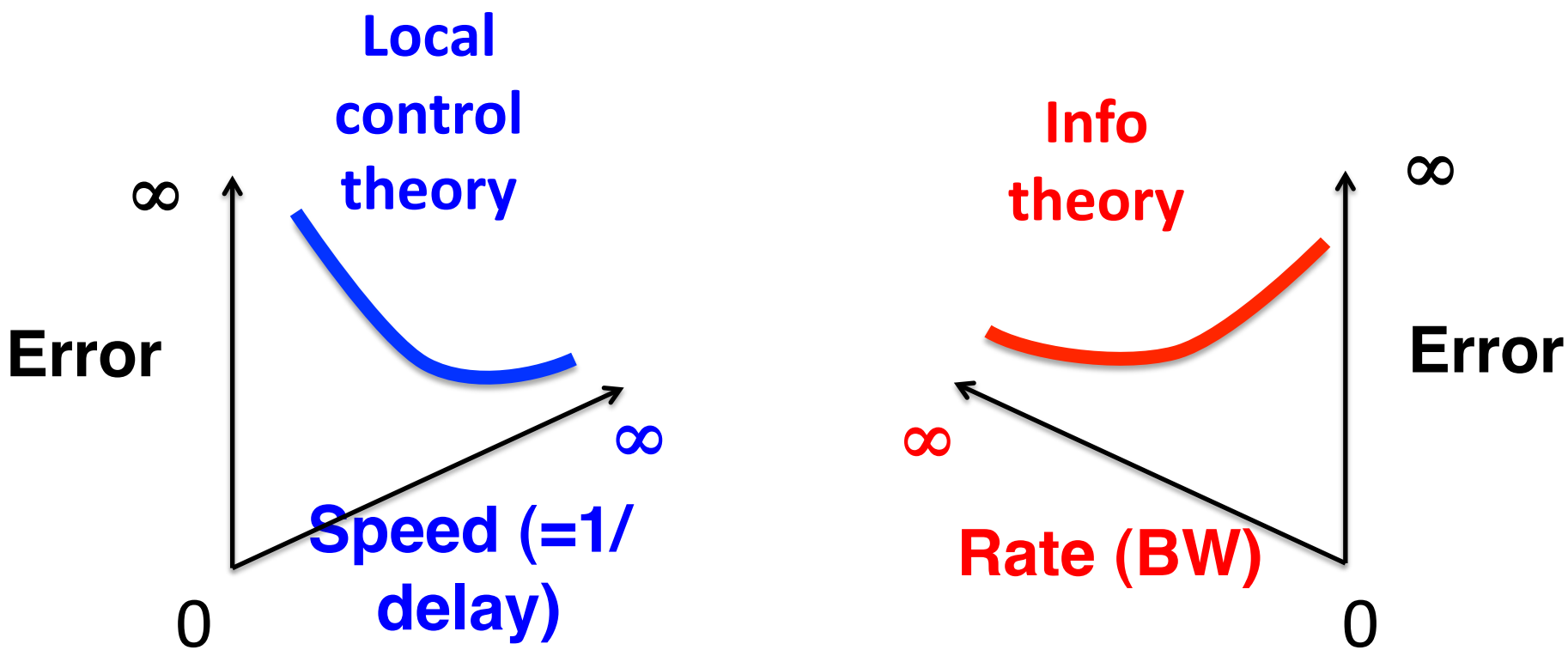


Due to
quantization,
loss, noise

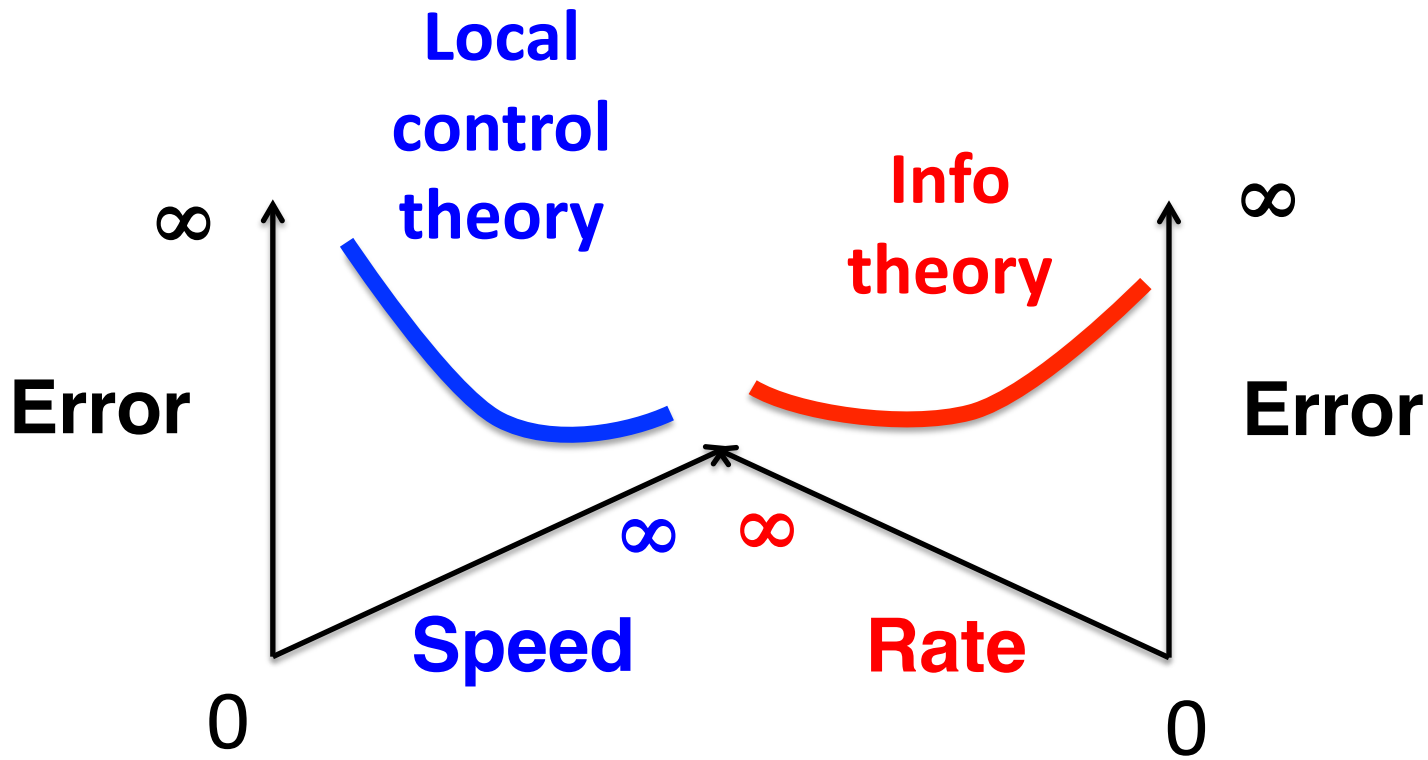
Communications



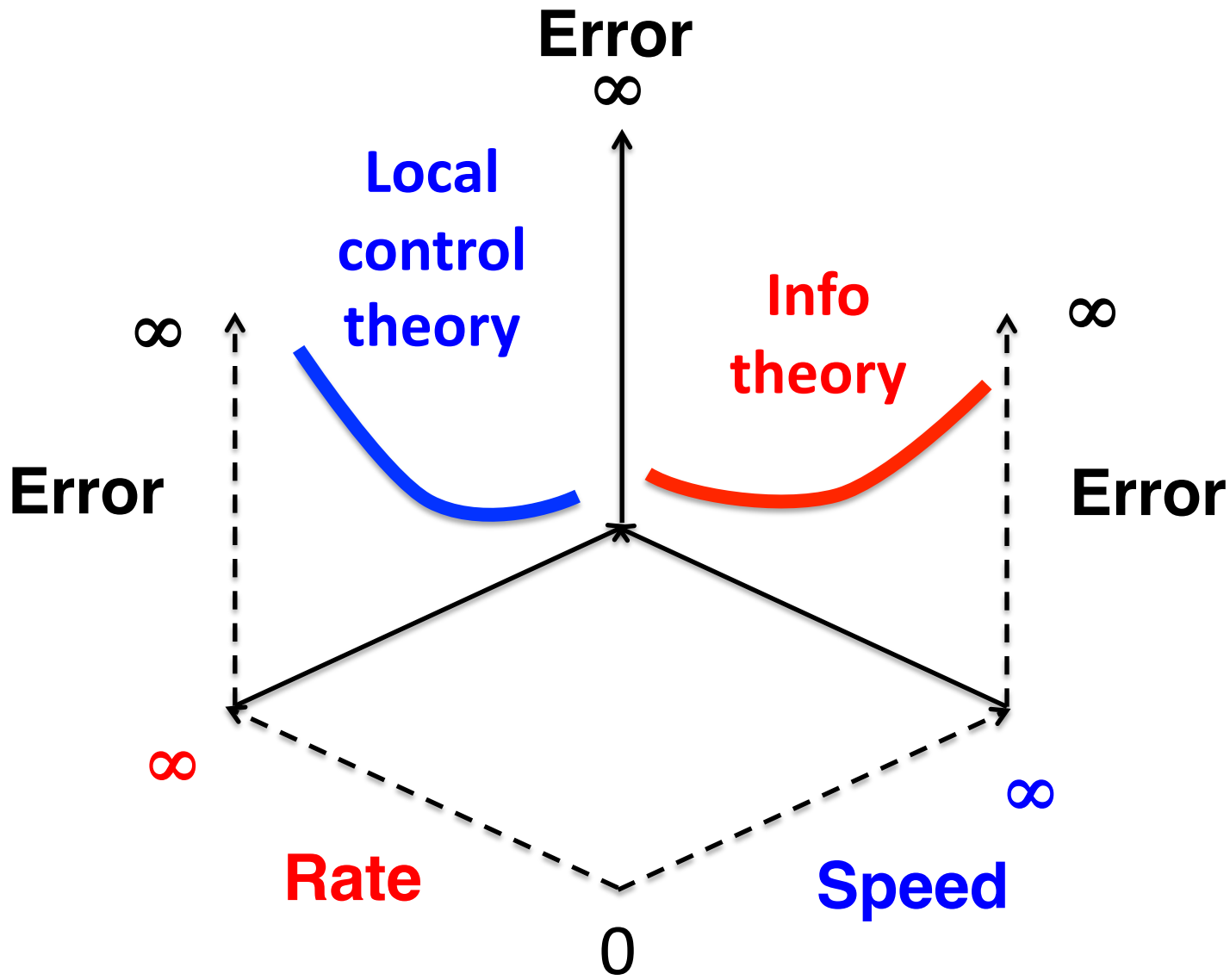
Communications



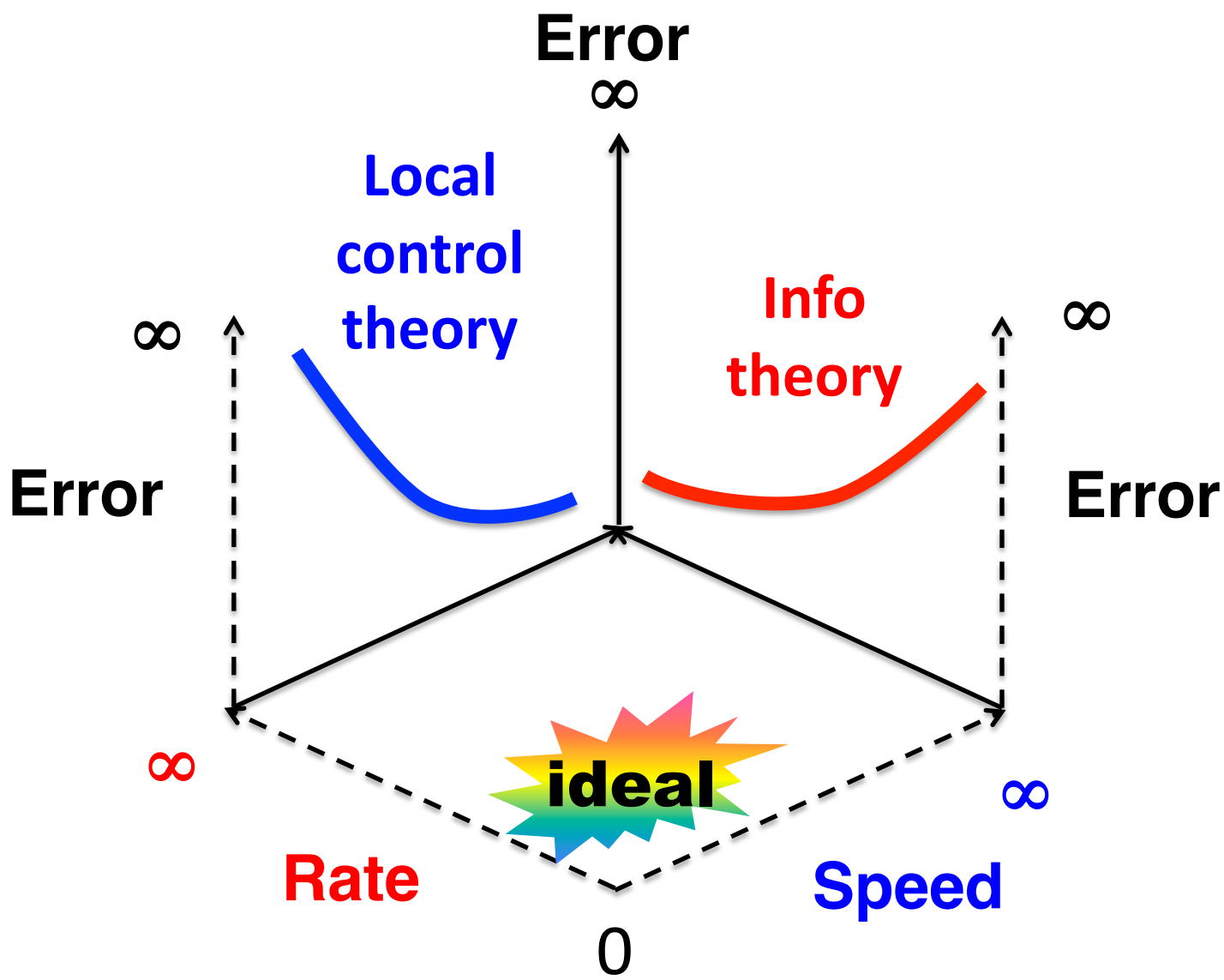
Communications



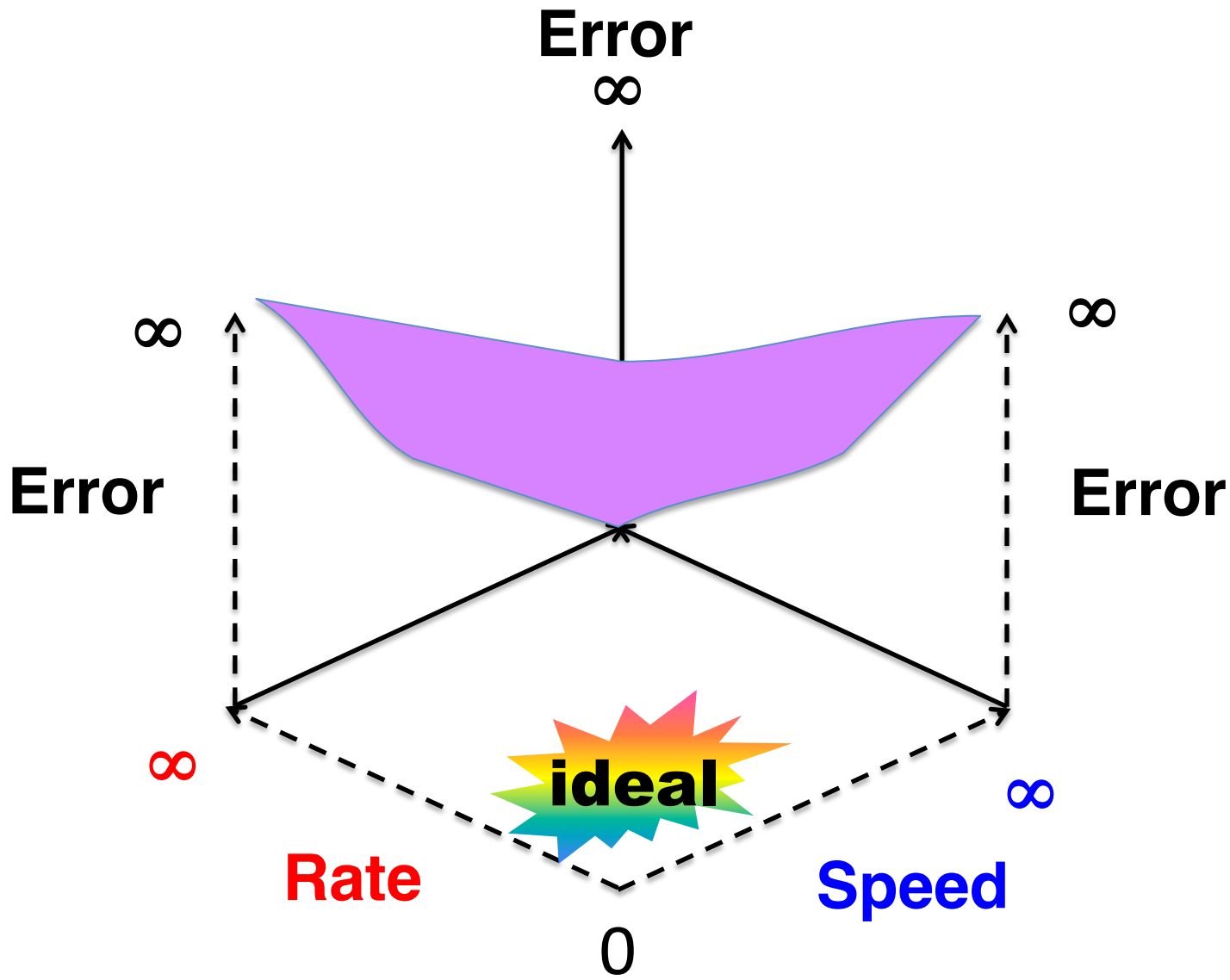
Communications



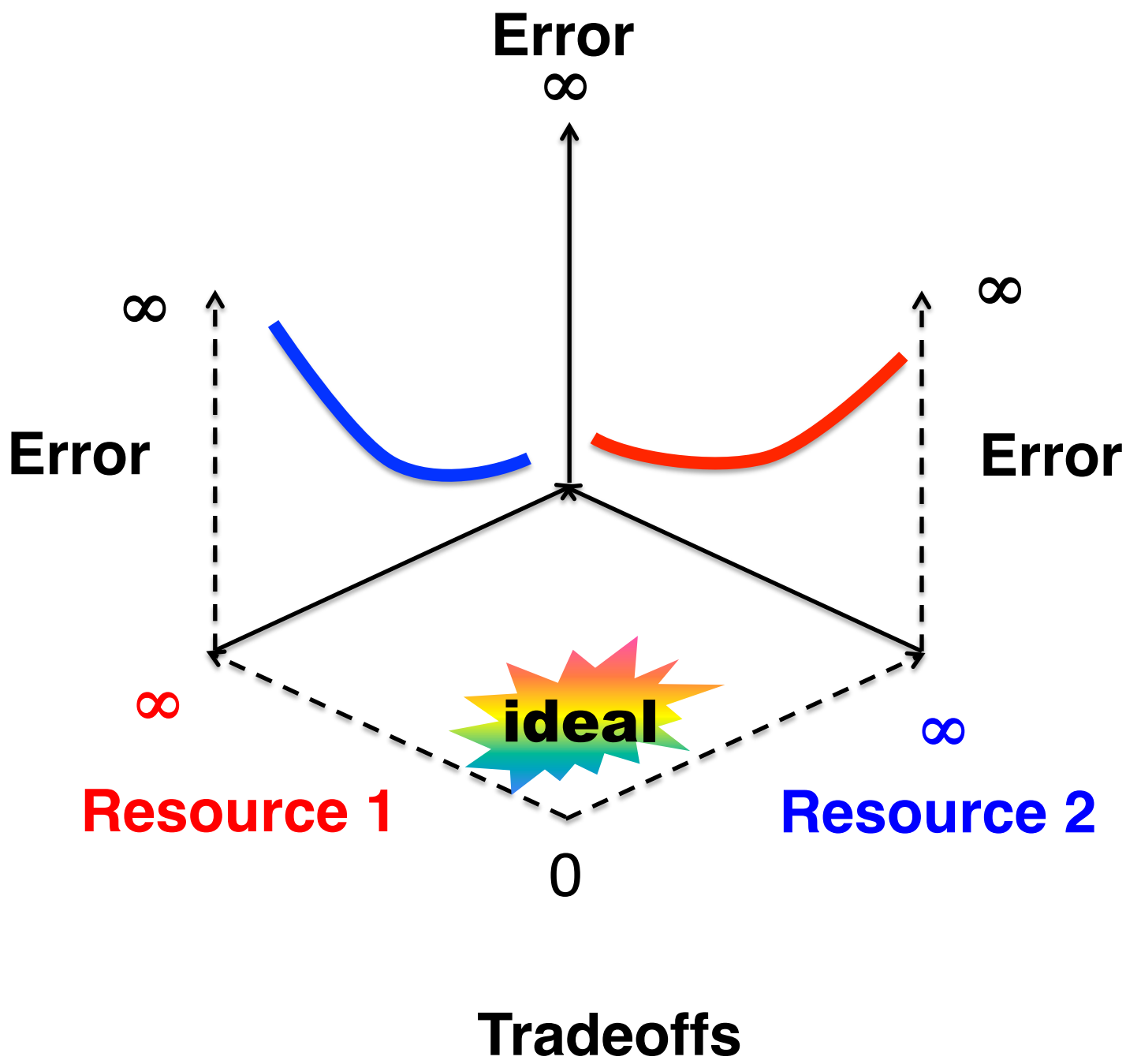
Communications



Communications

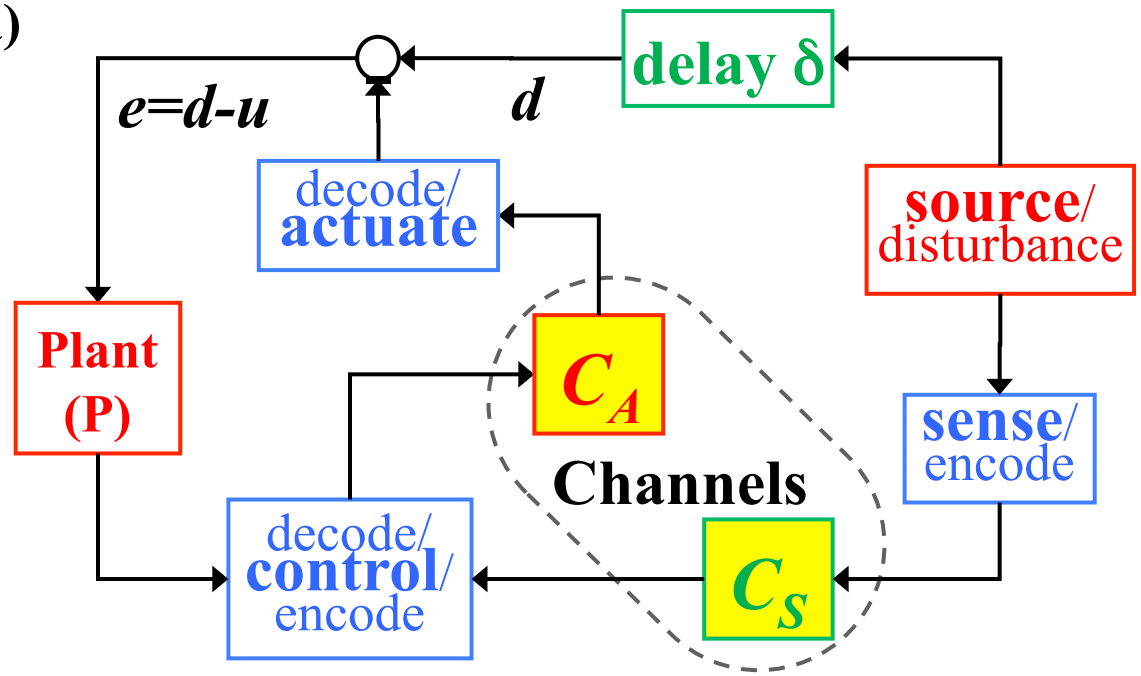


Local control


Error
∞
∞
Error
∞
Error
∞
ideal
∞
Resource 1
Resource 2
0
Tradeoffs

Control over limited channels (Martins et al)

a)



$$\int f(\omega)d\omega \triangleq \frac{1}{\pi}\int_0^\infty f(\omega)d\omega$$

b) $P(p) = \infty \quad p \geq 0$

c) $S(j\omega) \triangleq \dfrac{E(j\omega)}{D(j\omega)}$

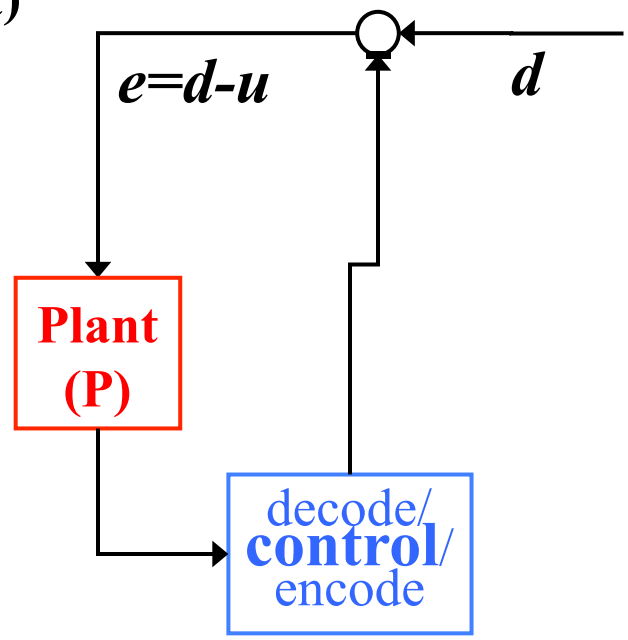
d) $\int \log|S|\,d\omega \geq p - C_S$

e) $\int \min\left(0, \log|S|\right)d\omega \geq p - C_A$

f) $P(z) = 0 \Rightarrow \quad \int \ln|S(j\omega)| \dfrac{z}{z^2+\omega^2}\,d\omega \geq \dfrac{1}{2}\ln\left|\dfrac{z+p}{z-p}\right| \quad \left(\geq \dfrac{p}{z} \text{ if } p < z\right)$

Control over limited channels (Martins et al)

a)



$$\int f(\omega)d\omega \triangleq \frac{1}{\pi}\int_0^\infty f(\omega)d\omega$$

b) $P(p) = \infty \quad p \geq 0$

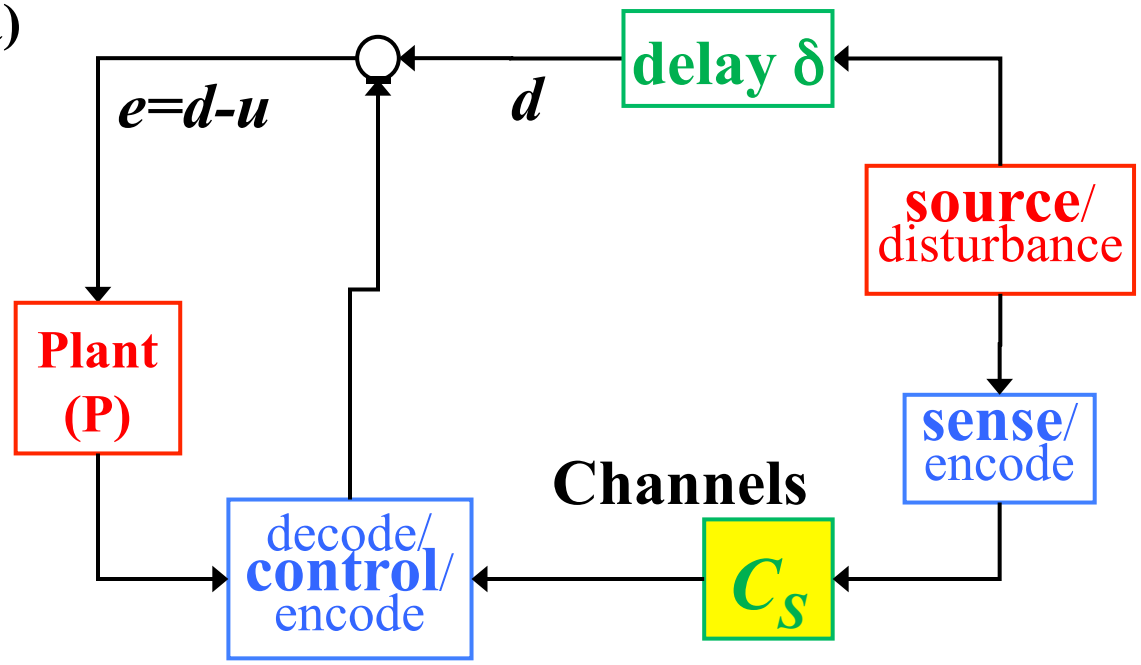
c) $S(j\omega) \triangleq \dfrac{E(j\omega)}{D(j\omega)}$

f) $P(z) = 0 \Rightarrow$

$$\int \ln\left|S(j\omega)\right| \frac{z}{z^2+\omega^2}\,d\omega \geq \frac{1}{2}\ln\left|\frac{z+p}{z-p}\right| \quad \left(\geq \frac{p}{z} \text{ if } p < z\right)$$

Control over limited channels (Martins et al)

a)



$$\int f(\omega)d\omega \triangleq \frac{1}{\pi}\int_0^\infty f(\omega)d\omega$$

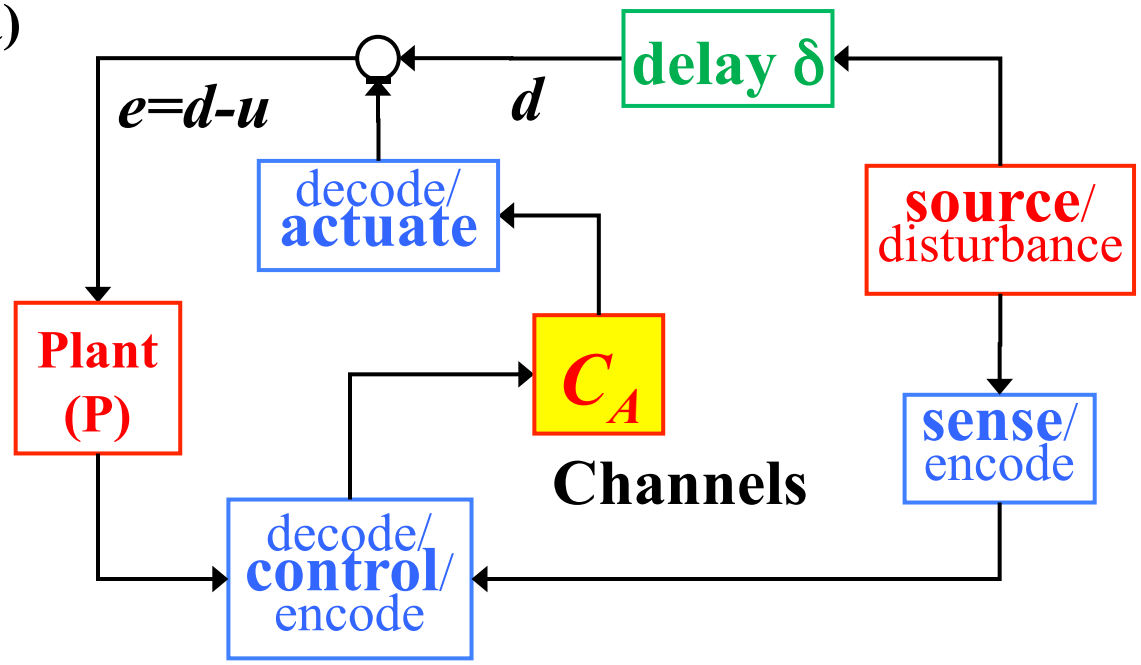
b) $P(p) = \infty \quad p \geq 0$

c) $S(j\omega) \triangleq \dfrac{E(j\omega)}{D(j\omega)}$

d) $$\int \log|S|\, d\omega \geq p - C_S$$

Control over limited channels (Martins et al)

a)



$$\int f(\omega)d\omega \triangleq \frac{1}{\pi}\int_0^\infty f(\omega)d\omega$$

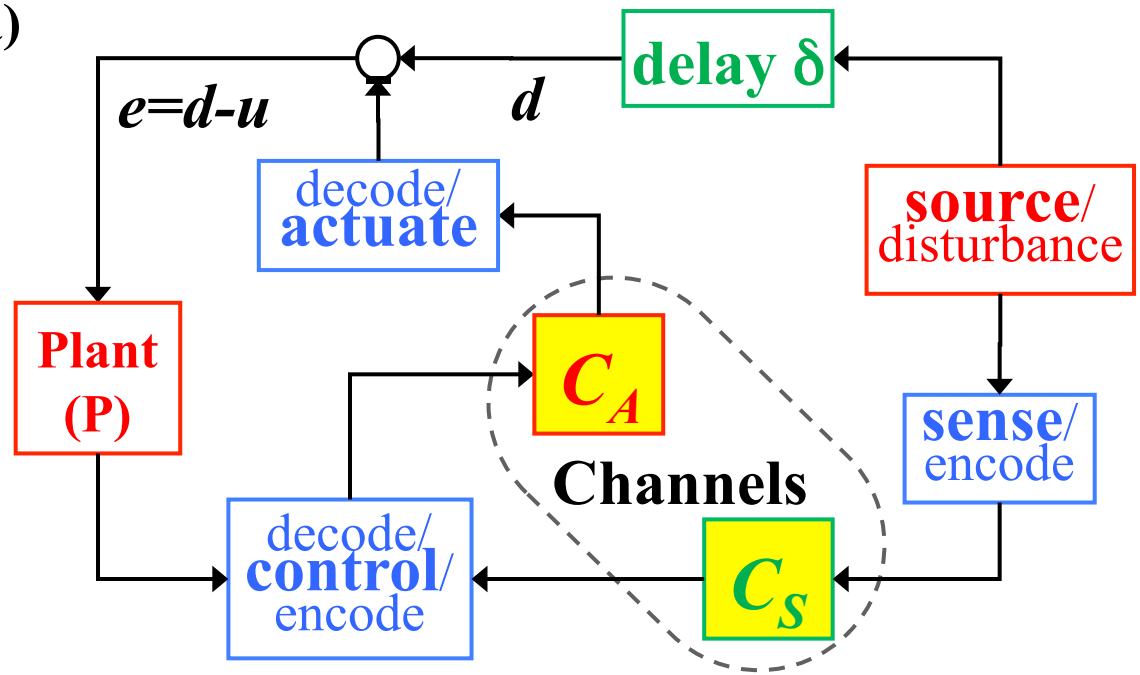
b) $P(p) = \infty \quad p \geq 0$

c) $S(j\omega) \triangleq \dfrac{E(j\omega)}{D(j\omega)}$

e) $$\int \min\left(0, \log|S|\right)d\omega \geq p - C_A$$

Control over limited channels (Martins et al)

a)



$$\int f(\omega)d\omega \triangleq \frac{1}{\pi}\int_0^\infty f(\omega)d\omega$$

b) $P(p) = \infty \quad p \geq 0$

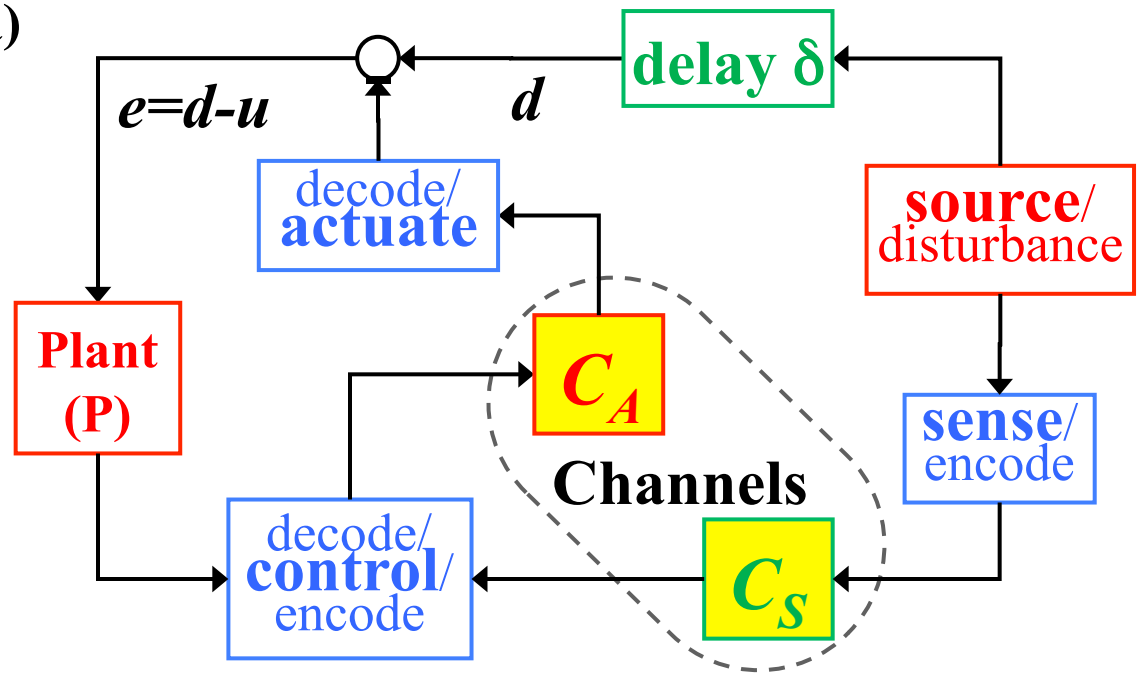
c) $S(j\omega) \triangleq \dfrac{E(j\omega)}{D(j\omega)}$

d) $$\int \log|S|\, d\omega \geq p - C_S$$

e) $$\int \min\left(0, \log|S|\right) d\omega \geq p - C_A$$

Control over limited channels (Martins et al)

a)



$$\int f(\omega)d\omega \triangleq \frac{1}{\pi}\int_0^\infty f(\omega)d\omega$$

b) $P(p) = \infty \quad p \geq 0$

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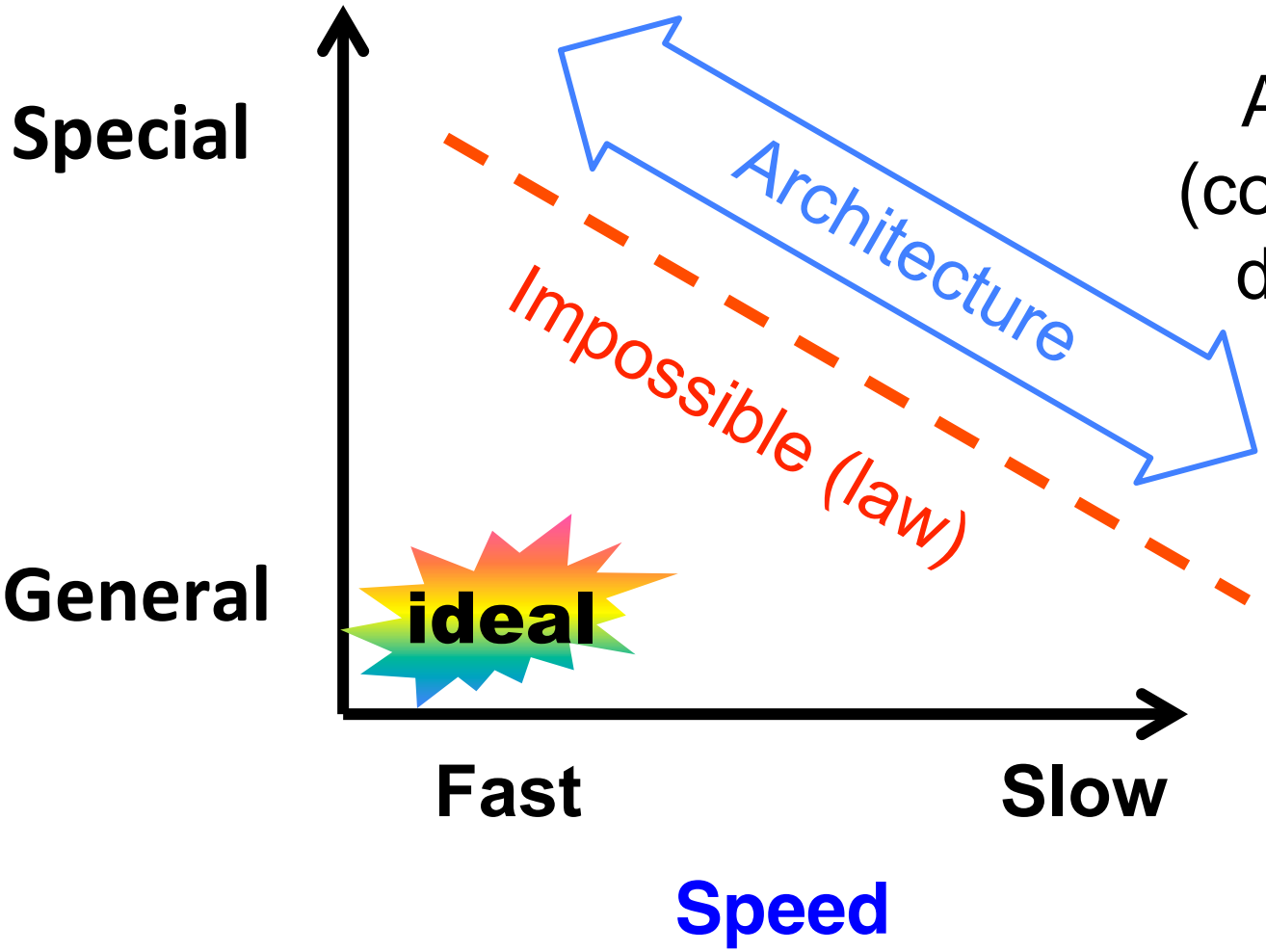
f) $P(z) = 0 \Rightarrow \quad \int \ln|S(j\omega)| \dfrac{z}{z^2+\omega^2}\,d\omega \geq \dfrac{1}{2}\ln\left|\dfrac{z+p}{z-p}\right| \quad \left(\geq \dfrac{p}{z} \text{ if } p < z\right)$

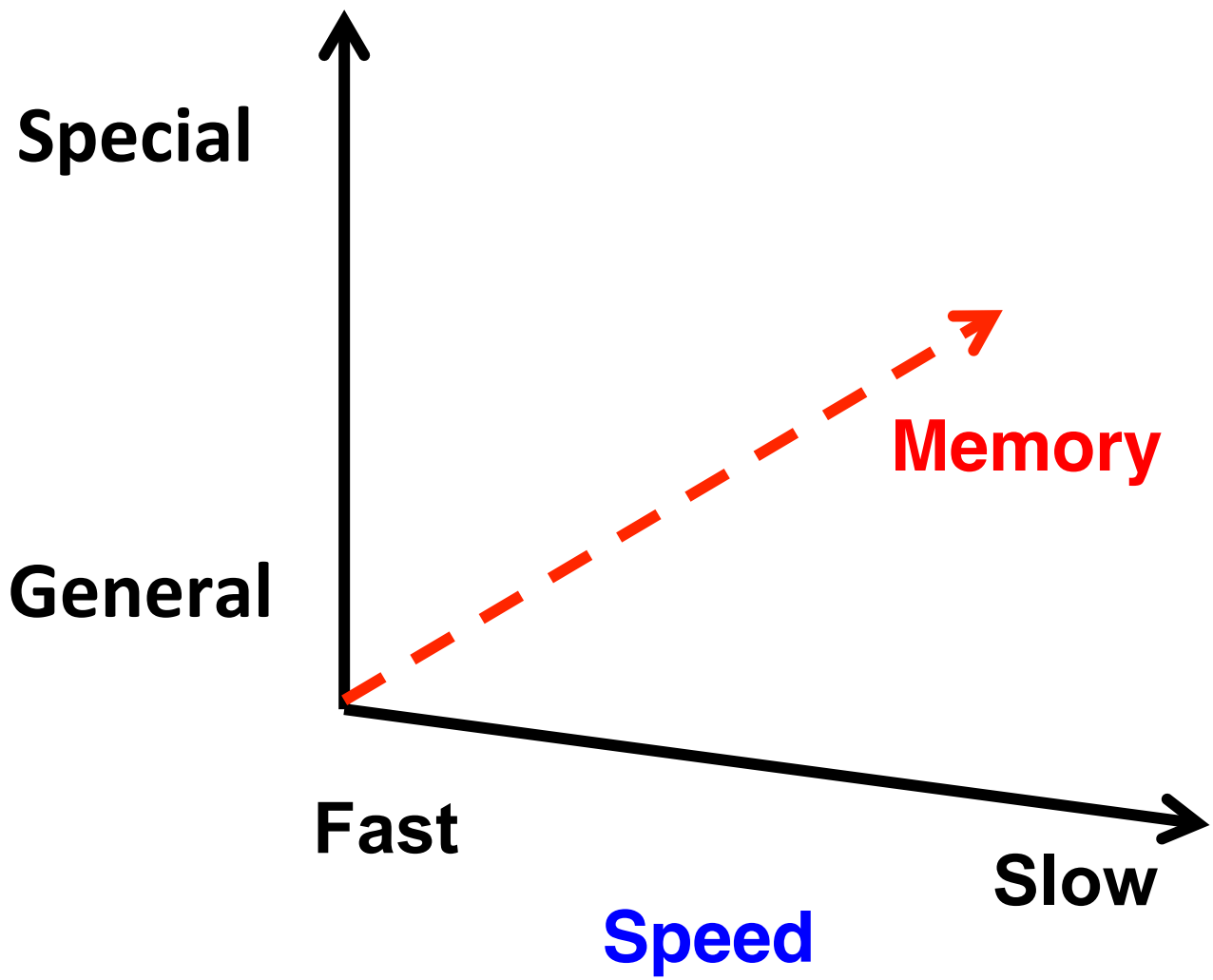
Universal laws and architectures (Turing)

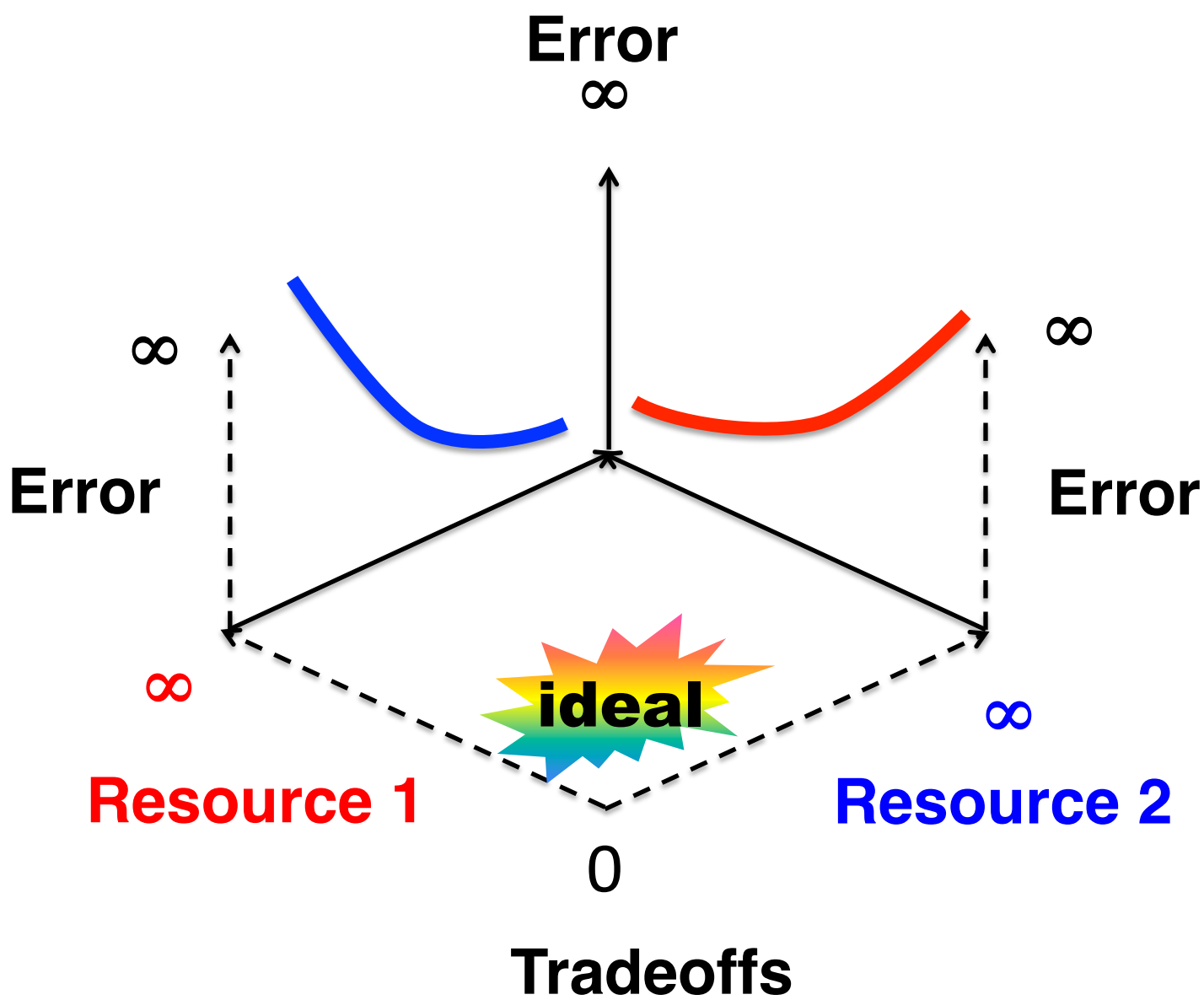


Architecture (constraints that deconstrain)

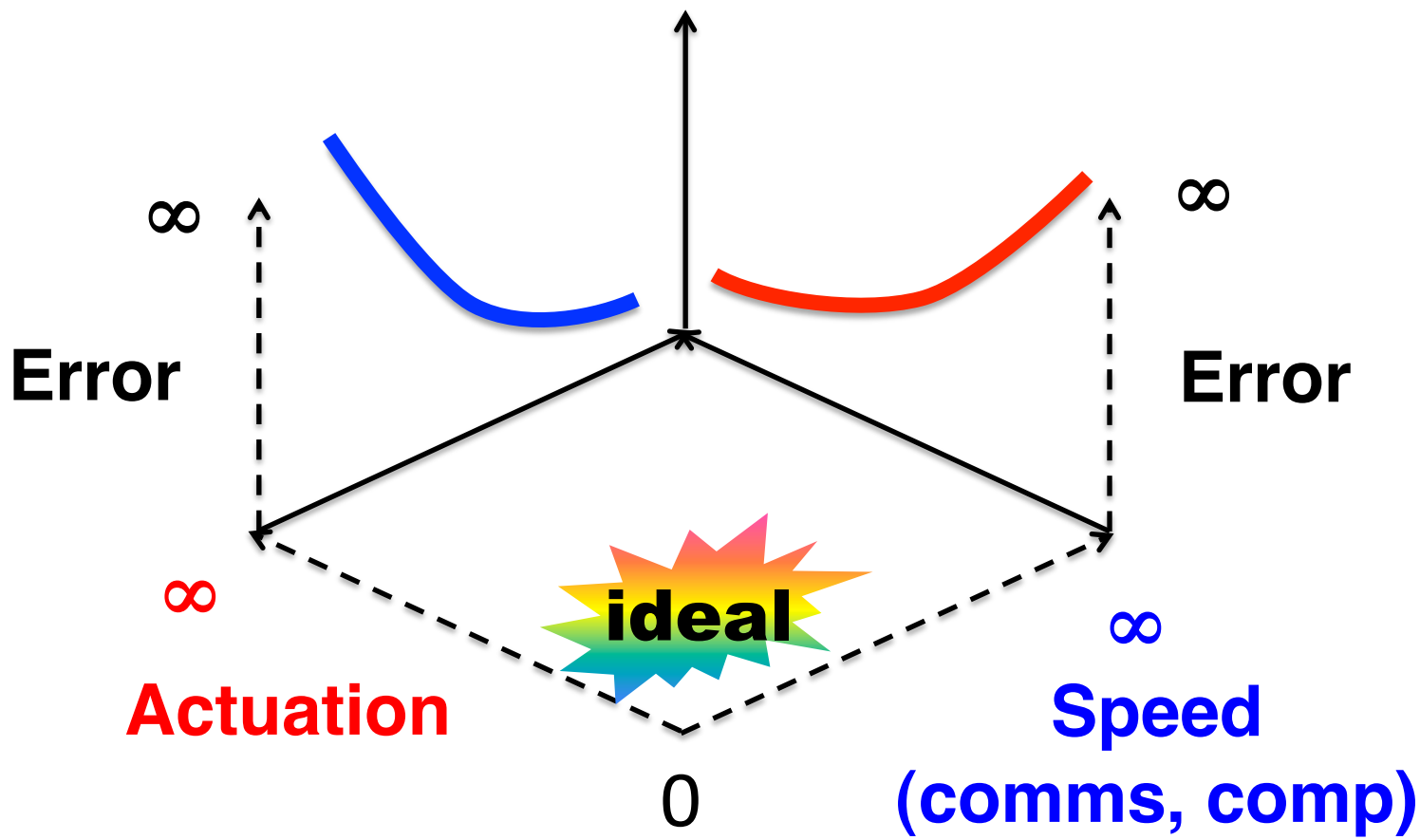
Memory is cheap, reusable, powerful.

Time is not.



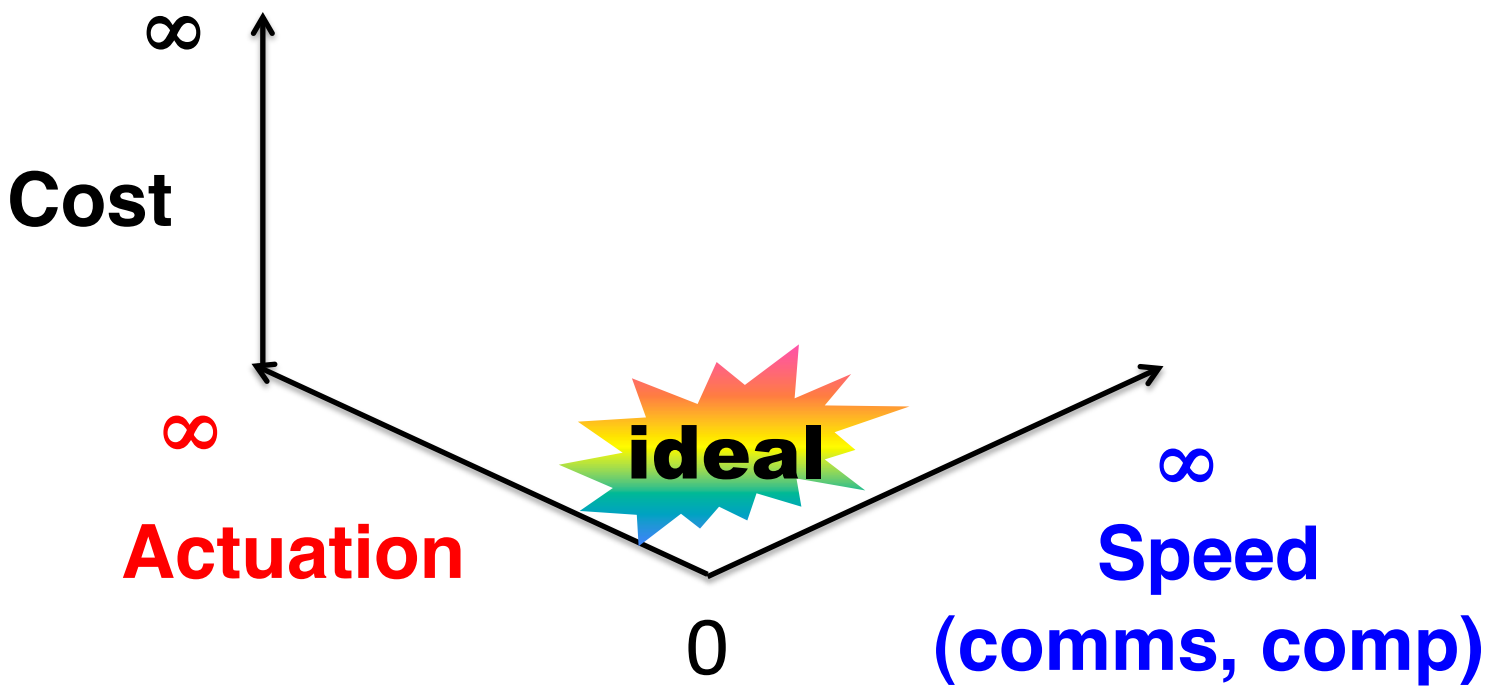

Error
∞
∞
Error
∞
Error
∞
ideal
∞
Resource 1
Resource 2
0
Tradeoffs

- **Cheap: memory, bandwidth, sensors**
- **Not : time (1/speed), actuators**
- Brains/bodies, cells, CyberPhySys, …



Critical Tradeoffs

All costs are ultimately “physical.”



Critical Tradeoffs