Dynamical structure and its uses for insight, discovery, and control

Shane Ross (CDS '04)

Engineering Science and Mechanics, Virginia Tech

www.shaneross.com

CDS@20

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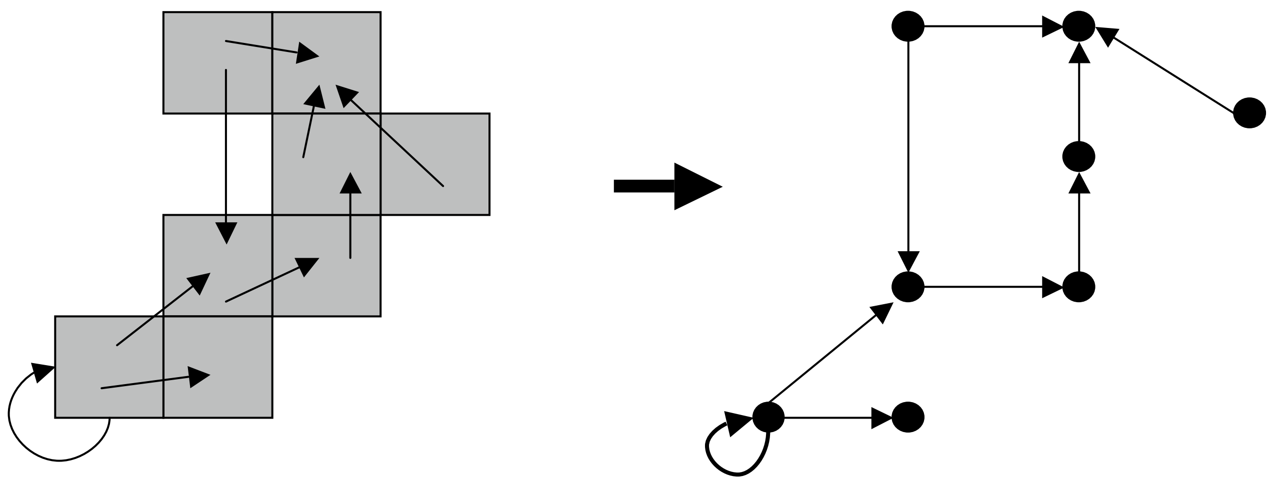
MultiSTEPS: MultiScale Transport in Environmental & Physiological Systems, IGERT www.multisteps.ictas.vt.edu

Motivation: application to data

- **Dynamical structure**: how phase space is connected / organized
- Fixed points, periodic orbits, or other invariant sets
  and their stable and unstable manifolds organize phase space
- Many systems defined from data or large-scale simulations
  — experimental measurements, observations
- e.g., from fluid dynamics, biology, social sciences
- Other tools (probabilistic, networks) could be useful in some settings



Phase space transport in 4+ dimensions

- Two examples

  — a biomechanical system

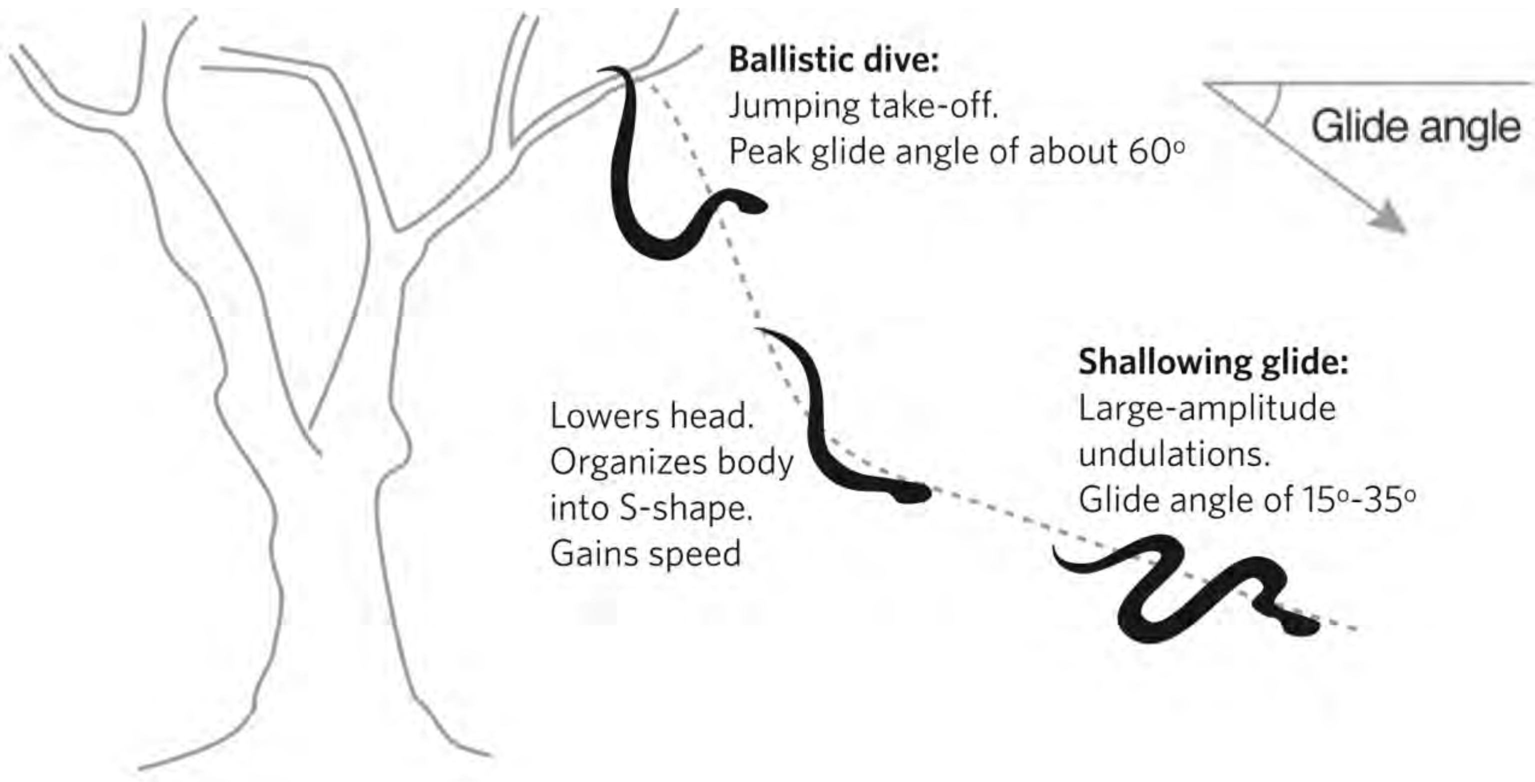
  — escape from a multi-dimensional potential well

- Then some examples from fluids and agriculture

Flying snakes

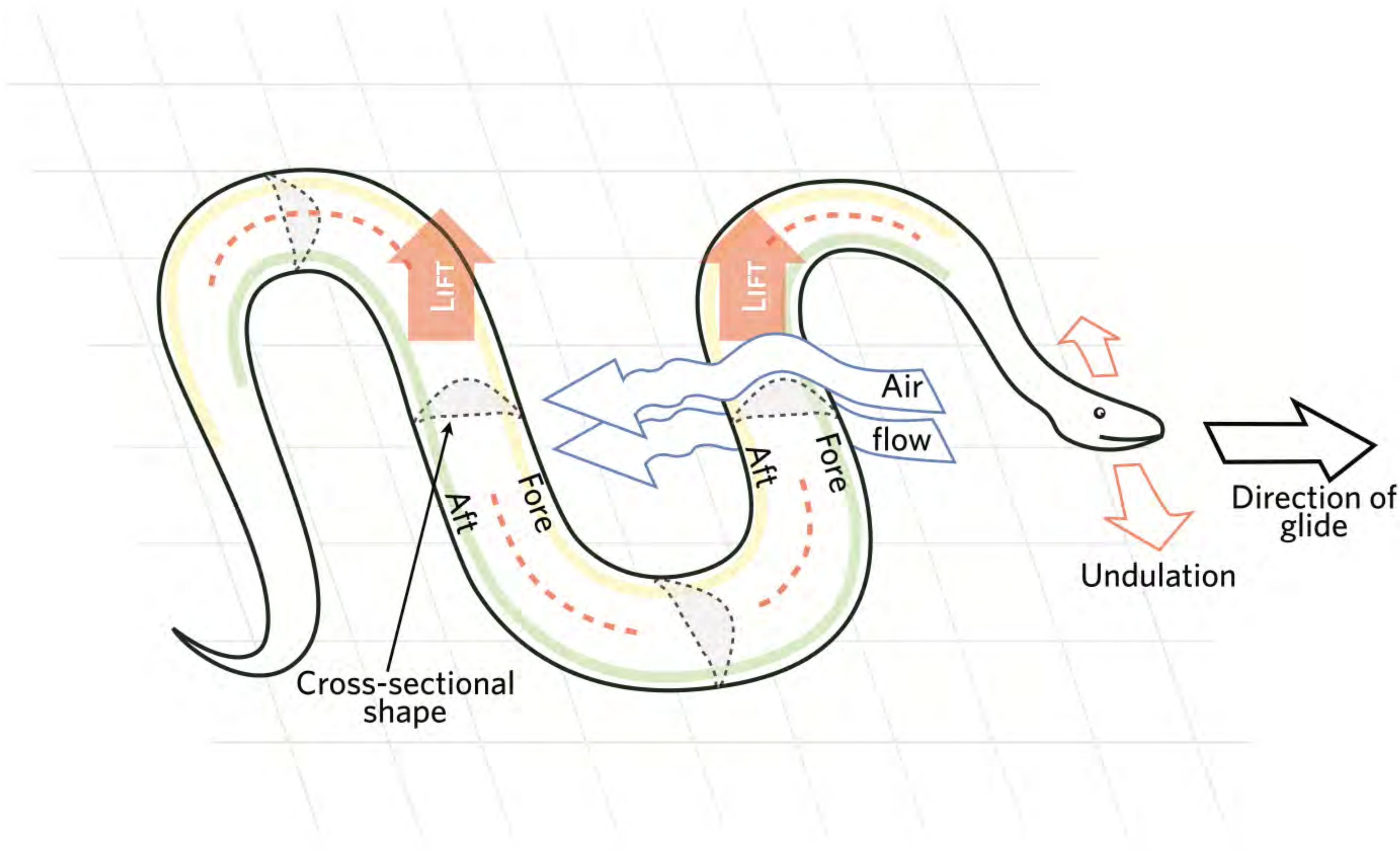
Joint work with Farid Jafari, Jake Socha, Pavlos Vlachos

Flying snakes



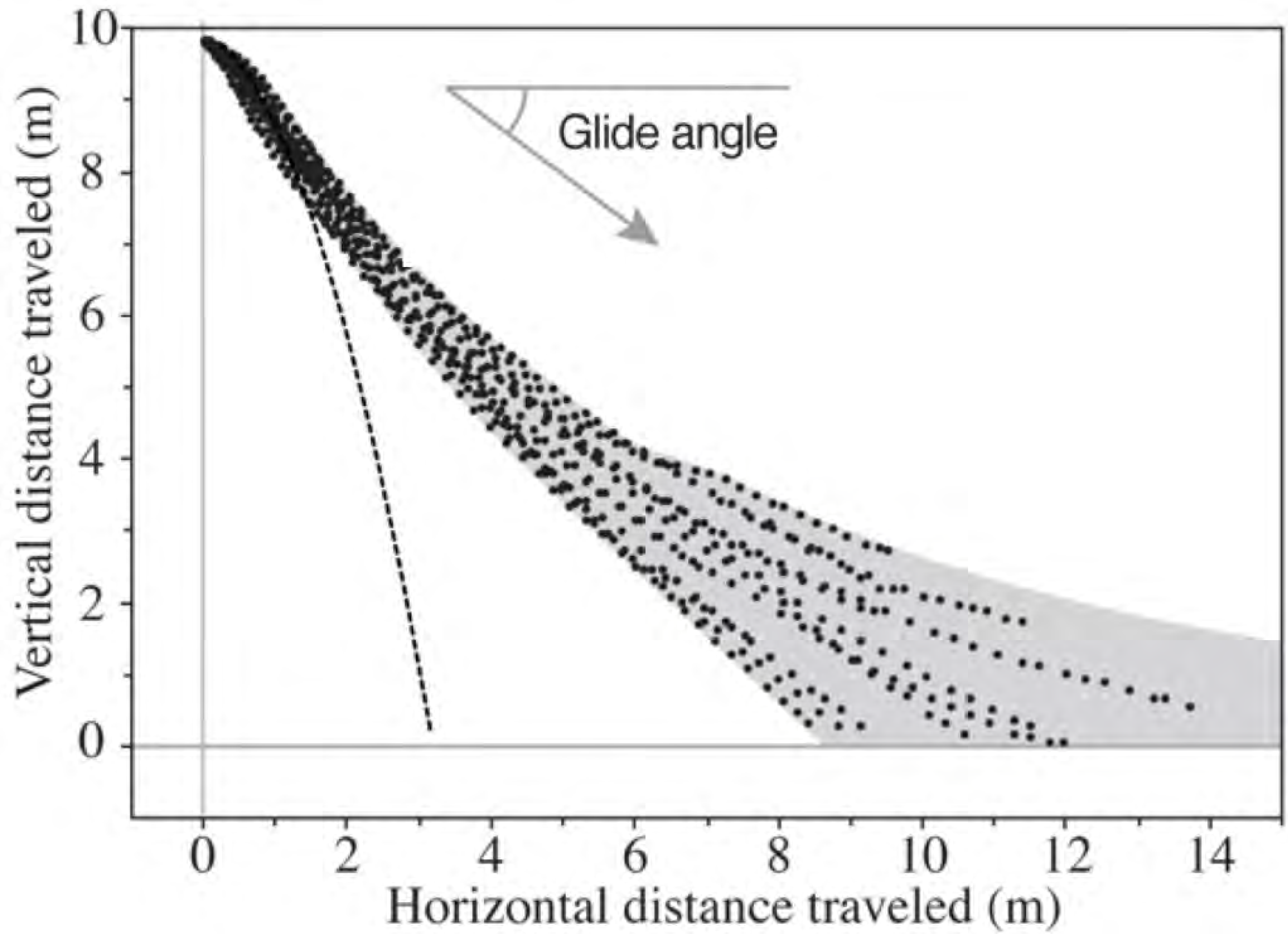
Krishnan, Socha, Vlachos, Barba [2014] Physics of Fluids

Flying snakes: undulation



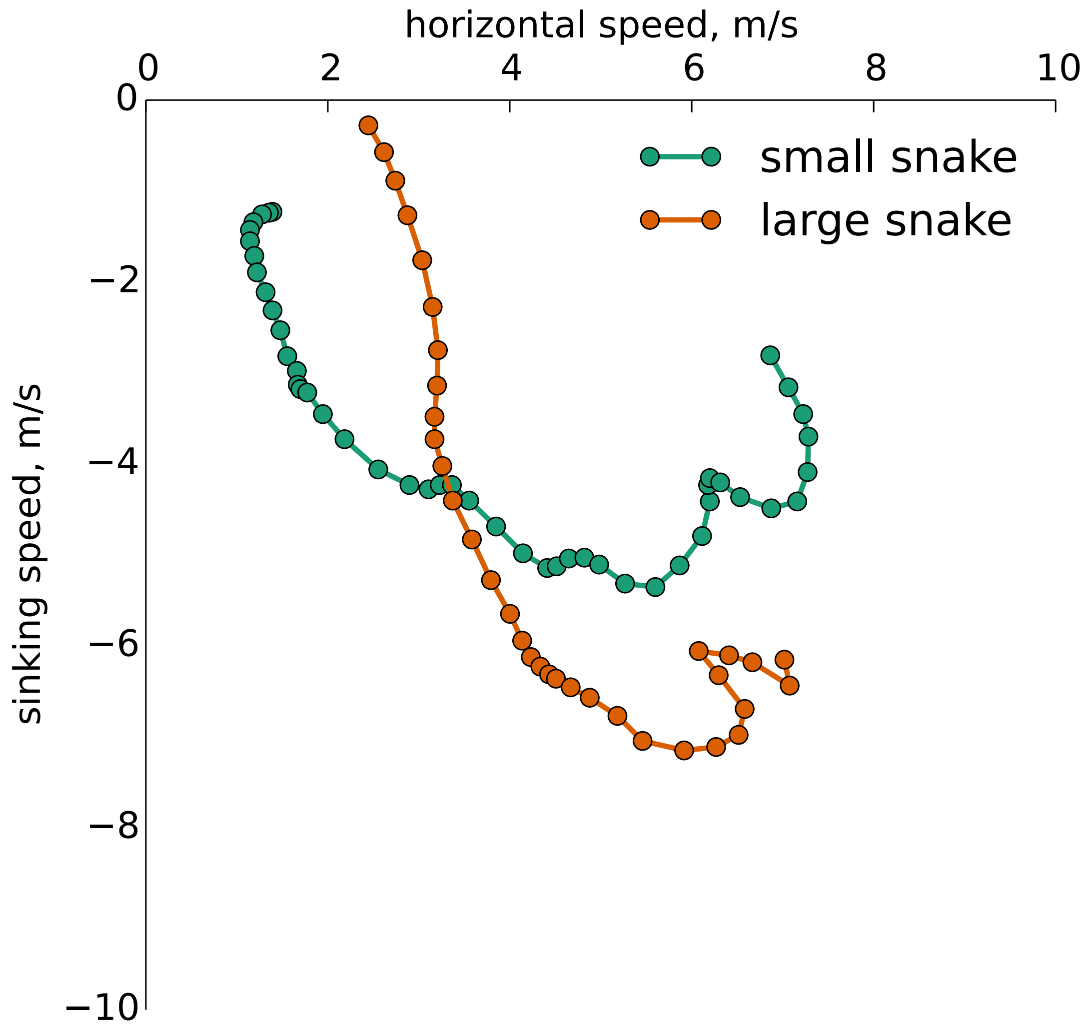
Krishnan, Socha, Vlachos, Barba [2014] Physics of Fluids

Flying snakes: experimental trajectories



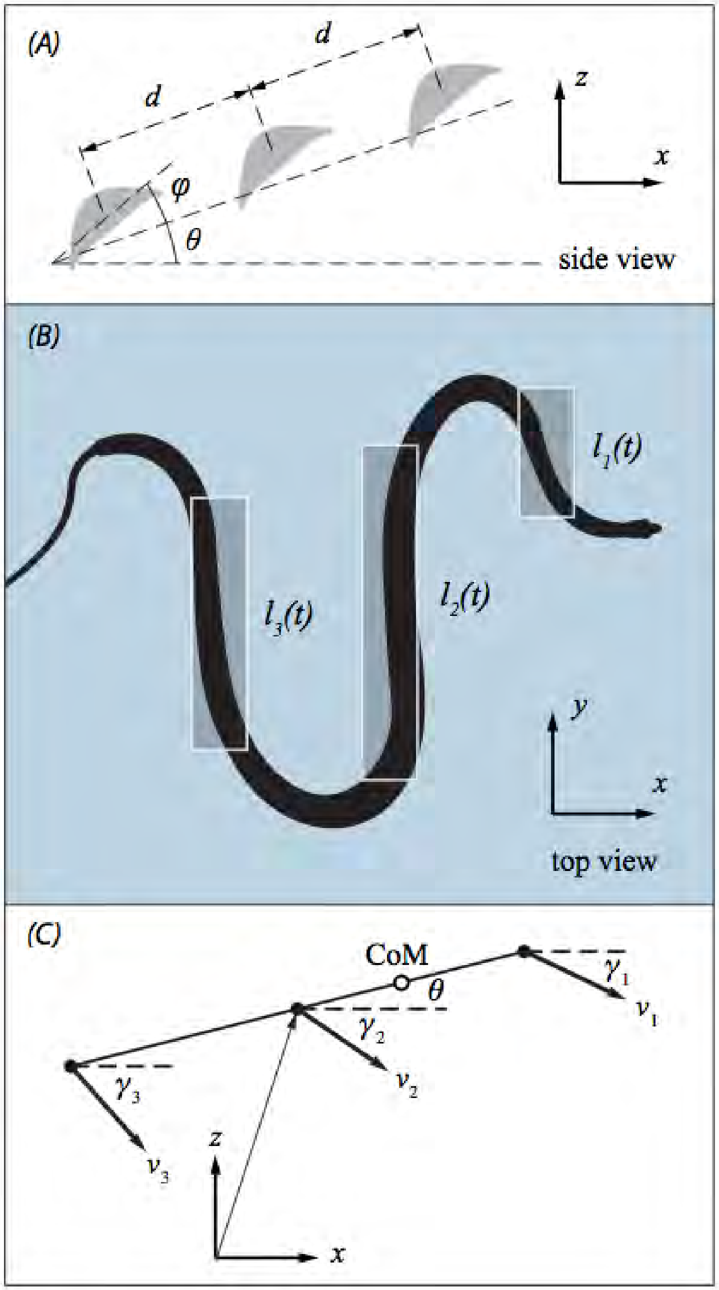
Socha [2011] Integrative and Comparative Biology

Flying snakes: velocity space



Socha [2011] Integrative and Comparative Biology

Flying snakes: minimal model



Consider a minimal model capturing the essential coupled translational-rotational dynamics — an undulating tandem wing configuration.

Given by 4-dimensional time-periodic system

$$\begin{aligned}\dot{v}_x &= u_1(\theta, \Omega, v_x, v_z, t)\\ \dot{v}_z &= u_2(\theta, \Omega, v_x, v_z, t)\\ \dot{\theta} &= u_3(\Omega) = \Omega\\ \dot{\Omega} &= u_4(\theta, \Omega, v_x, v_z, t)\end{aligned}$$

with translational kinematics $\dot{x} = v_x$, $\dot{z} = v_z$.

System is passively stable in pitch θ with equilibrium manifold $\{\Omega = 0\}$.

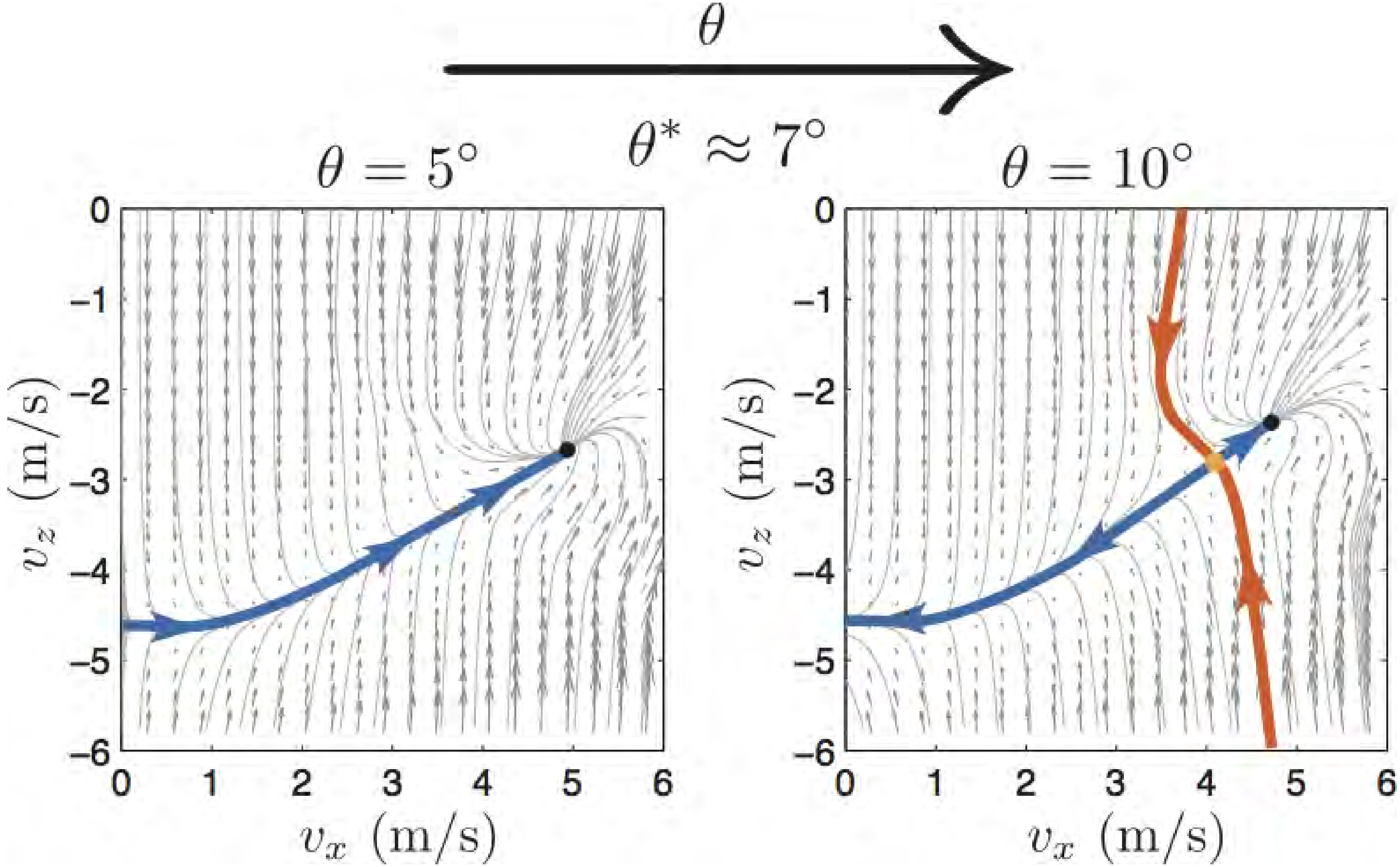
Translational dynamics are more complicated, but there does seem to be a 'shallowing manifold'.

Jafari, Ross, Vlachos, Socha [2014] Bioinspir. & Biomim.

Flying snakes: achieving equilibrium glide

Flying snakes: falling like a stone

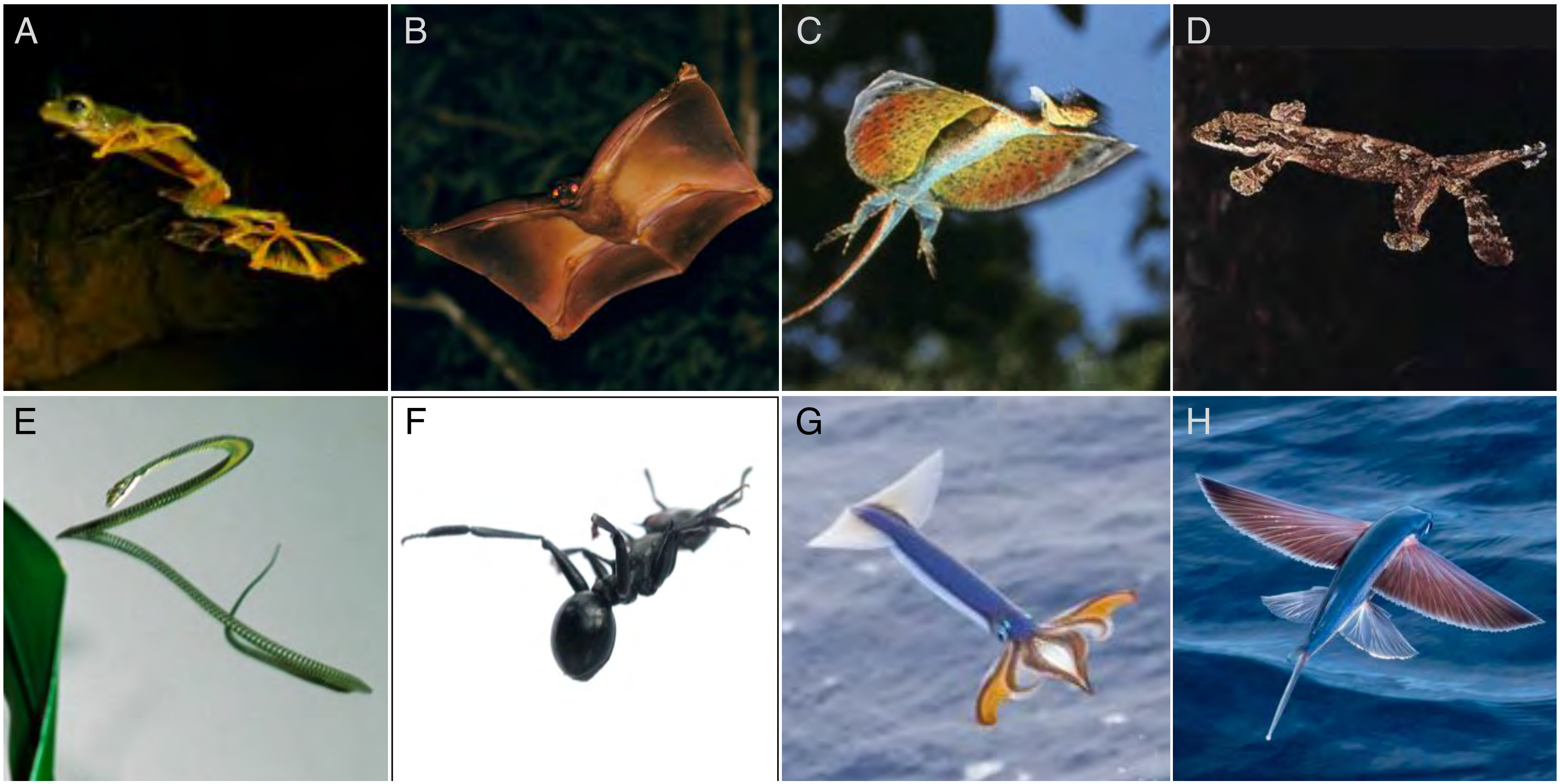
Flying snakes: separatrix behavior



saddle-node bifurcation at θ^* along shallowing manifold

Animal gliders

Common framework for understanding the diversity of animal gliders



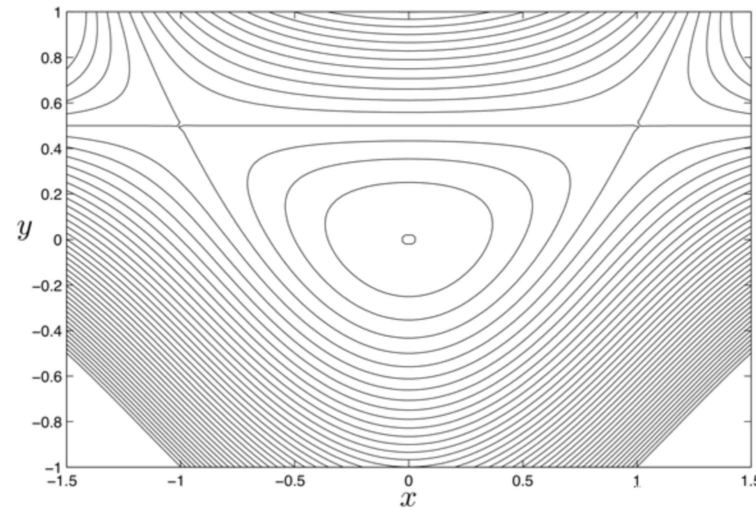
work underway with Jake Socha and Isaac Yeaton

Ship motion and capsize

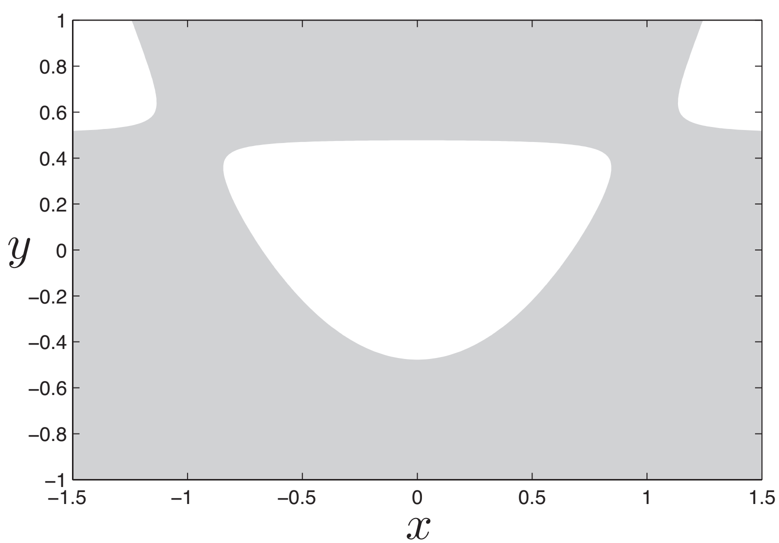
Ship motion and capsize

Tubes leading to capsize

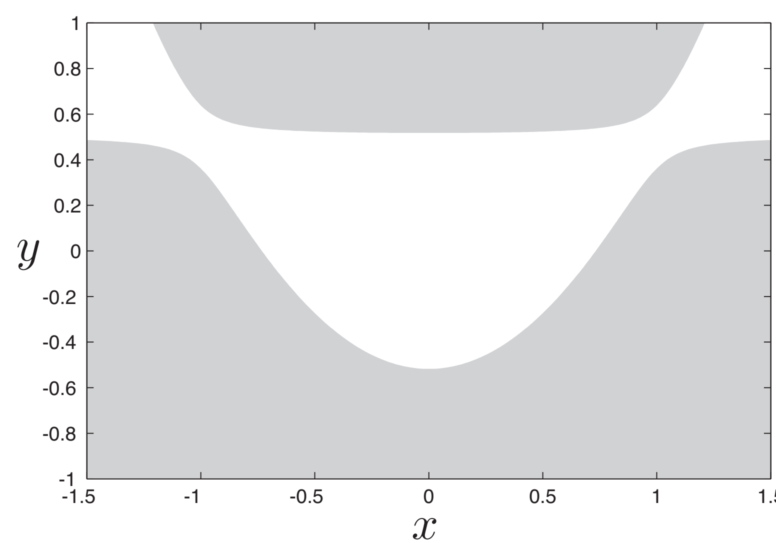
- Model built around Hamiltonian,
  $H = p_x^2/2 + R^2 p_y^2/4 + V(x,y)$,
  where $x = \mathrm{roll}$ and $y = \mathrm{pitch}$ are coupled



$V(x,y)$

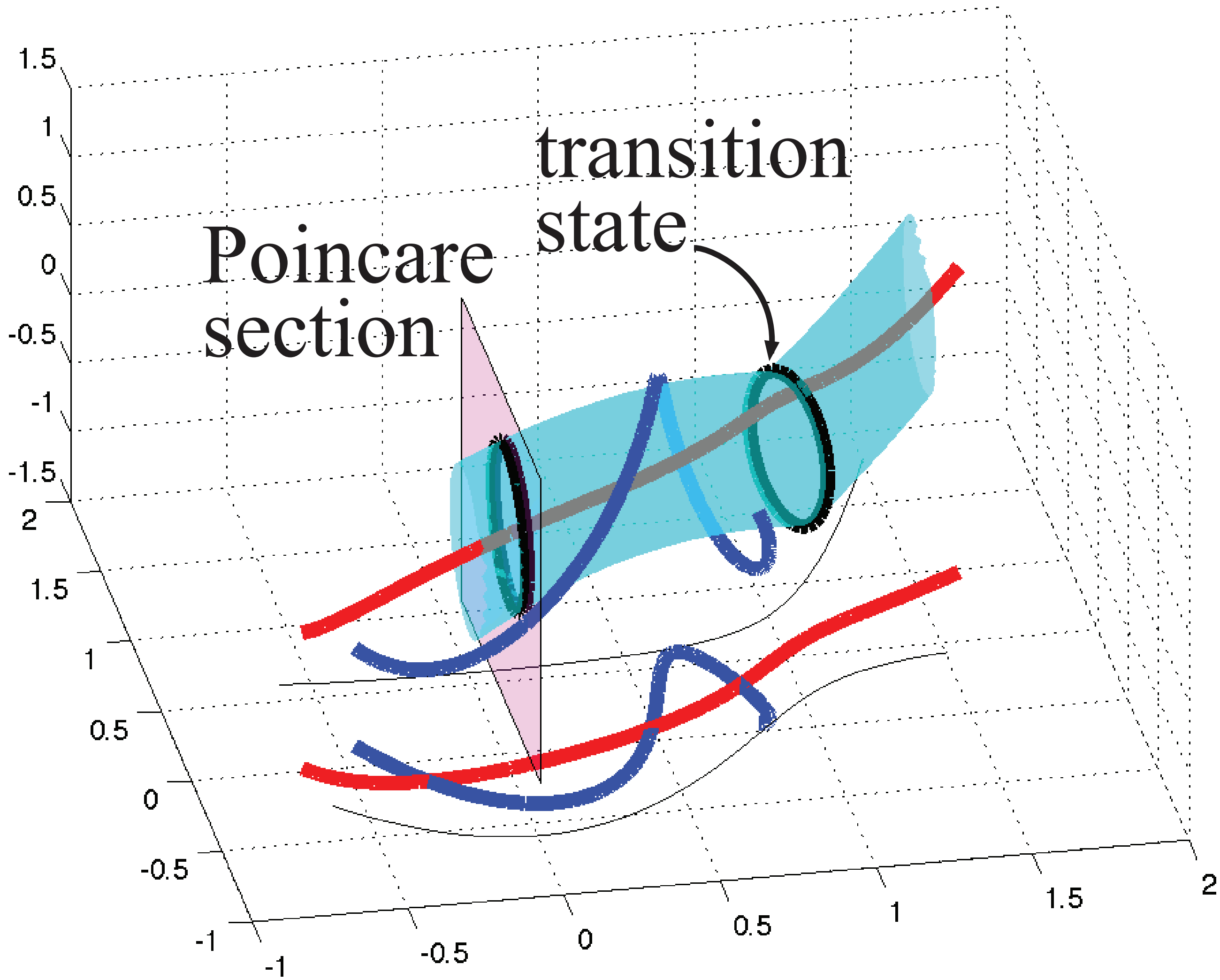


$E < E_c$



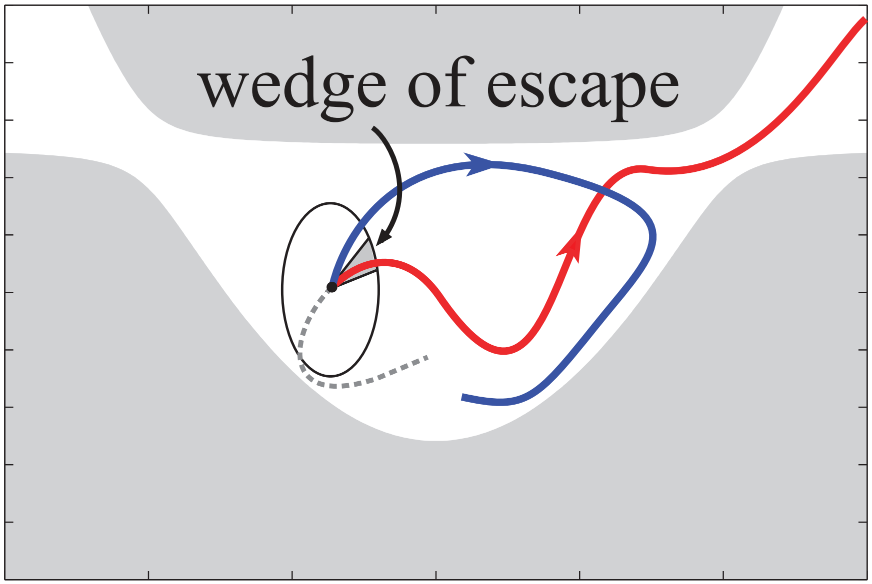
$E > E_c$

Tubes leading to capsize



Tubes leading to capsize

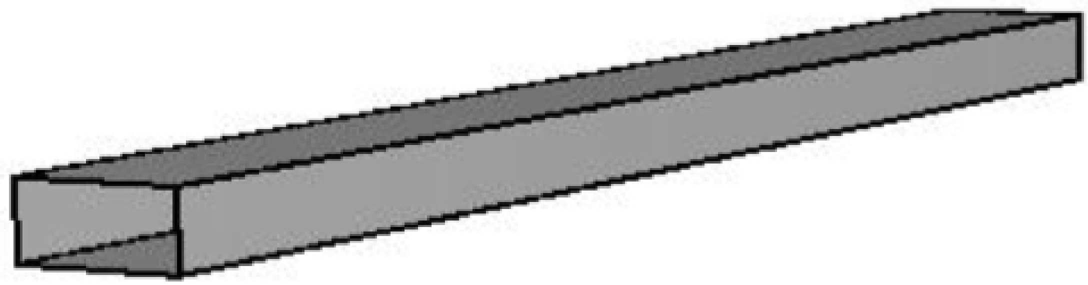
- Wedge of trajectories leading to imminent capsize

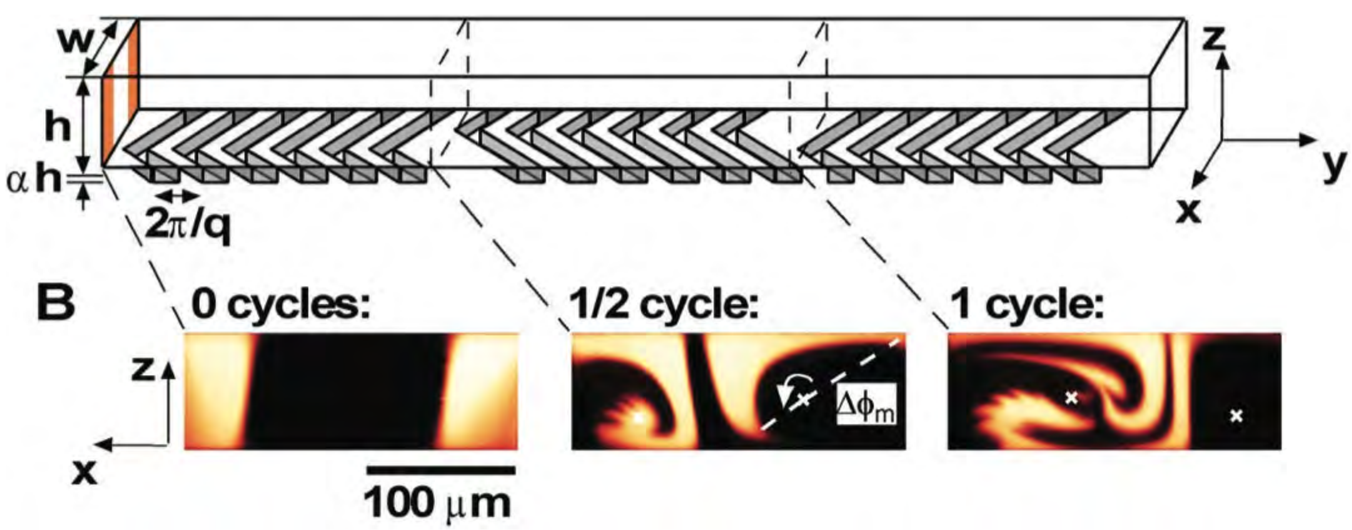


- Tubes are a useful paradigm for predicting capsize even in the presence of random forcing, e.g., random ocean waves
- Could inform **control schemes to avoid capsize** in rough seas

2D fluid example – almost-cyclic behavior

- A microchannel mixer: microfluidic channel with spatially periodic flow structure, e.g., due to grooves or wall motion[1]
- How does behavior change with parameters?

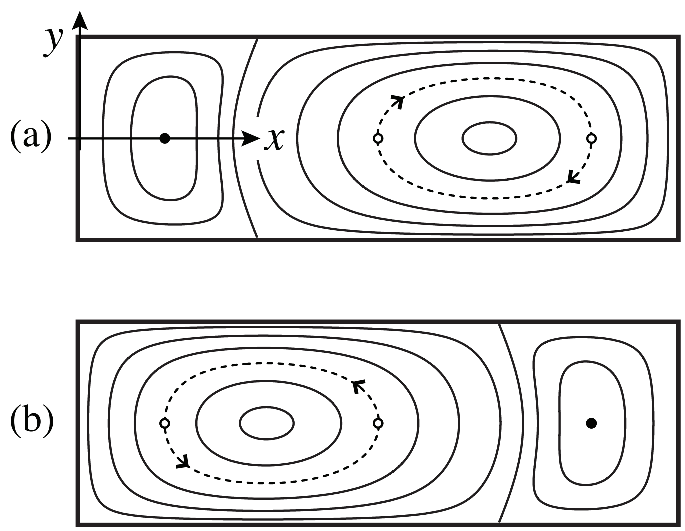




[1]Stroock et al. [2002], Stremler et al. [2011]

2D fluid example – almost-cyclic behavior

- A microchannel mixer: modeled as periodic Stokes flow



streamlines for $\tau_f = 1$

tracer blob ($\tau_f > 1$)

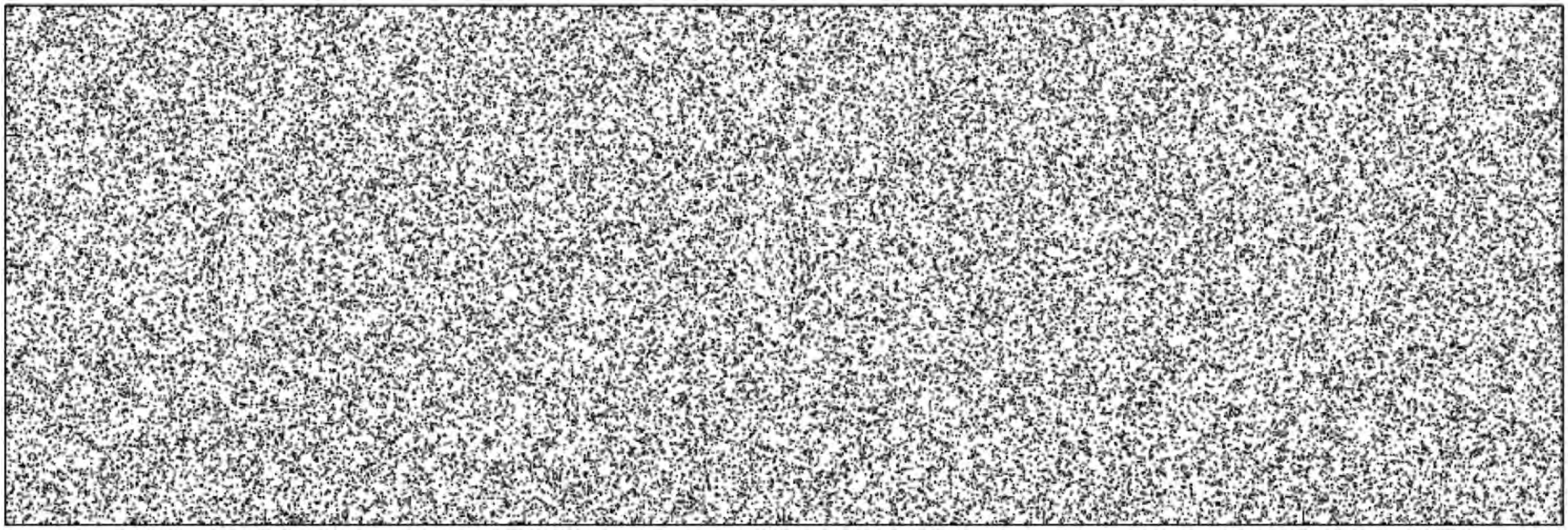
- piecewise constant vector field (repeating periodically)

  top streamline pattern during first half-cycle (duration $\tau_f/2$)

  bottom streamline pattern during second half-cycle (duration $\tau_f/2$), then repeat

- System has parameter τ_f, period of one cycle of flow, which we treat as a bifurcation parameter — there's a critical point $\tau_f^* = 1$

2D fluid example – almost-cyclic behavior



Poincaré section for $\tau_f < 1 \Rightarrow$ no obvious structure!

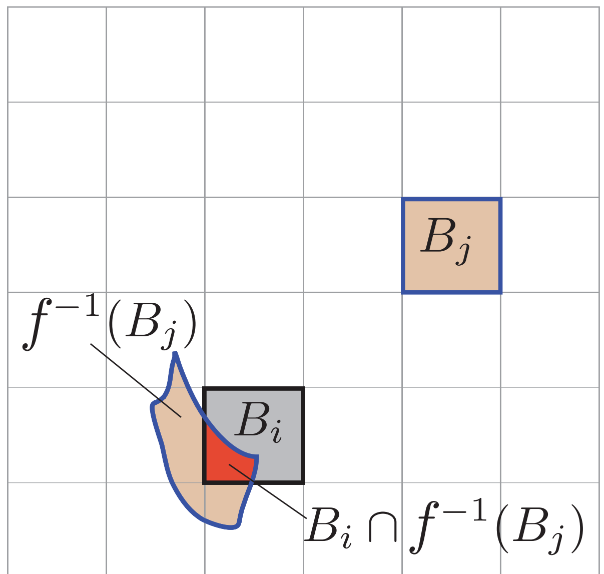
- Poincaré map: Over large range of parameter, no obvious cyclic behavior
- So, is the phase space featureless?

Almost-invariant sets / almost-cyclic sets

- No, we can identify **almost-invariant sets** (AISs) and **almost-cyclic sets** (ACSs)[1]
- Create box partition of phase space $\mathcal{B} = \{B_1, \ldots B_q\}$, with q large
- Consider a q-by-q **transition (Ulam) matrix**, P, where

$$P_{ij} = \frac{m(B_i \cap f^{-1}(B_j))}{m(B_i)},$$

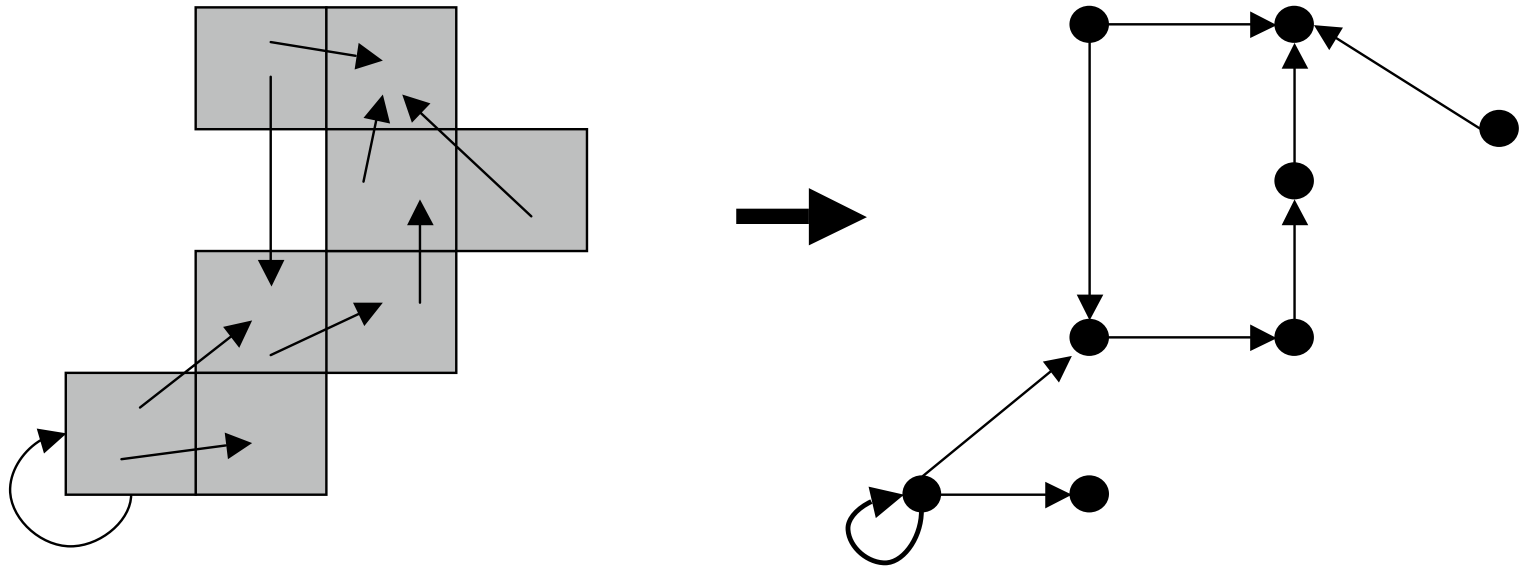
the *transition probability* from B_i to B_j using, e.g., $f = \phi_t^{t+T}$, often computed numerically



- P approximates $\mathcal{P}$, Perron-Frobenius transfer operator — which evolves densities, ν, over one iterate of f, as $\mathcal{P}\nu$
- Typically, we use a reversibilized operator R, obtained from P

[1] Dellnitz & Junge [1999], Froyland & Dellnitz [2003]

Identifying AISs by graph- or spectrum-partitioning



- P admits graph representation where nodes correspond to boxes B_i and transitions between them are edges of a directed graph
- Graph partitioning methods can be applied[1]
- can obtain AISs/ACSs and transport between them
- spectrum-partitioning as well (eigenvectors of large eigenvalues)[2]

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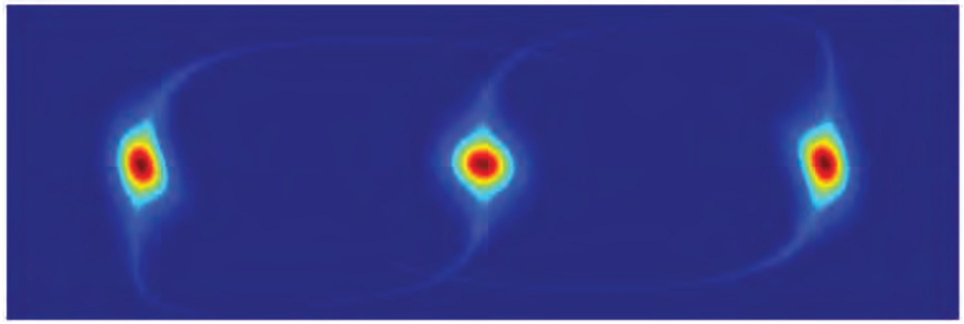
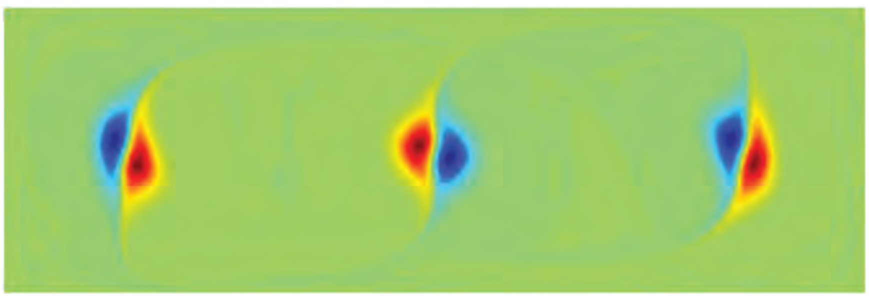
[1]Dellnitz, Junge, Koon, Lekien, Lo, Marsden, Padberg, Preis, Ross, Thiere [2005] Int. J. Bif. Chaos
[2]Dellnitz, Froyland, Sertl [2000] Nonlinearity

Identifying AISs by graph- or spectrum-partitioning

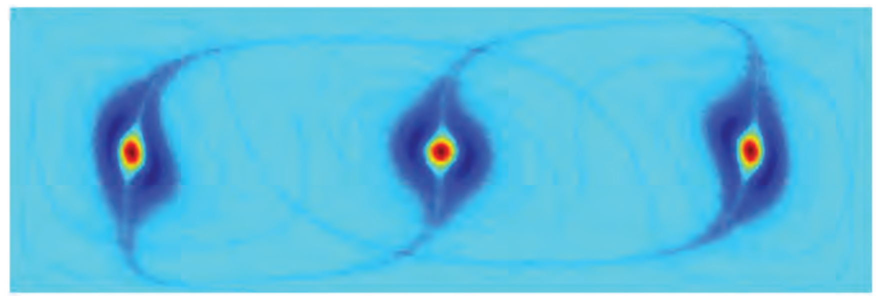
Top eigenvectors of transfer operator reveal structure



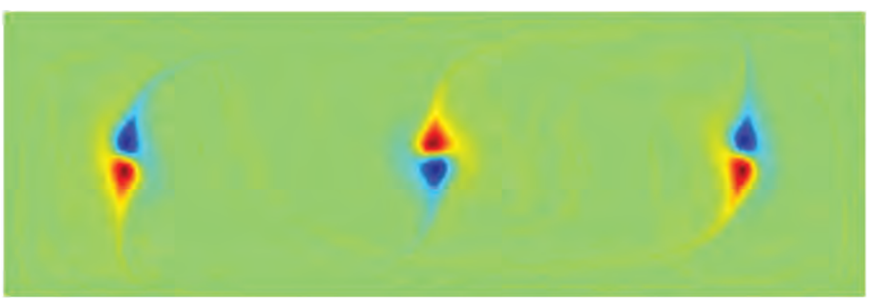
ν_2



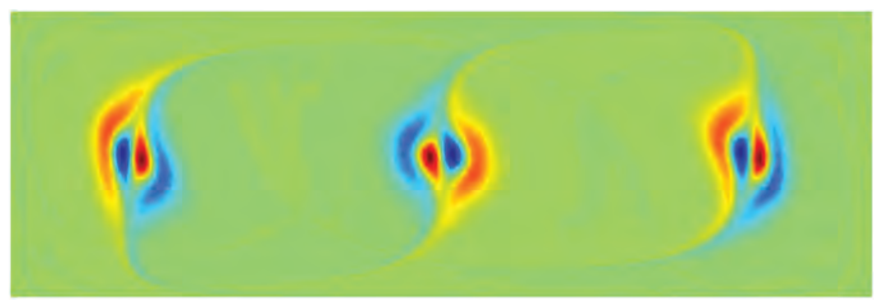
ν_3



ν_4

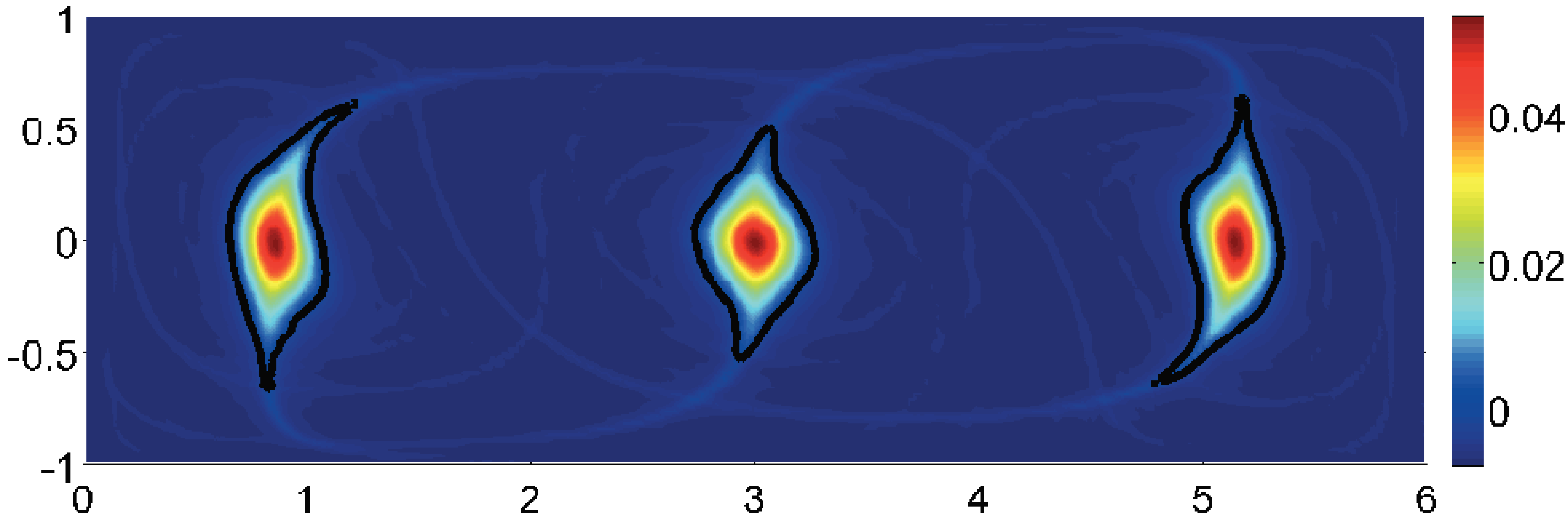


ν_5



ν_6

Almost-cyclic sets stir fluid like rods



The zero contour (black) is the boundary between the two almost-invariant sets.

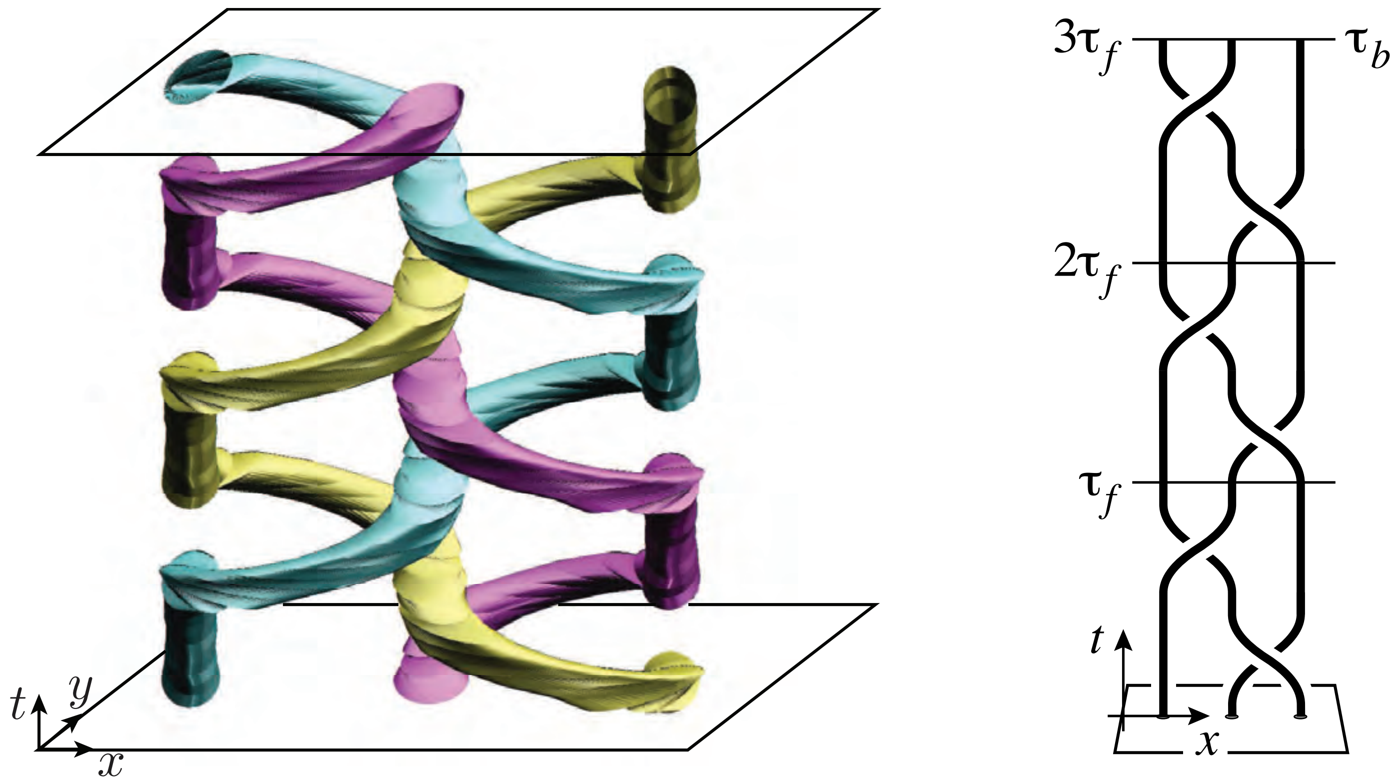
- Three-component AIS made of 3 ACSs each of period 3

Almost-cyclic sets stir fluid like rods

Almost-cyclic sets, in effect, stir the surrounding fluid like 'ghost rods'

In fact, there's a theorem (Thurston-Nielsen classification theorem) that provides a topological lower bound on the mixing based on braiding in space-time

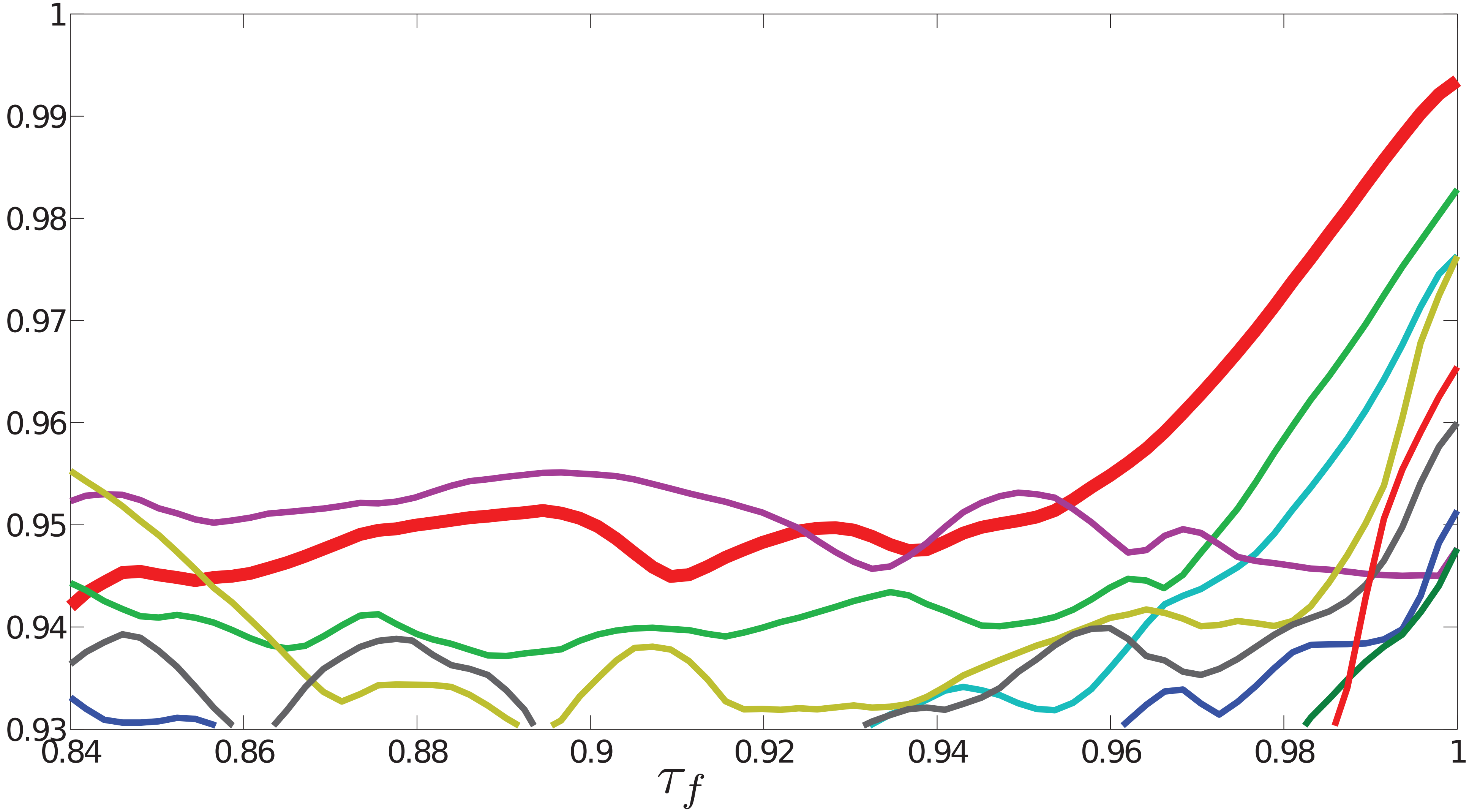
Almost-cyclic sets stir fluid like rods



Thurston-Nielsen theorem applies only to periodic points
— But seems to work, even for approximately cyclic blobs of fluid[1]

[1] Stremler, Ross, Grover, Kumar [2011] Phys. Rev. Lett.

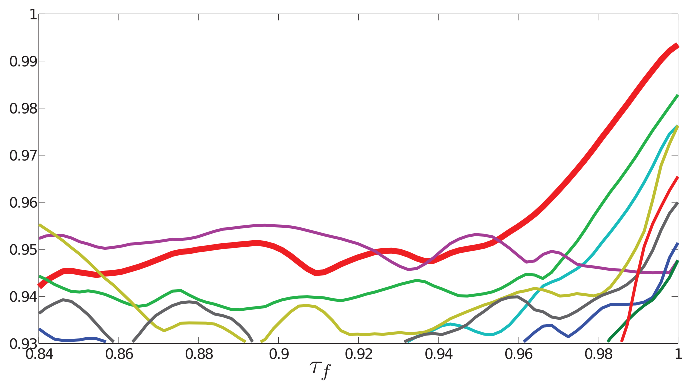
Eigenvalues/eigenvectors vs. parameter



Top eigenvalues of transfer operator as parameter τ_f changes

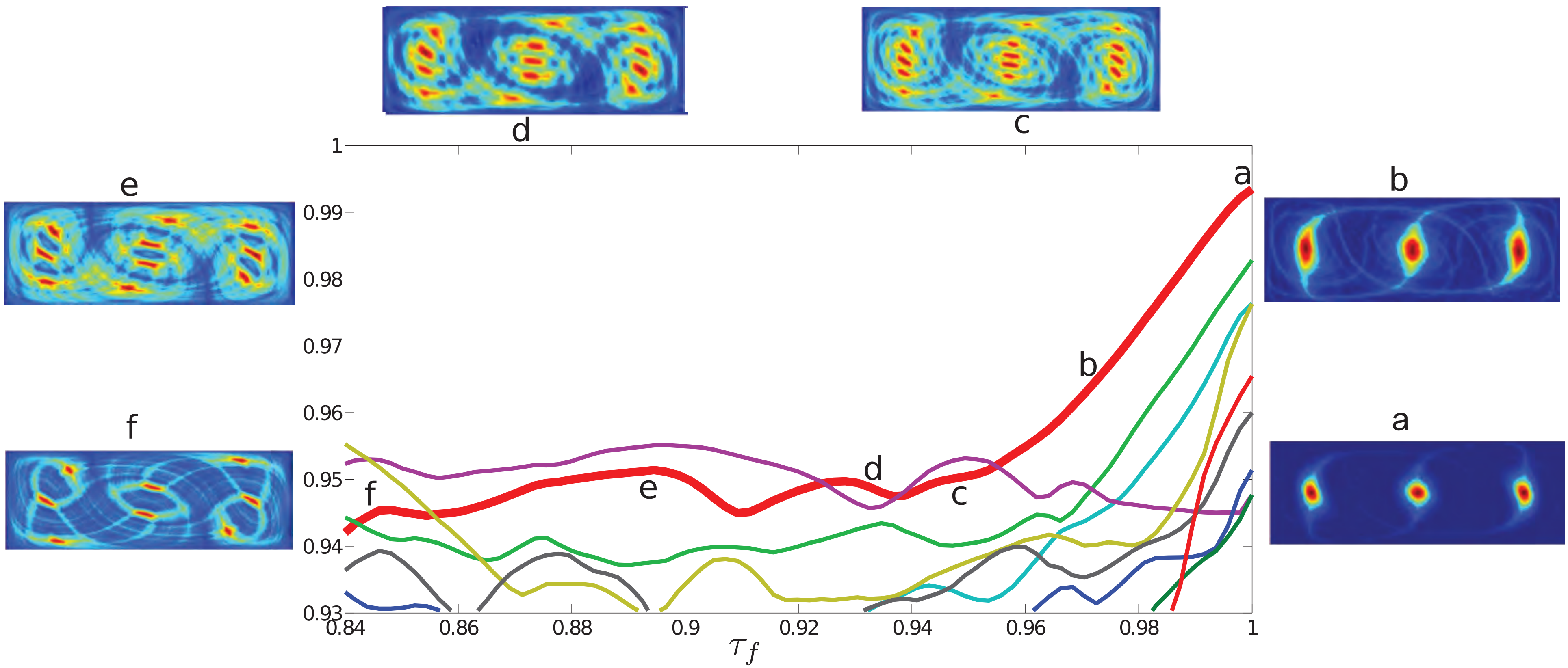
Lines colored according to continuity of eigen**vector**

Eigenvalues/eigenvectors vs. parameter



Genuine eigenvalue crossings? Eigenvalues generically avoid crossings if there is no symmetry present (Dellnitz, Melbourne, 1994)

Eigenvalues/eigenvectors vs. parameter



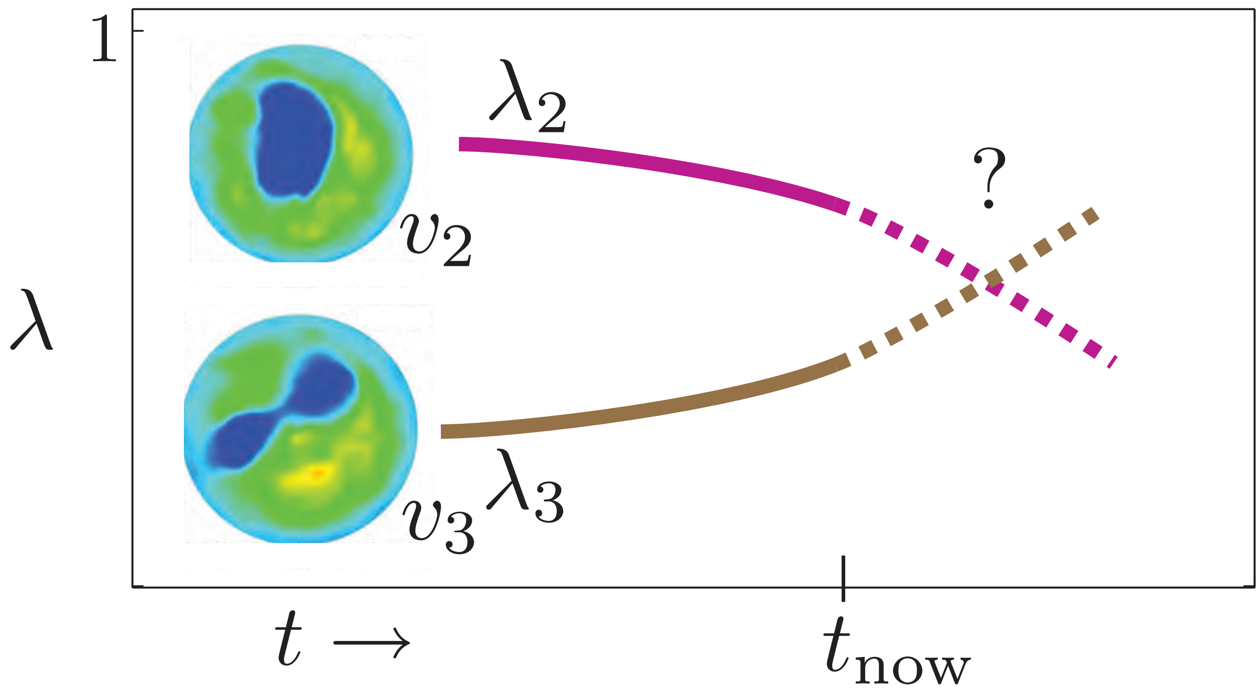
change in eigenvector along thick red branch (a to f), as τ_f decreases.

Grover, Ross, Stremler, Kumar [2012] Chaos

Predict critical transitions in geophysical transport?

Ozone data (Lekien and Ross [2010] Chaos)

Predict critical transitions in geophysical transport?



- Different eigenmodes can correspond to dramatically different behavior.
- Some eigenmodes increase in importance while others decrease
- Can we predict dramatic changes in system behavior?
- e.g., predicting major changes in geophysical transport patterns??

Chaotic fluid transport: aperiodic setting

- Identify regions of high sensitivity of initial conditions
- The finite-time Lyapunov exponent (FTLE),

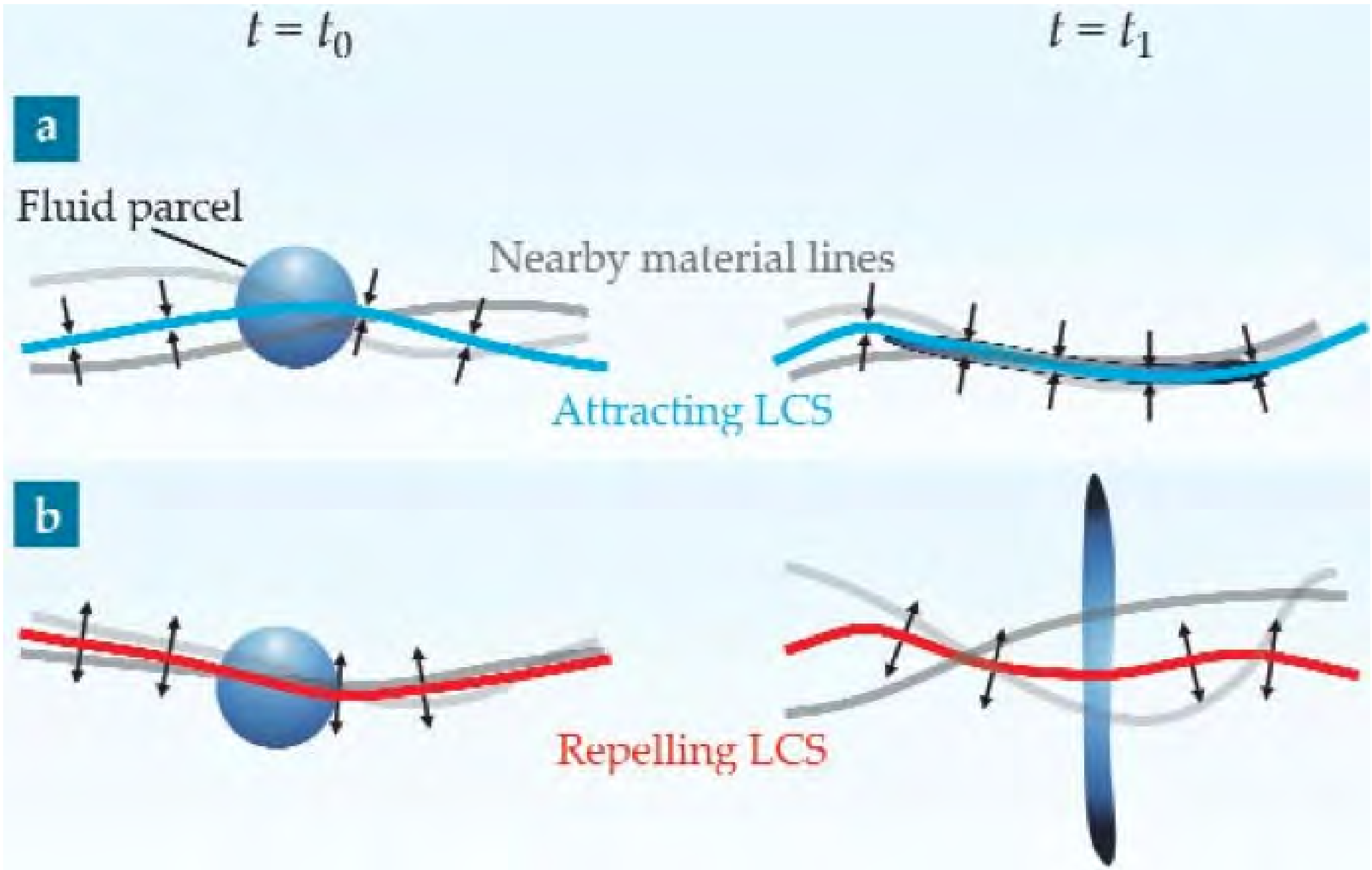
$$\sigma_t^T(x) = \frac{1}{|T|} \log \left\| D\phi_t^{t+T}(x) \right\|$$

  measures the maximum stretching rate over the interval T of trajectories starting near the point x at time t
- Ridges of σ_t^T reveal hyperbolic codim-1 surfaces; finite-time stable/unstable manifolds; '**Lagrangian coherent structures**' or **LCSs**[2]

[2] cf. Bowman, 1999; Haller & Yuan, 2000; Haller, 2001; Shadden, Lekien, Marsden, 2005

Repelling and attracting structures

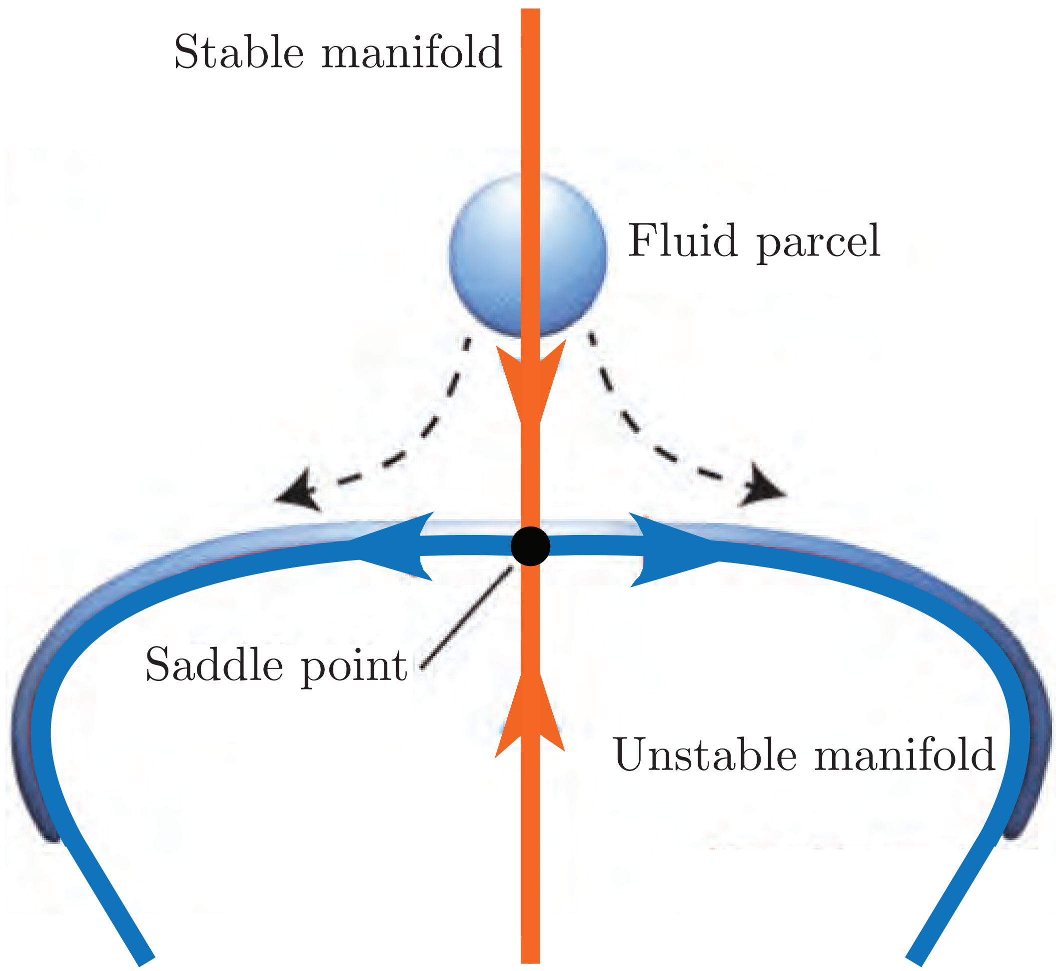
- attracting structures for $T < 0$
  repelling structures for $T > 0$



Peacock and Haller [2013]

Repelling and attracting structures

- Stable manifolds are repelling structures
  Unstable manifolds are attracting structures



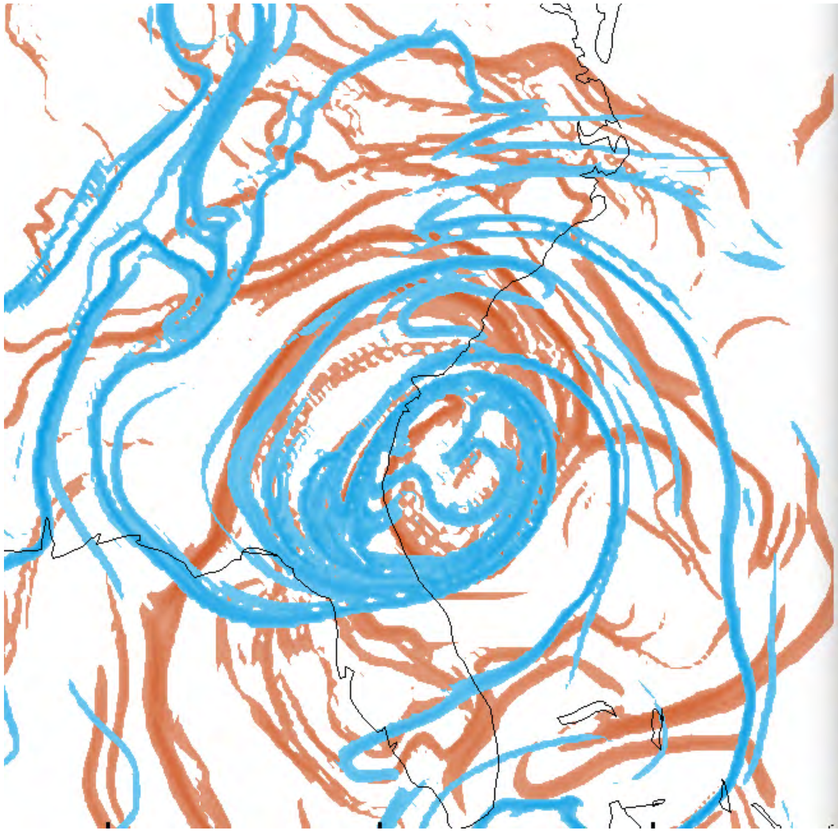
Peacock and Haller [2013]

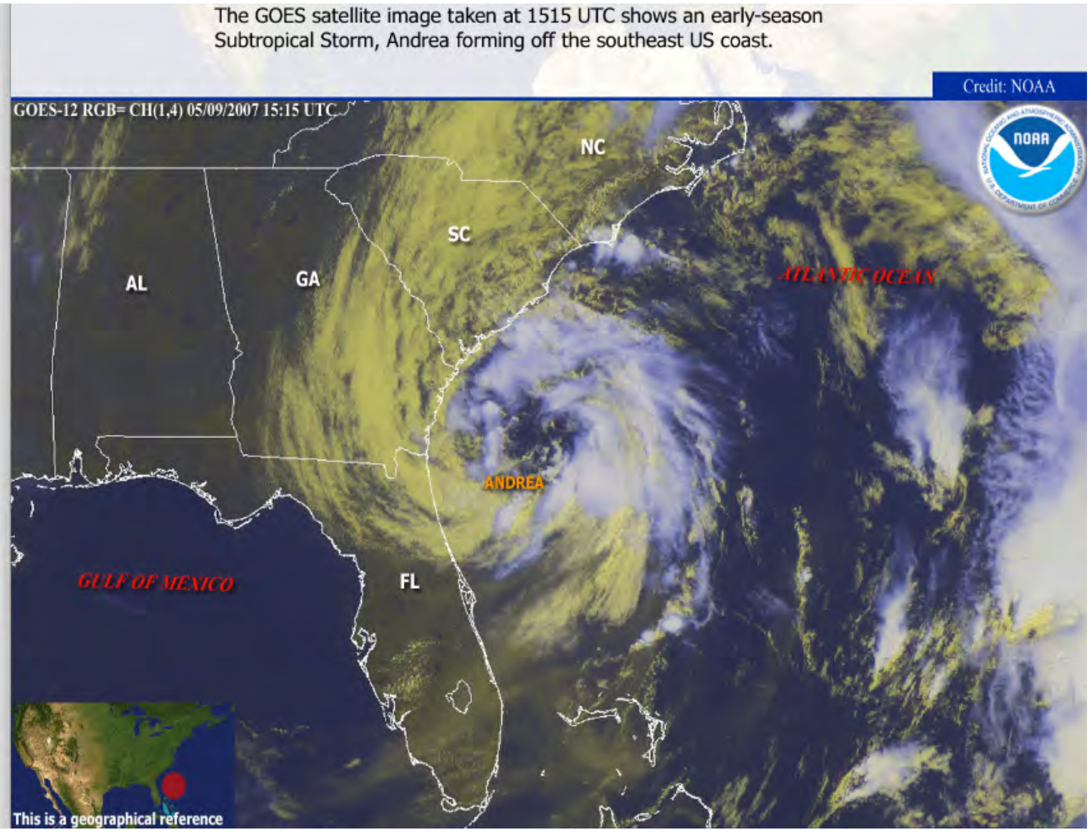
Atmospheric flows: continental U.S.

LCSs: orange = repelling, blue = attracting

2D curtain-like structures bounding air masses

Atmospheric flows and lobe dynamics





orange = repelling LCSs, blue = attracting LCSs

satellite

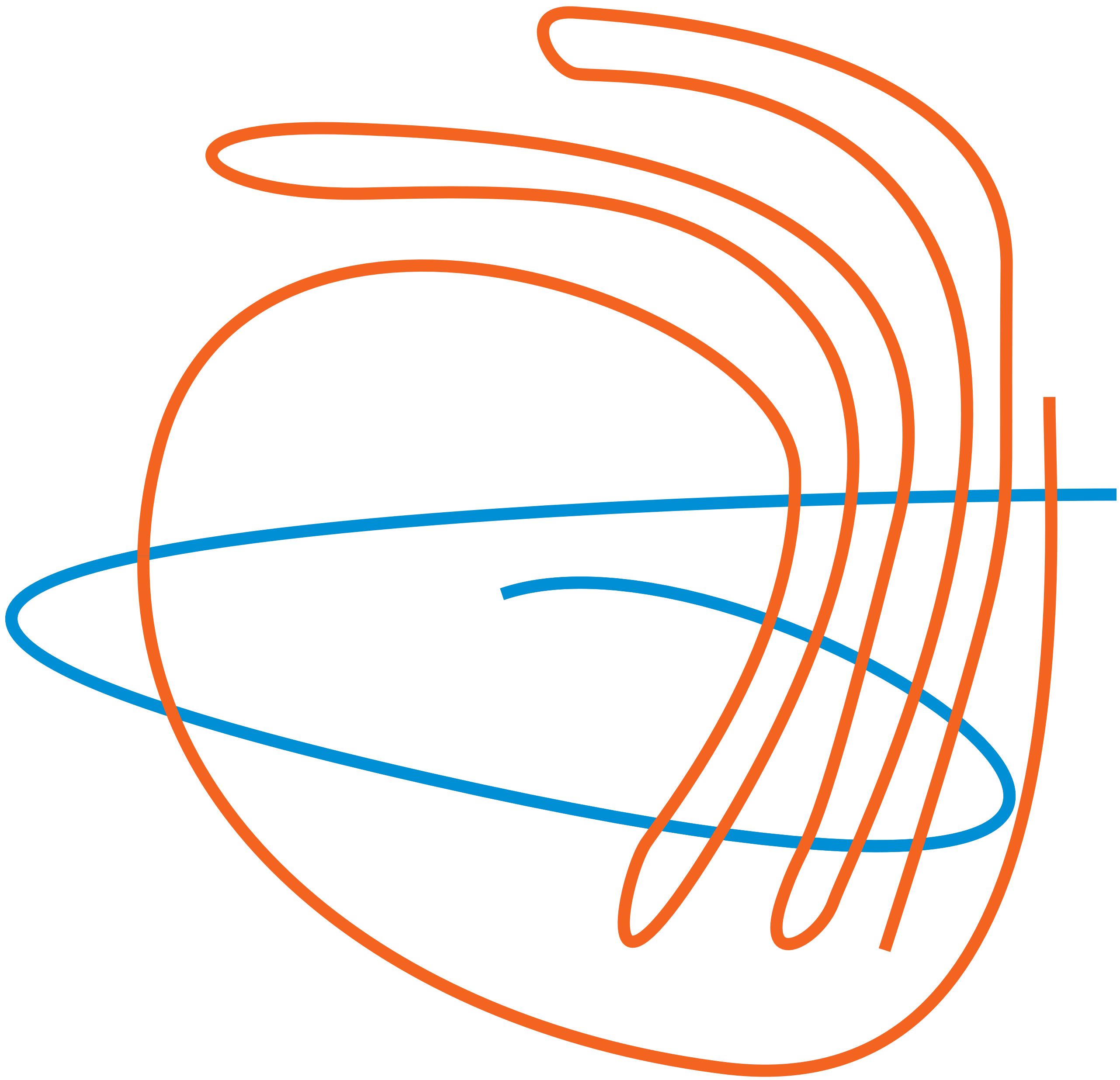
Andrea, first storm of 2007 hurricane season

cf. Sapsis & Haller [2009], Du Toit & Marsden [2010], Lekien & Ross [2010], Ross & Tallapragada [2012]

Atmospheric flows and lobe dynamics

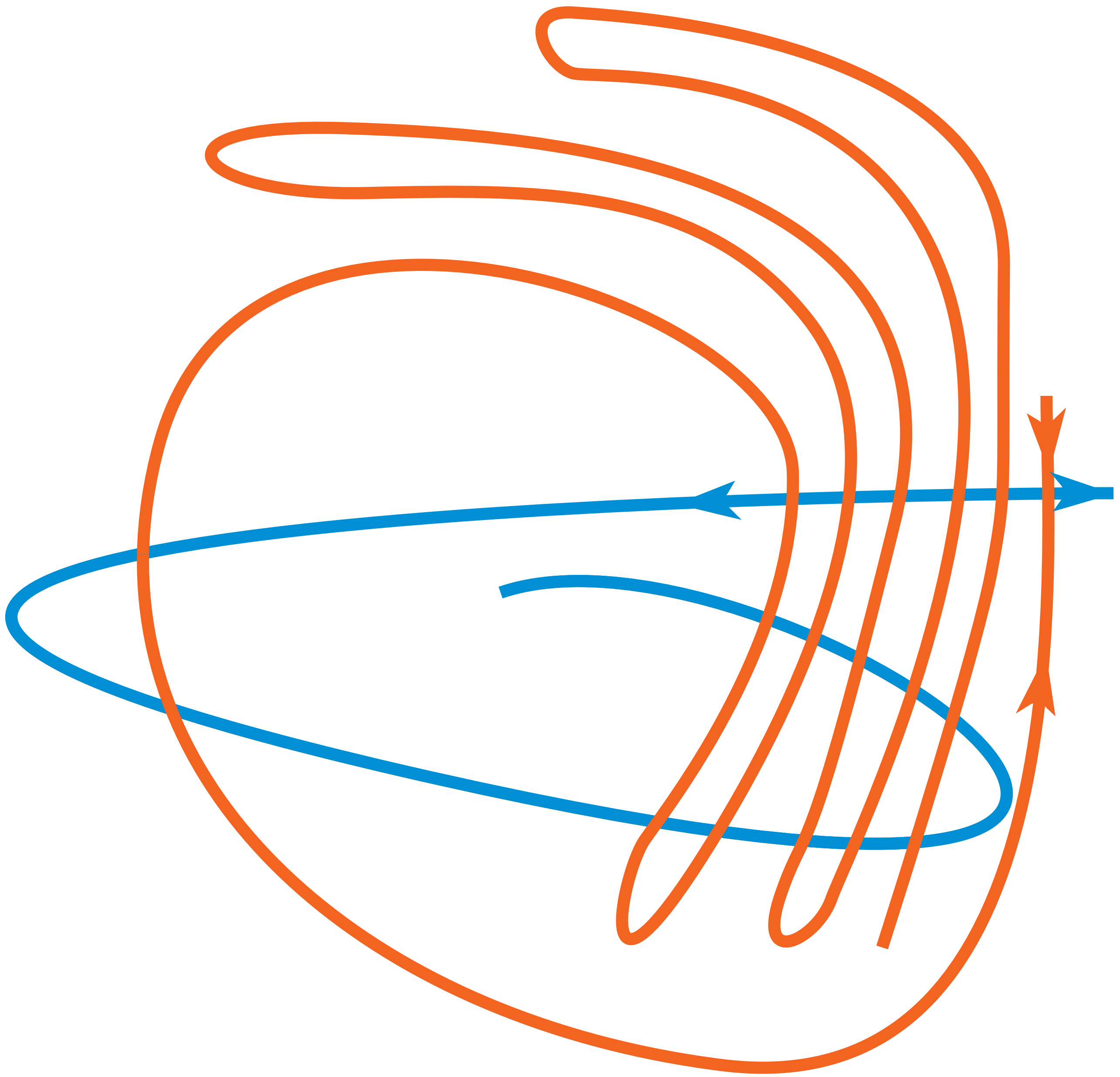
Andrea at one snapshot; LCS shown (orange = repelling, blue = attracting)

Atmospheric flows and lobe dynamics



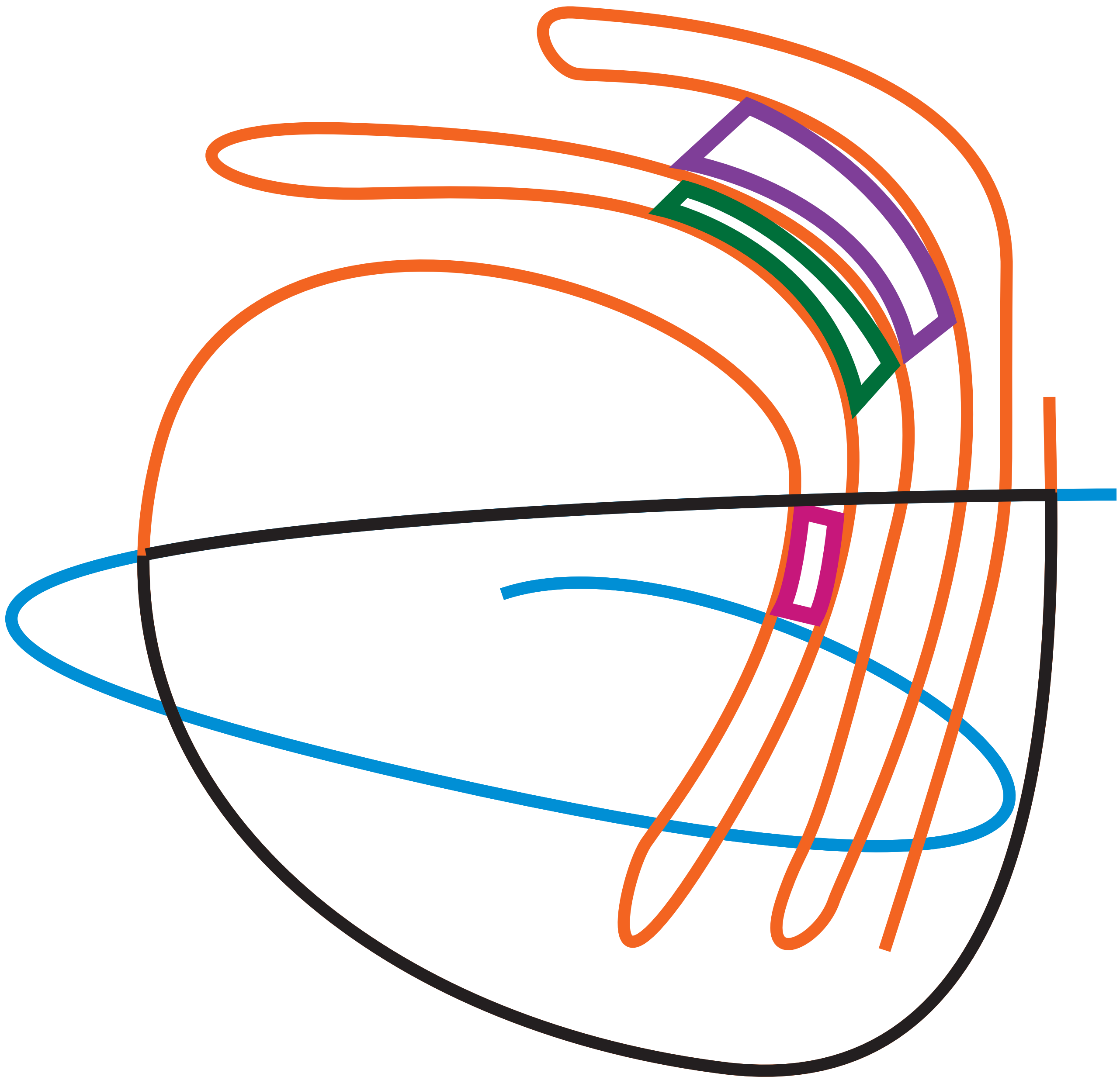
orange = repelling (stable manifold), blue = attracting (unstable manifold)

Atmospheric flows and lobe dynamics



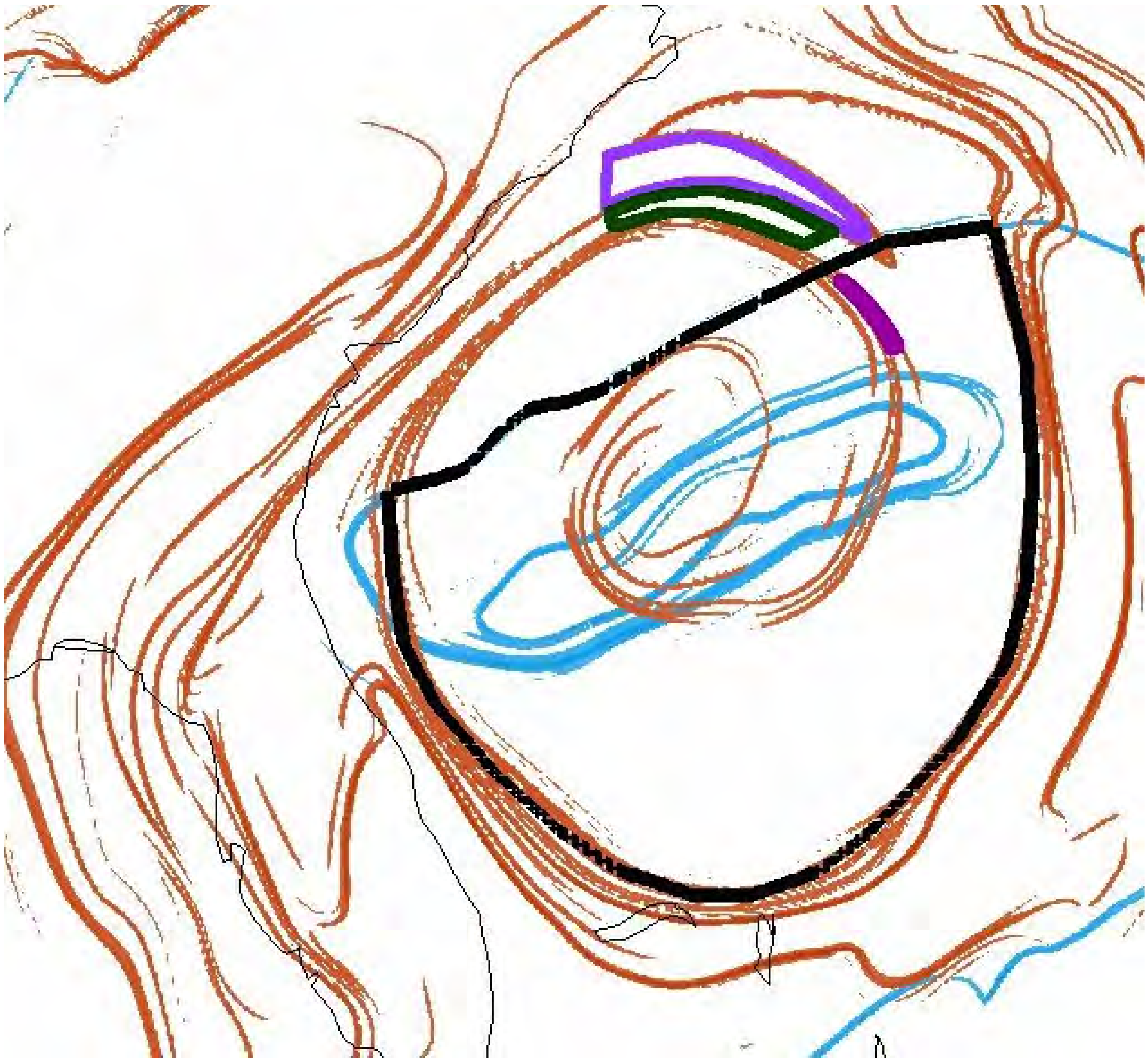
orange = repelling (stable manifold), blue = attracting (unstable manifold)

Atmospheric flows and lobe dynamics



Portions of lobes colored; magenta = outgoing, green = incoming, purple = stays out

Atmospheric flows and lobe dynamics

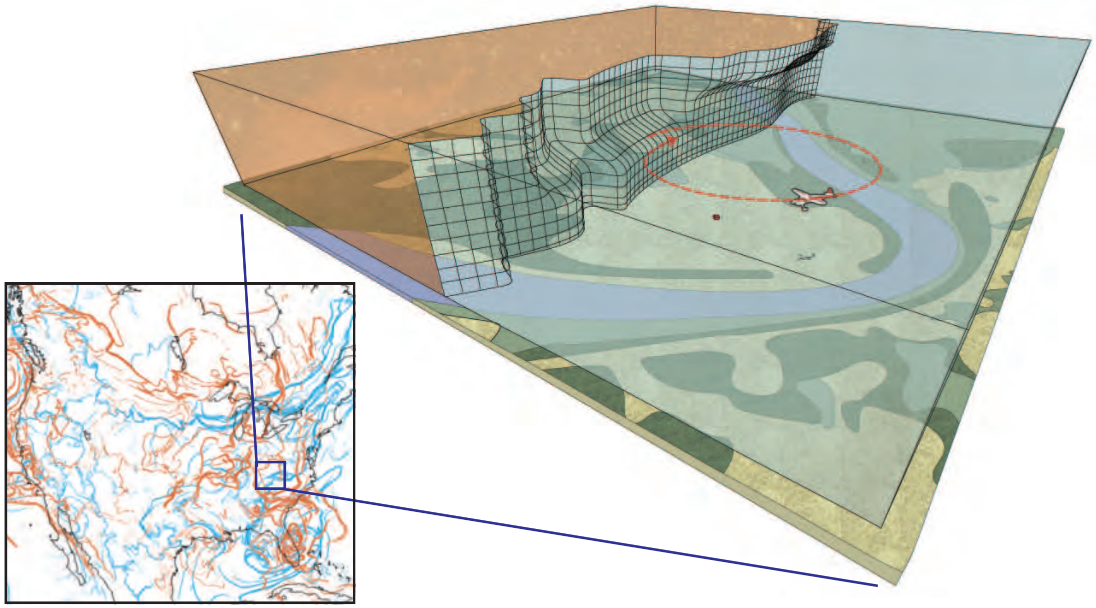


Portions of lobes colored; magenta = outgoing, green = incoming, purple = stays out

Atmospheric flows and lobe dynamics

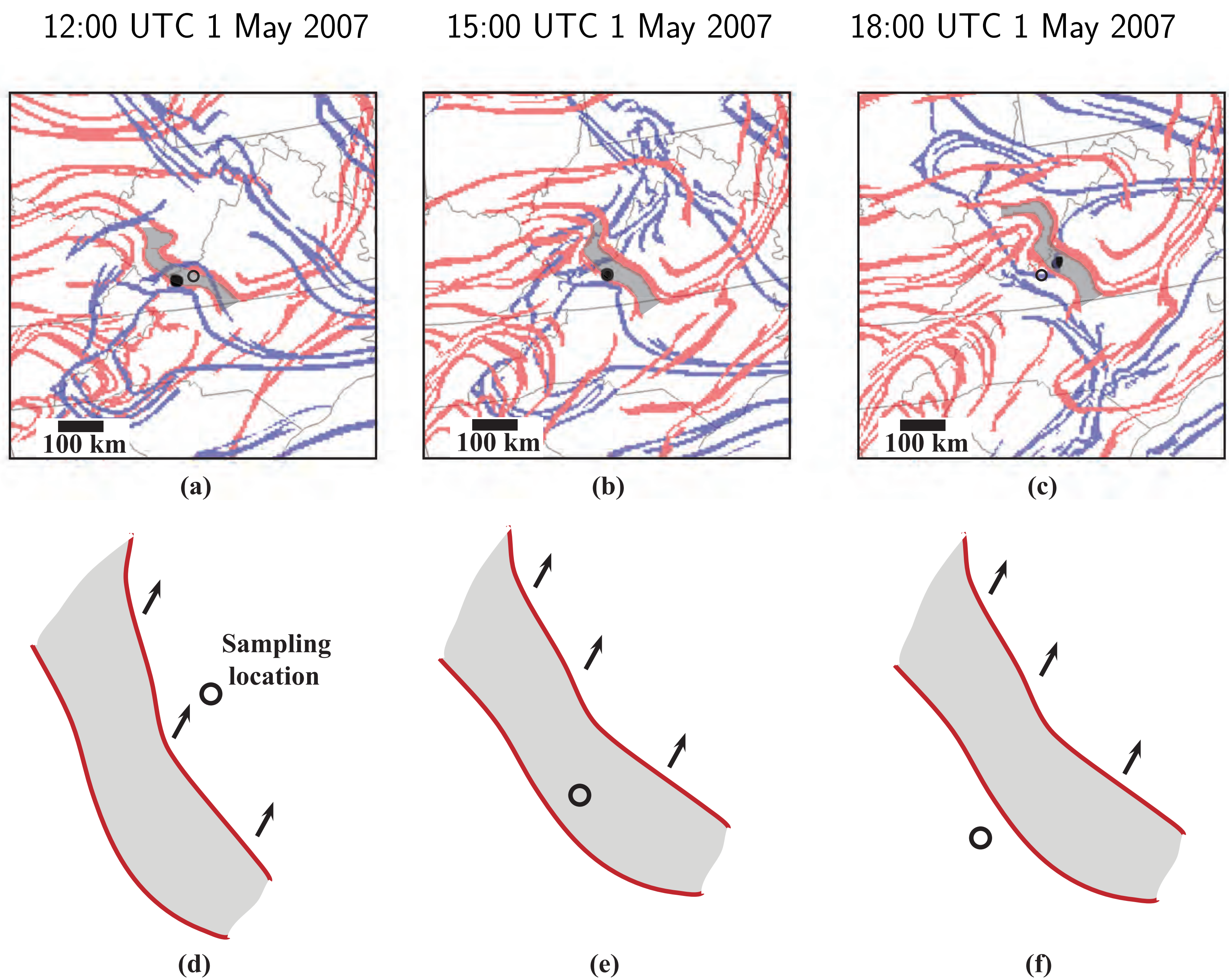
Sets behave as lobe dynamics dictates

Airborne diseases moved about by coherent structures



Joint work with David Schmale, Plant Pathology / Agriculture at Virginia Tech

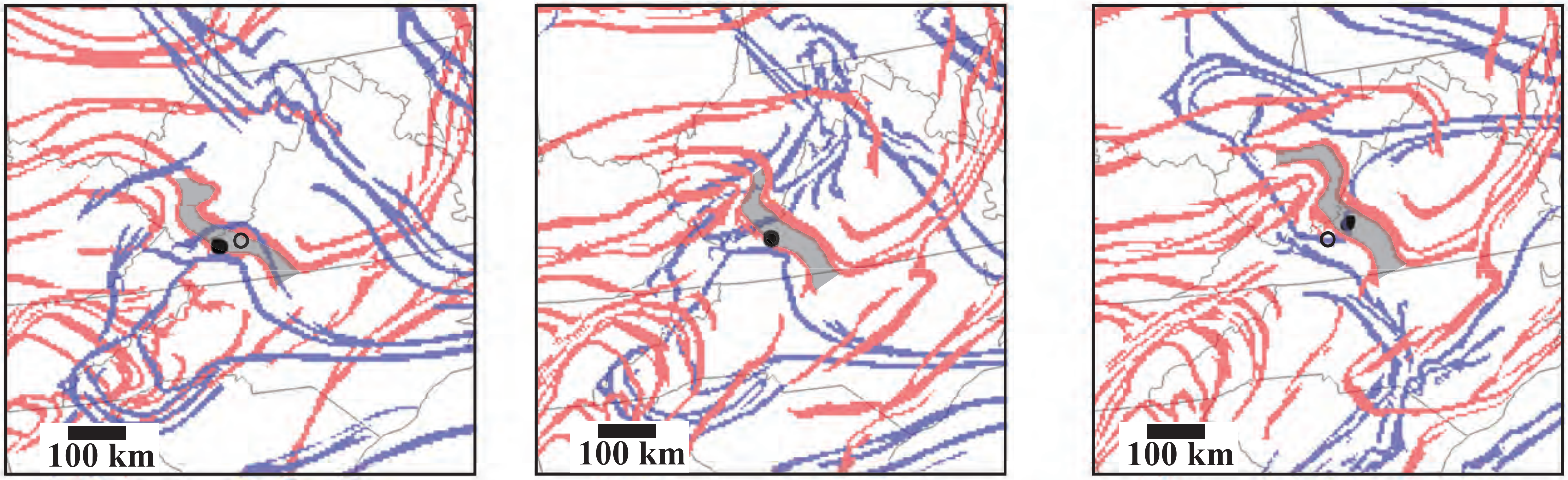
Coherent filament with high pathogen values



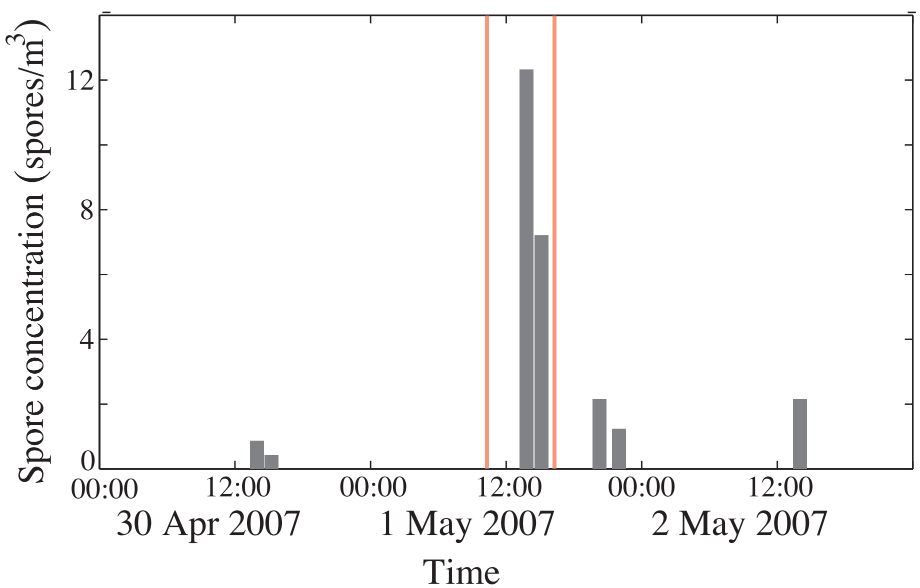
Tallapragada et al [2011] Chaos; Schmale et al [2012] Aerobiologia; BozorgMagham et al [2013] Physica D

Coherent filament with high pathogen values

12:00 UTC 1 May 2007 | 15:00 UTC 1 May 2007 | 18:00 UTC 1 May 2007



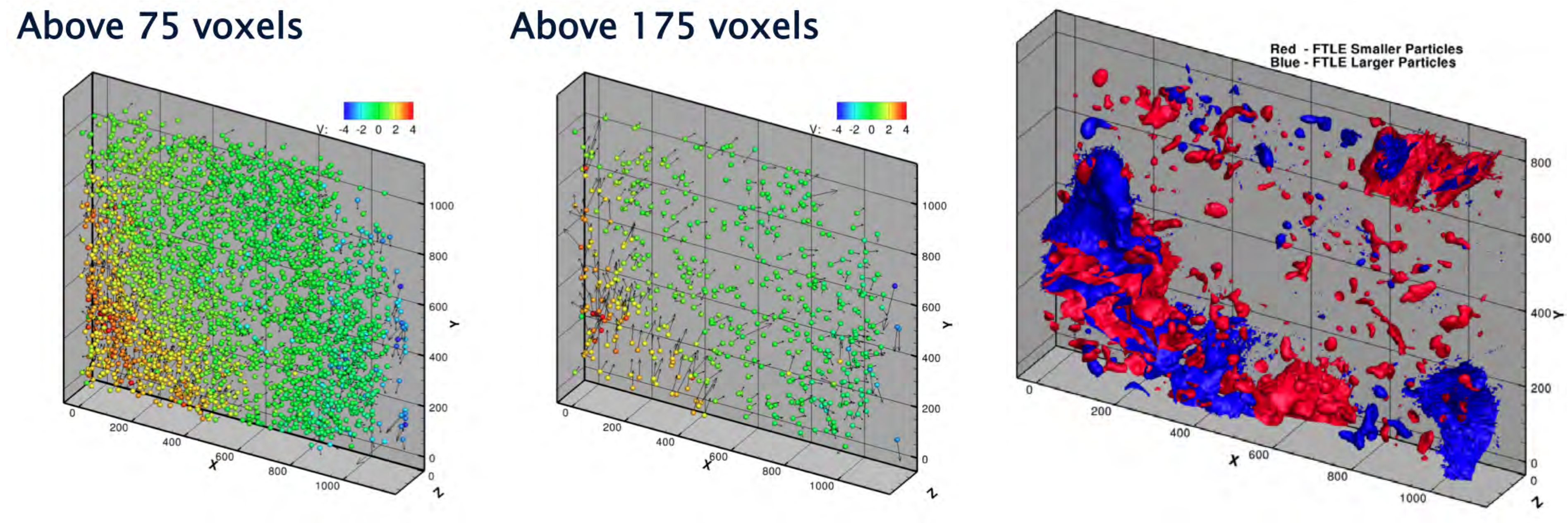
(a) (b) (c)



Tallapragada et al [2011] Chaos; Schmale et al [2012] Aerobiologia; BozorgMagham et al [2013] Physica D

Laboratory fluid experiments

3D Lagrangian structure for non-tracer particles:
— Inertial particle patterns (do not follow fluid velocity)

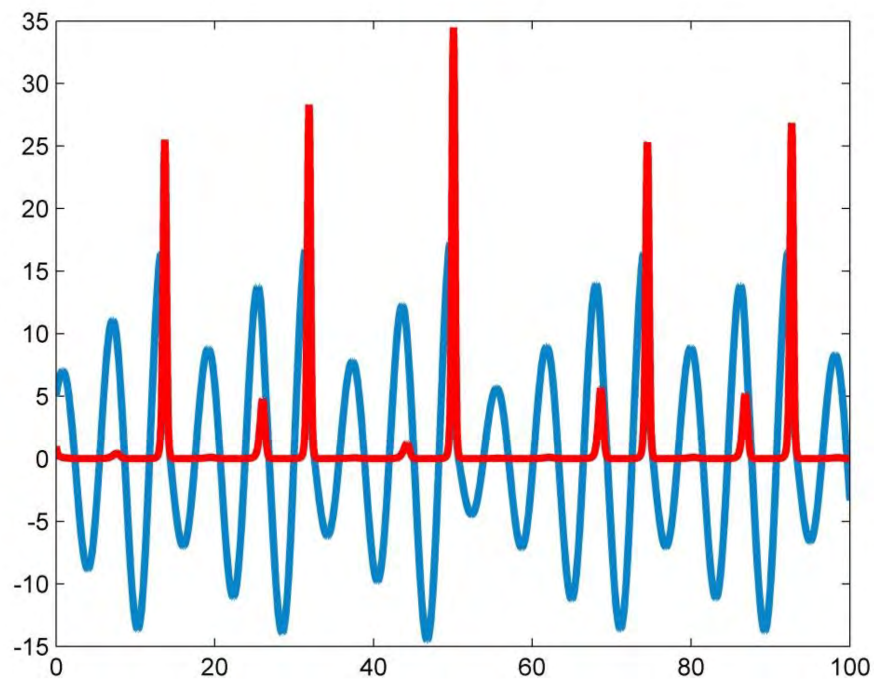


e.g., allows further exploration of physics of multi-phase flows[3]

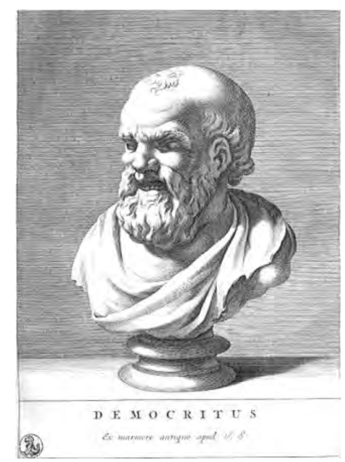
[3]Raben, Ross, Vlachos [2014,2015] Experiments in Fluids

Detecting causality

- Ultimate goal: detecting causality between two time series,



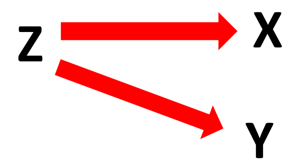
I would rather discover one causal law than be King of Persia.
Democritus (460-370 B.C.)



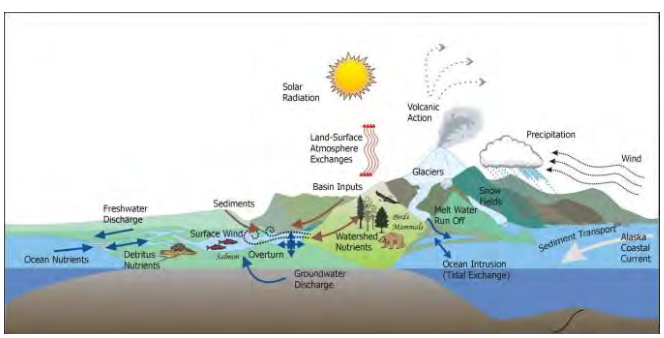
Detecting causality

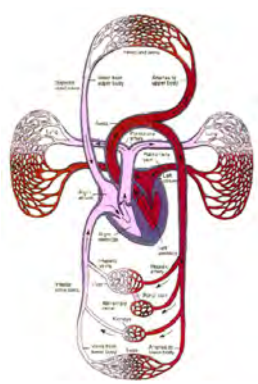
- We have just two time series,
  - Which signal is the driver,
  - Causality direction,

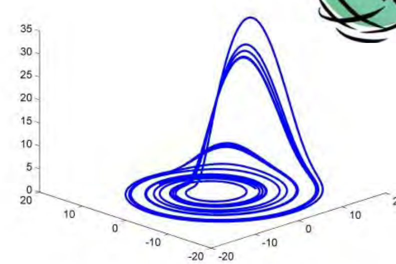
  - Direct causality vs. common external forcing,



  - …
- Signals from:
  - Measurements: temperature, pressure, salinity, velocity, …
  - Maps,
  - ODE's, PDE's, …







Detecting causality – cross-mapping approach

- If two signals are from a same n-D manifold, then there would be some correspondence between shadow manifolds (reconstructed phase spaces),

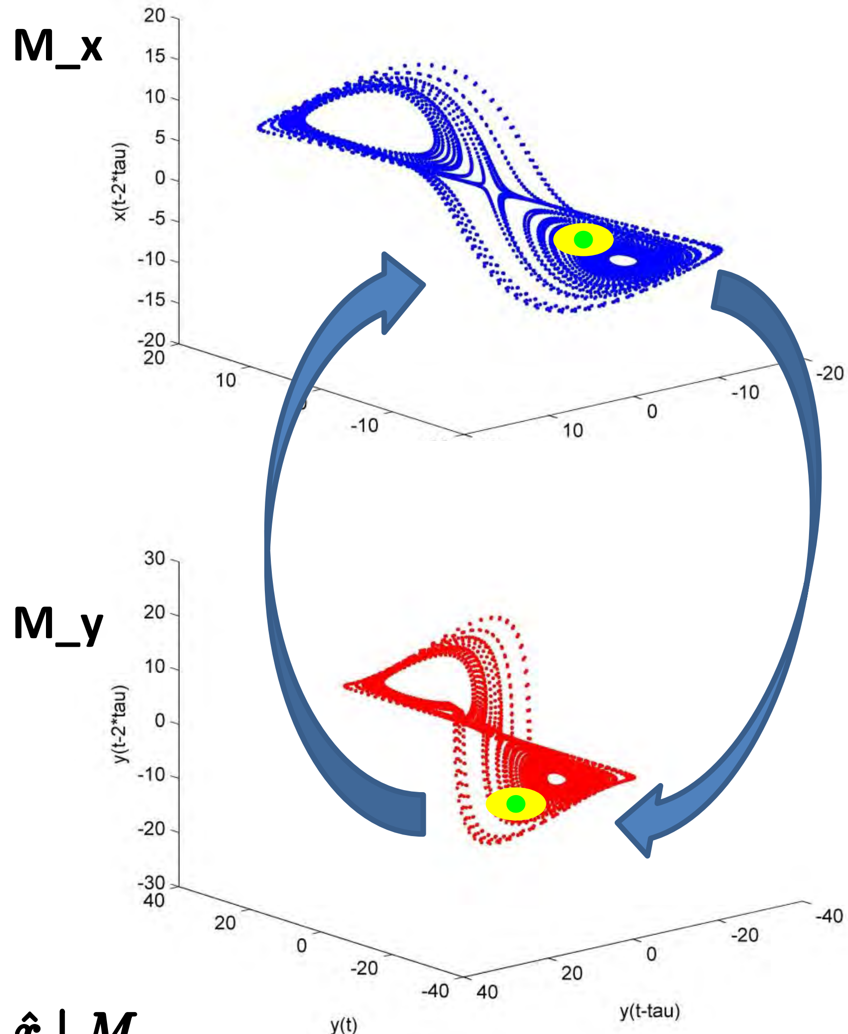
Estimating states across manifolds using nearest neighbors:

- If x(t) causally influences y(t) then **signature** of x(t) inherently exists in y(t),

$$\dot{\mathbf{y}}(t) = \bar{f}(\mathbf{x}, \mathbf{y}, \ldots)$$

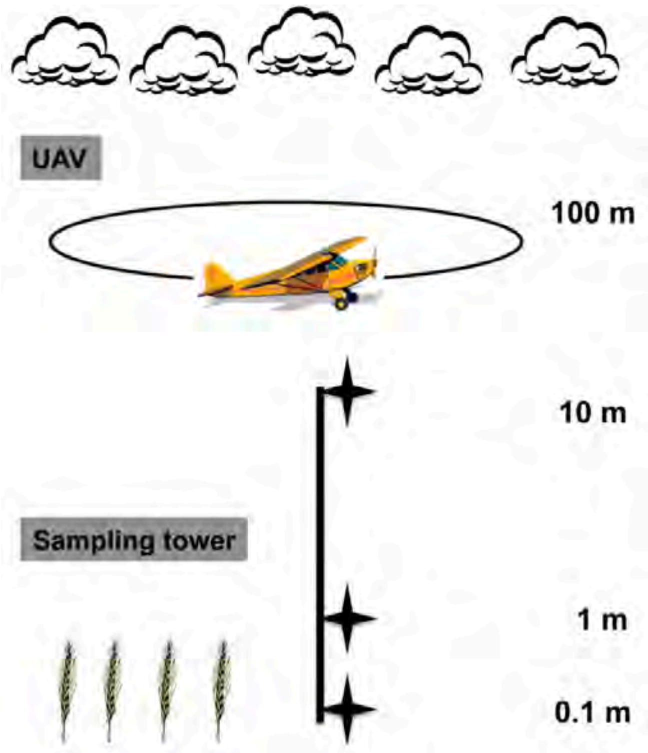
$$y(t+1) = \bar{g}(x(t), y(t))$$

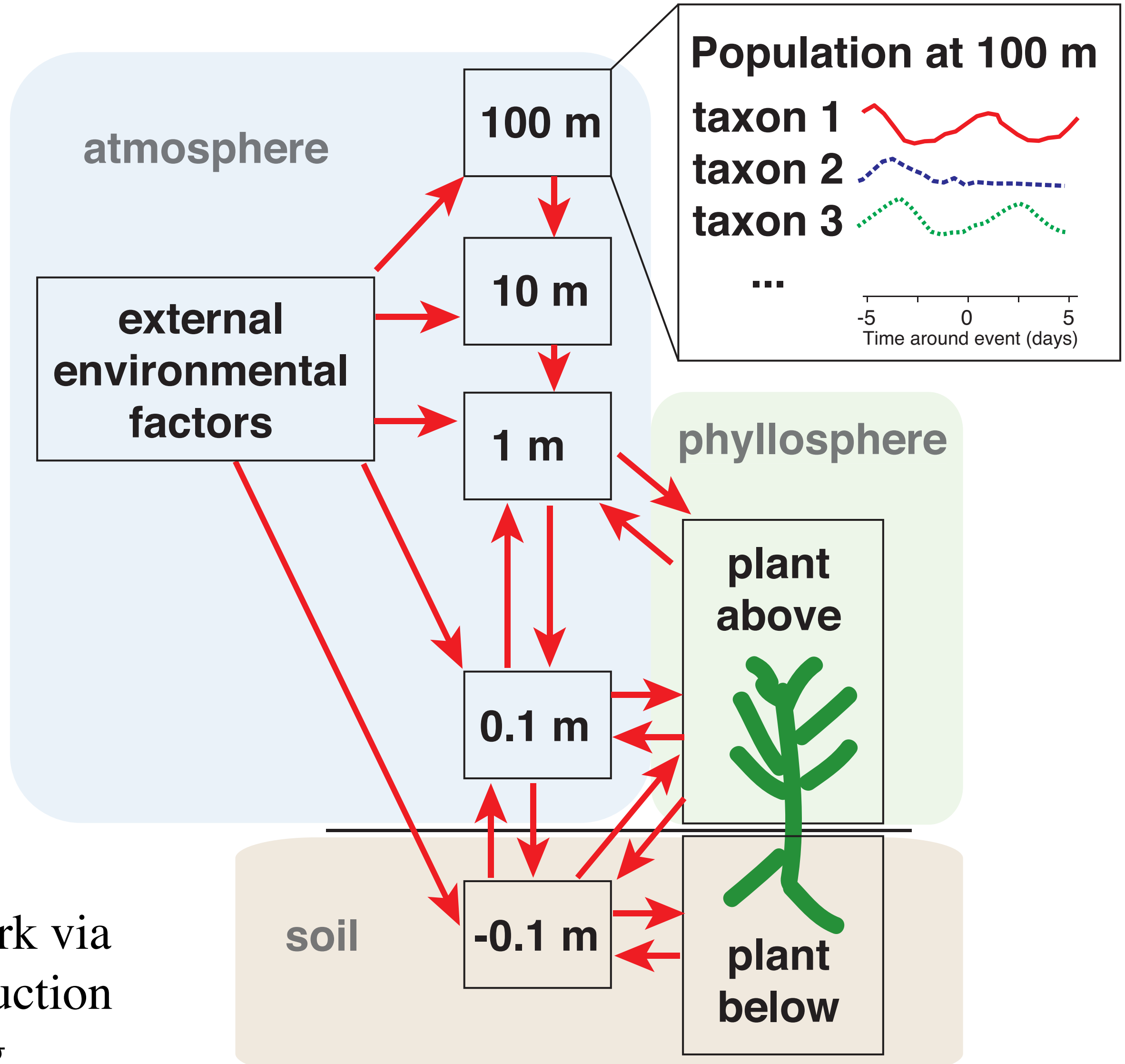
- **If so, historical record of y(t) values can reliably estimate the state of x** → $\hat{x} \mid M_y$



Sugihara et al. 2012

Detecting causality – agricultural example





Determining the causal network via nonlinear state space reconstruction and convergent cross mapping

Phase space geometry — looking forward

- Many inter-related concepts
  - apply to data-based finite-time settings — just more interesting
  - almost-invariant sets, almost-cyclic sets, braids, LCS, transfer operators, phase space transport networks, dependence on parameters, separatrices, basins of stability
- Opportunities:
  - use in control
  - value-added way of viewing and comparing data
  - detecting causality
- Applications:
  - agriculture, ecology
  - predicting critical transitions in geophysical flow patterns
  - comparative biomechanics, ...