Opinion Exchange Dynamics

Omer Tamuz
MIT Math / Caltech HSS

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- Modeling tools: probability, dynamics, game theory.

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- **Convergence**? **Aggregation of information**?
- Modeling tools: probability, dynamics, game theory.
- Not models like in Physics.

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  - Bayesian agents.

Questions

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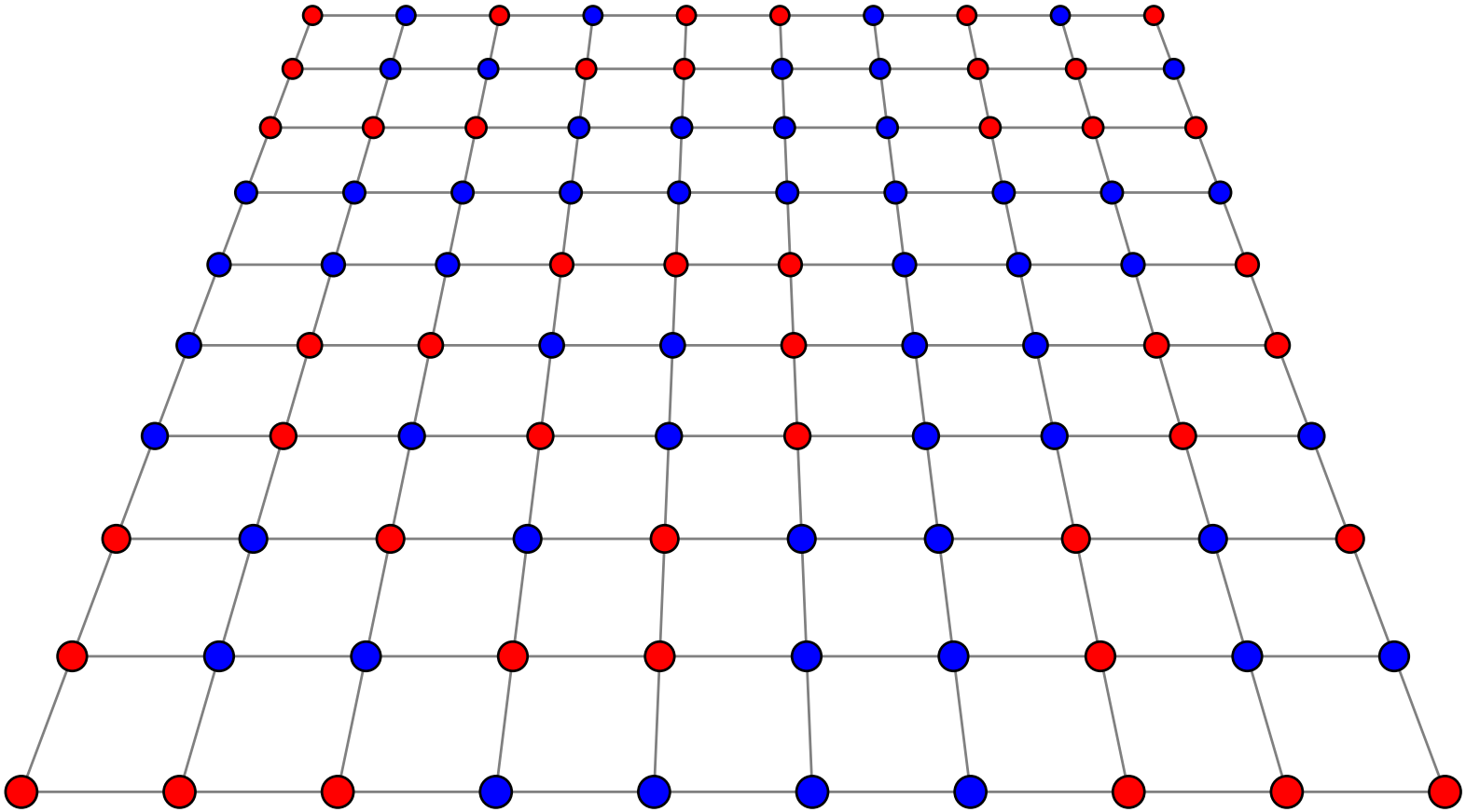
- If so, then does everyone converge to the **correct** choice?
- Otherwise, does the **majority** converge to the correct choice?
- When society is large then there's **enough information** out there to determine the correct choice with high probability.

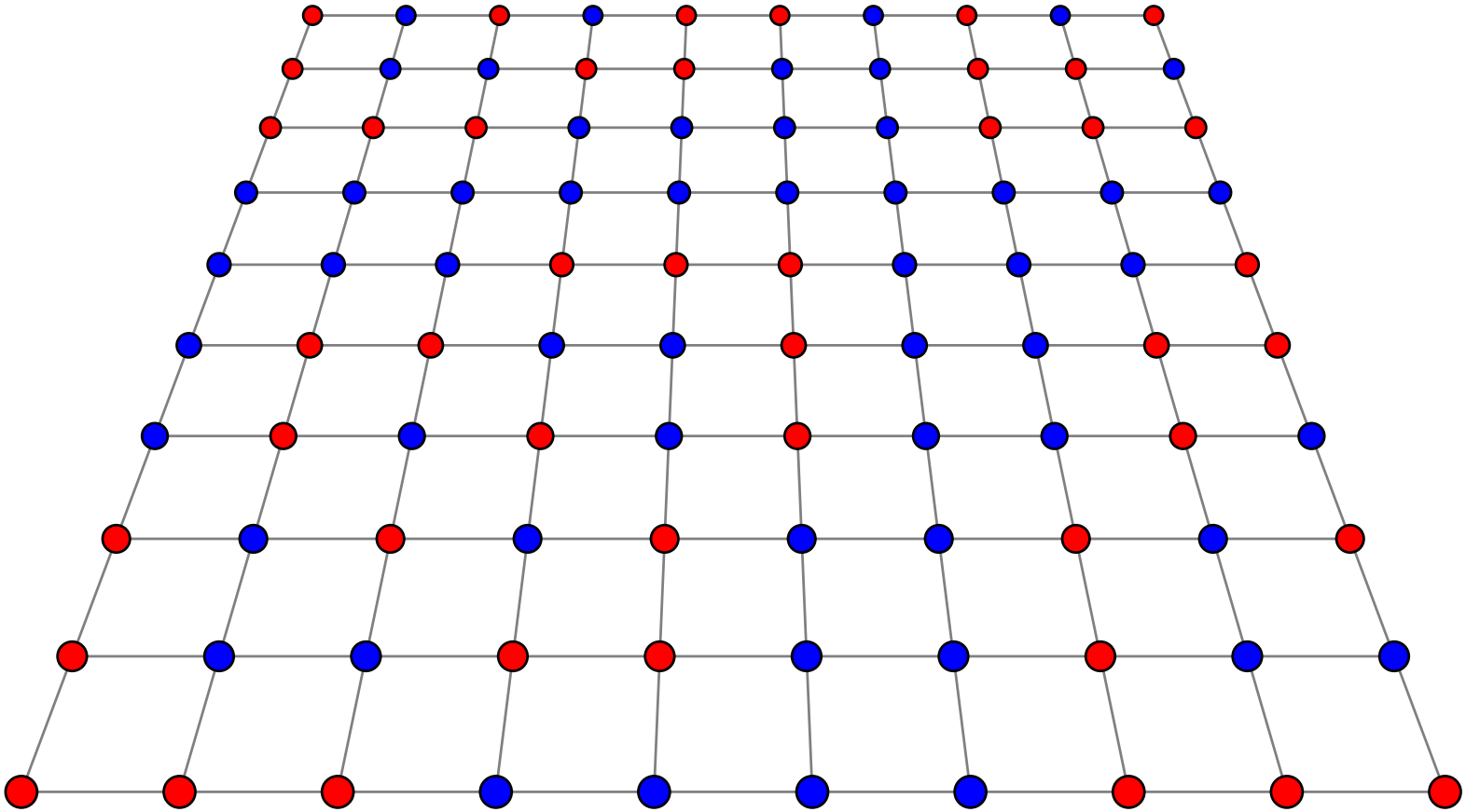
Conformist automata

Conformist automata



- At each time period, each person updates her choice to match the majority of her neighbors.

Conformist automata



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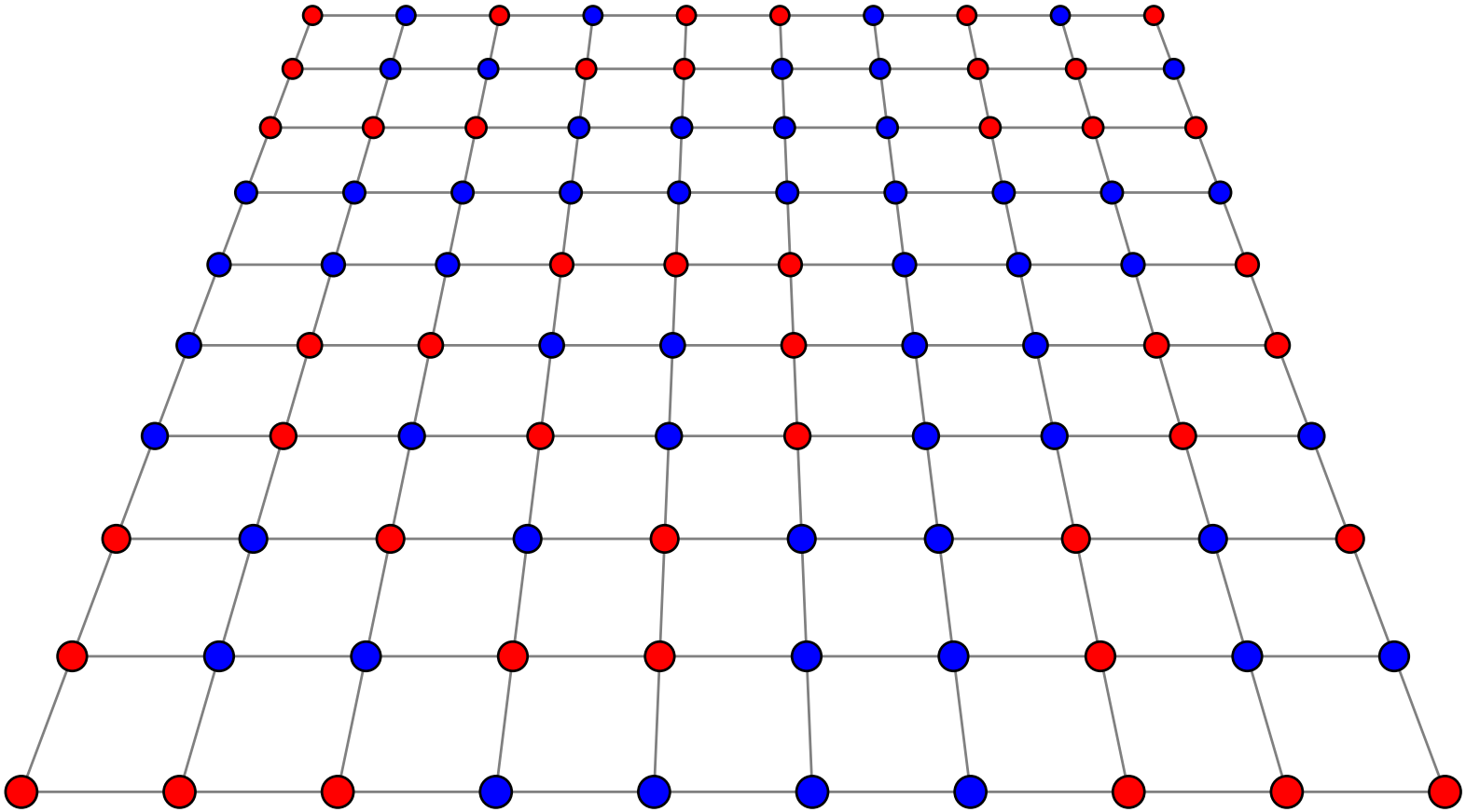

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- **Deterministic dynamical system**.

Convergence

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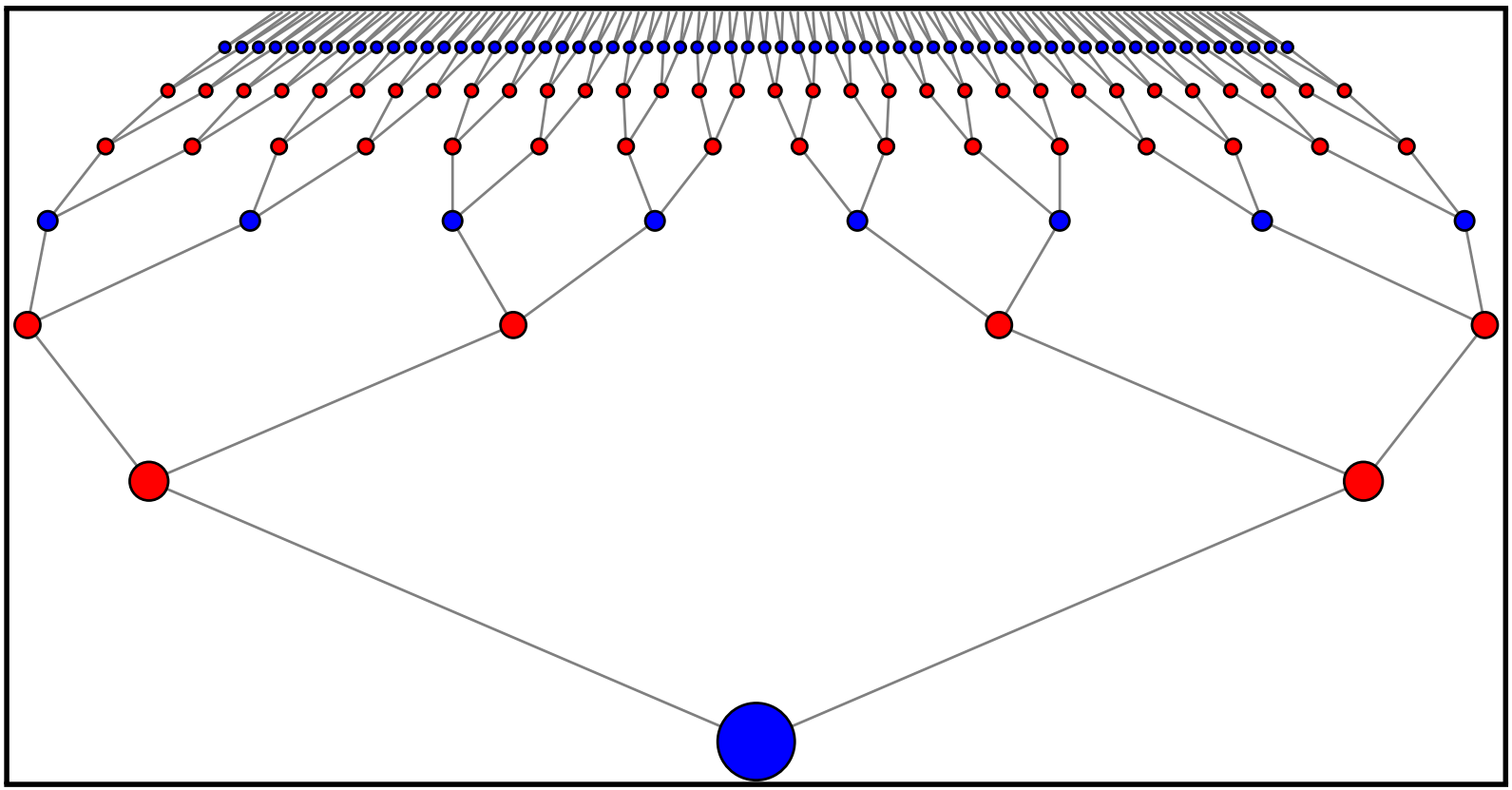
Theorem (Goles & Olivos '80)

Every periodic orbit has period two.

- In other words: everyone eventually makes the same choice they made **two time periods ago**.

- What about infinite graphs?



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Theorem (T. & Tessler, 2014)

The total number of times that i chooses differently than **two time periods ago** *is at most*

$$\frac{d+1}{d-1} \cdot d \cdot \sum_{r=0}^{\infty} \left(\frac{d+1}{d-1} \right)^{-r} n_r(i).$$

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$$\frac{d+1}{d-1} \cdot d \cdot \sum_{r=0}^{\infty} \left(\frac{d+1}{d-1} \right)^{-r} n_r(i).$$

- Sufficient but not necessary.

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- Take a graph in which everyone converges (to period two).
- Does the majority converge to the **correct** choice?
- **Not in general**. Berger '01: a sequence of finite graphs of size going to infinity, in which a group of 18 **imposes its opinion**, if all are in agreement.

The majority opinion

- A **transitive graph** "looks the same from the point of view of each vertex".

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On **transitive finite graphs** *the majority converges to the correct choice with high probability.*

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On **transitive finite graphs** *the majority converges to the correct choice with high probability.*

Theorem (Benjamini et al. 2014)

On **transitive unimodular infinite graphs** *the "majority" converges to the correct choice with probability one.*

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- Conjecture: information flows well on any bounded degree graph.

The majority opinion

- Information flows well on **"egalitarian"** graphs.
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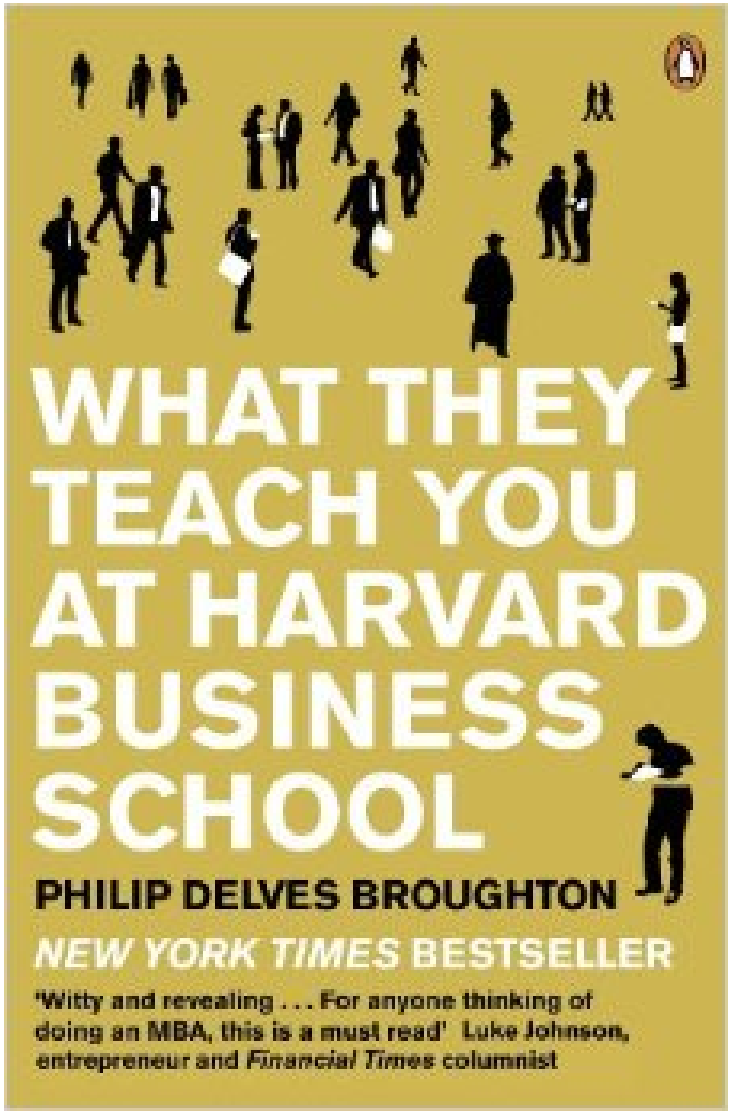
- Related results with Yakov Babicheno on more complicated automata.

Bayesian agents



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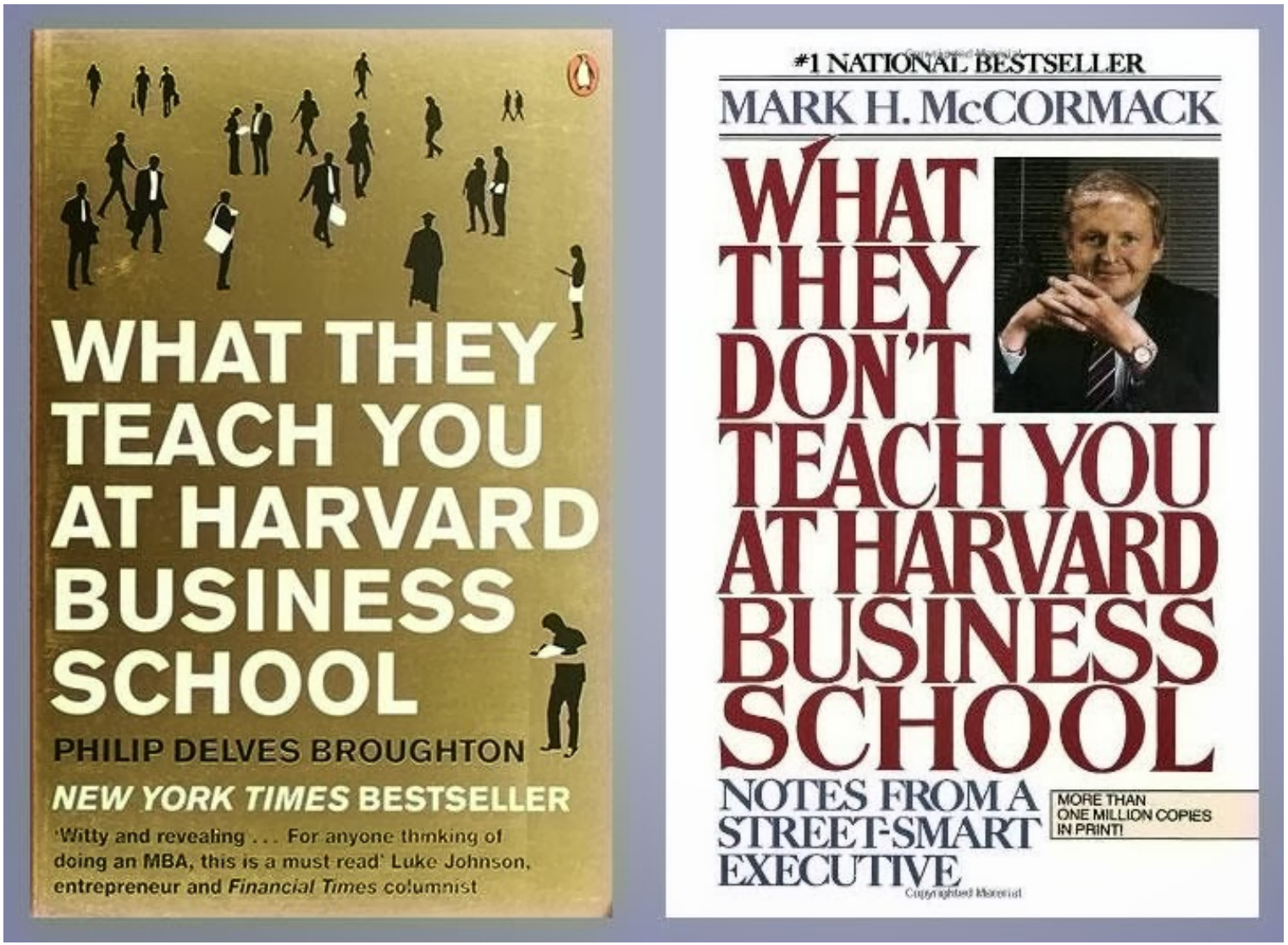

Bayesian agents

A Repeated Bayesian Game

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- $A_i(t) \in \{\text{red}, \text{blue}\}$.
- Agent i knows X_i, neighbors' previous actions.
- Dynamics.

$$A_i(t) = \underset{a \in \{\text{red},\text{blue}\}}{\operatorname{argmax}} \; \mathbb{P}[a | \text{what } i \text{ knows at time } t].$$

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- **It depends on the graph.**

Locally connected graphs

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Definition

G is L-**locally-connected** if for every edge (i, j) there is a path from j back to i of length at most L.

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- 1-locally-connected is undirected. **Relaxation of undirectedness.**

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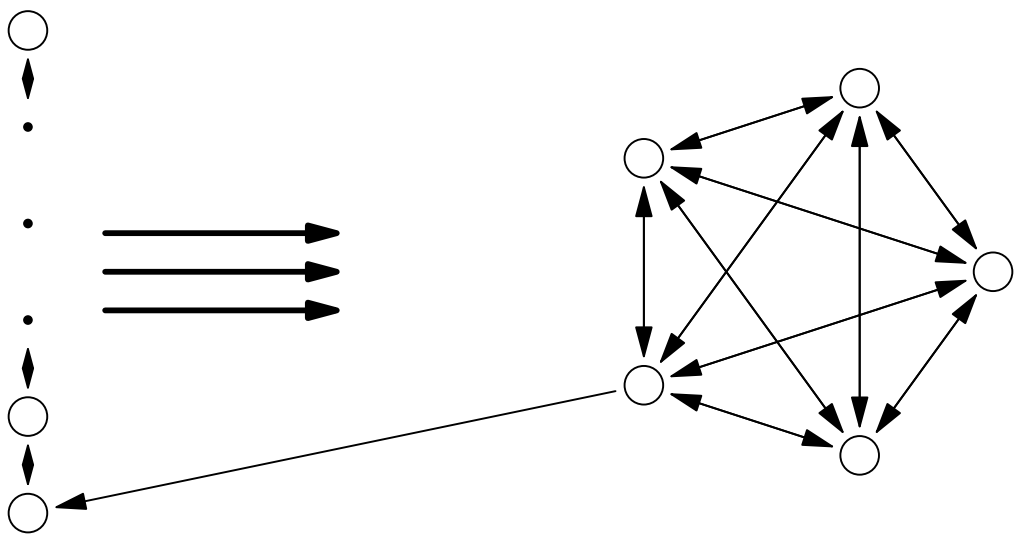
- Relaxation of **transitivity**, notion of **egalitarianism**.

The Royal Family

- The peasants all observe the royal family on TV.



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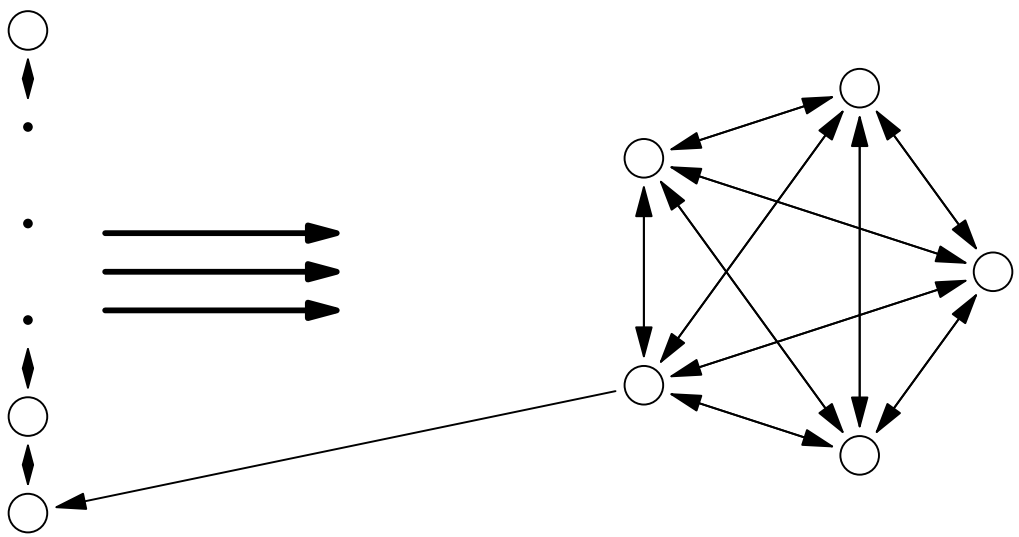

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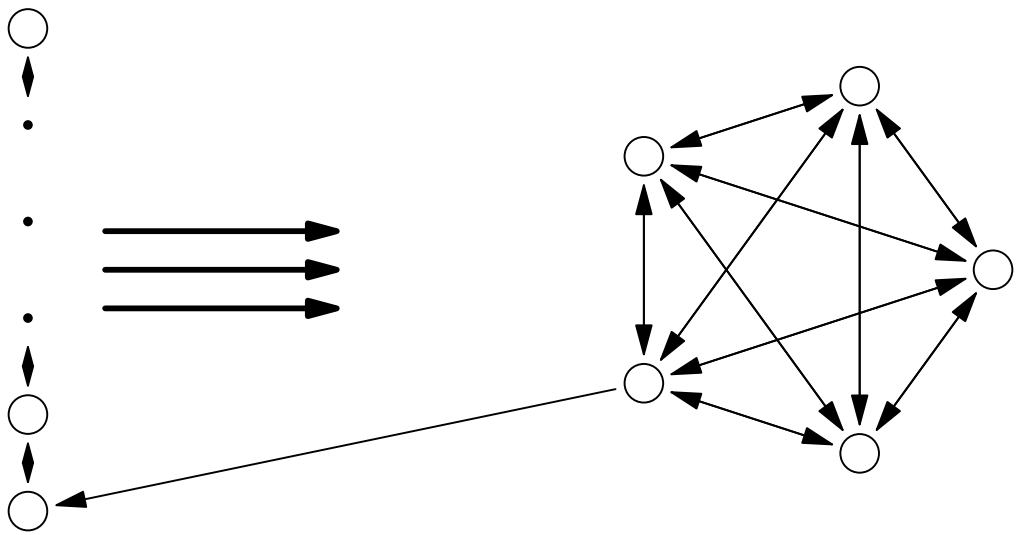
- After two time periods everyone follows it.



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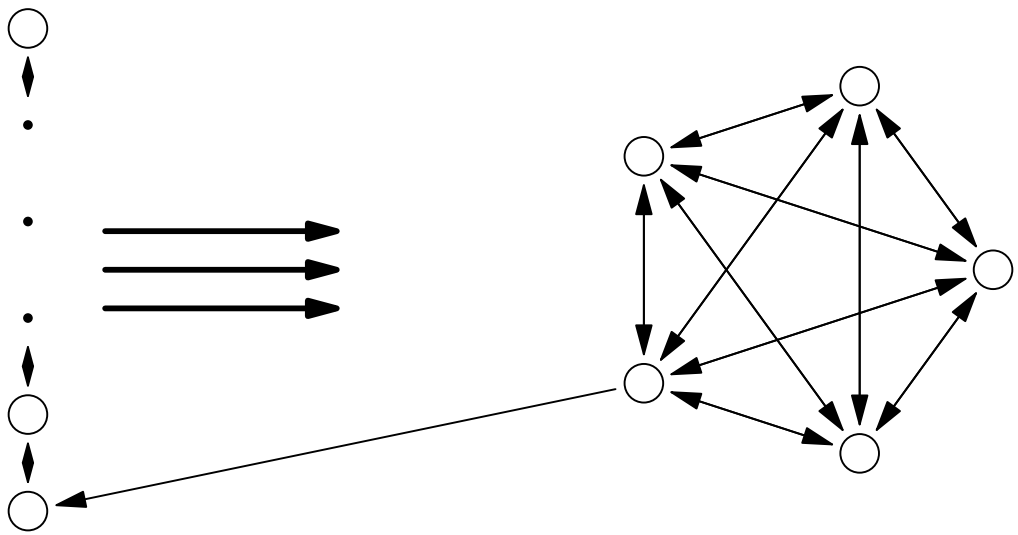
- **But it still may be wrong.**



Learning

Theorem (Mossel, Sly & T. 2012)

On every infinite, L-locally connected graph, everyone eventually makes the correct choice.

Information flows well on **"egalitarian"** graphs.

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- Thanks!