Smart Grid

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Smart grid team (impact of CDS)

Caltech

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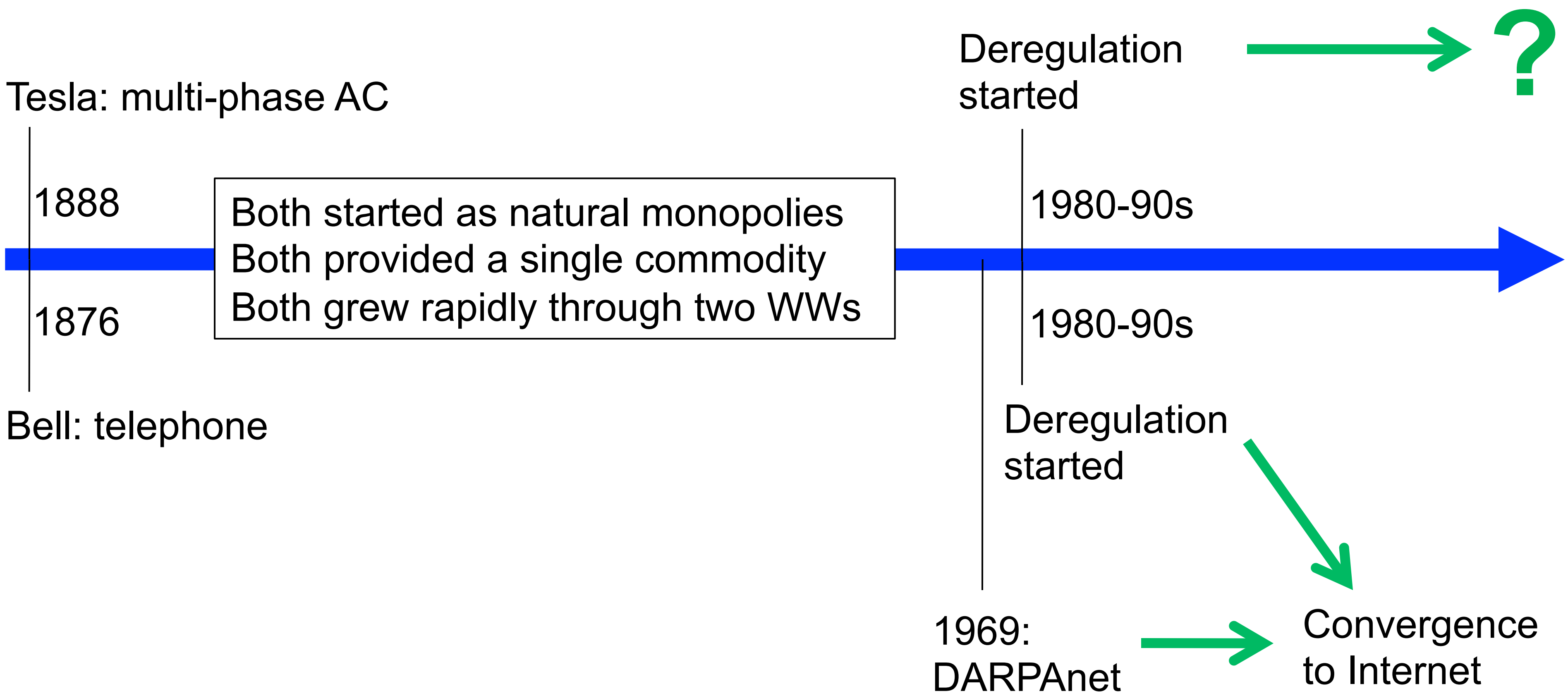
Big picture

how should we evolve our energy system (grid)?

Watershed moment

Power network will undergo similar **architectural transformation** that phone network went through in the last two decades



Watershed moment

Industries will be destroyed & created

AT&T, MCI, McCaw Cellular, Qualcom

Google, Facebook, Twitter, Amazon, eBay, Netflix

Infrastructure will be reshaped

Centralized intelligence, vertically optimized

Distributed intelligence, layered architecture

What will drive power network transformation ?

Four drivers

Renewables for sustainability

Electrification of transportation

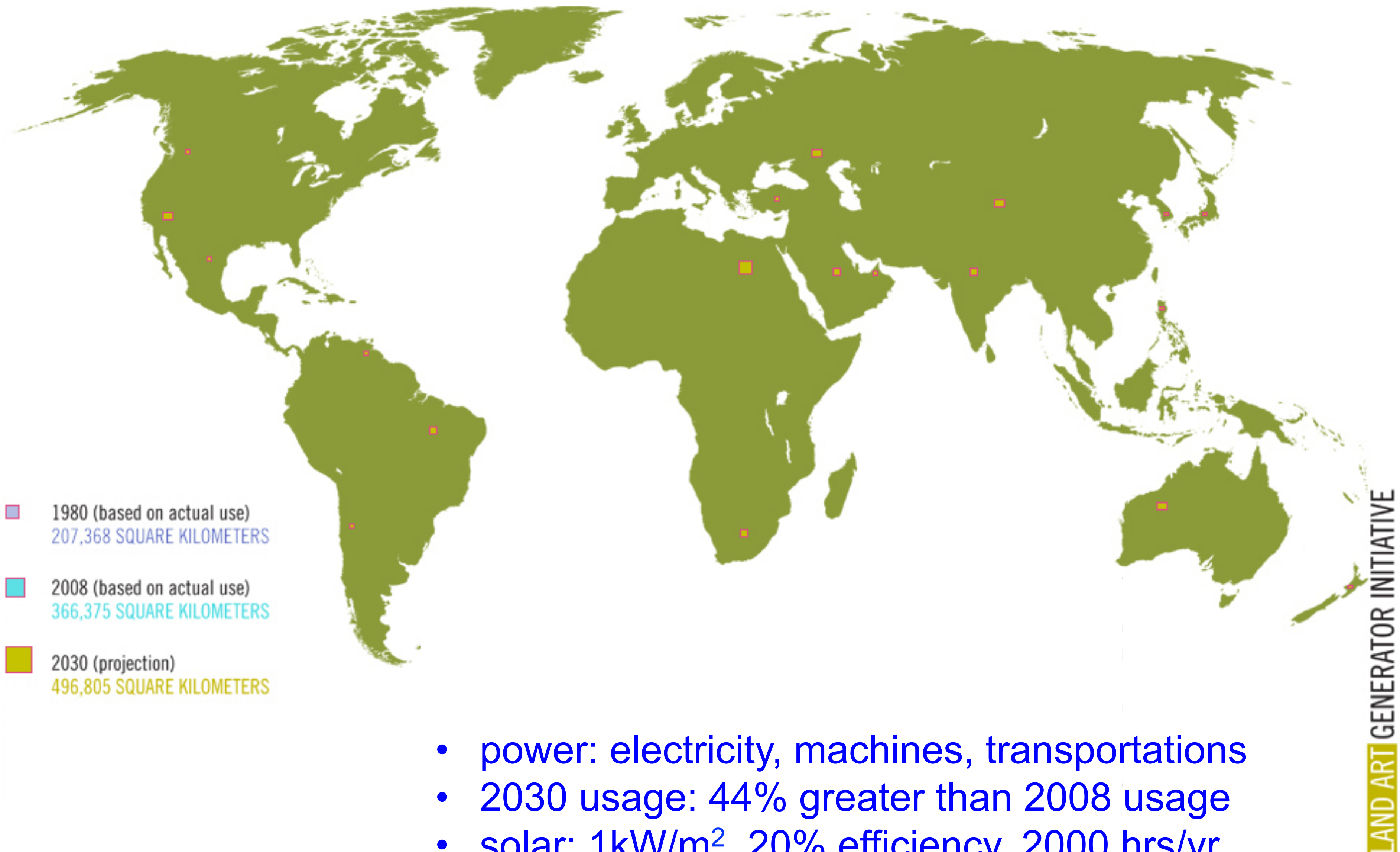
challenges

Advances in power electronics

Deployment of sensing, control, comm

enablers

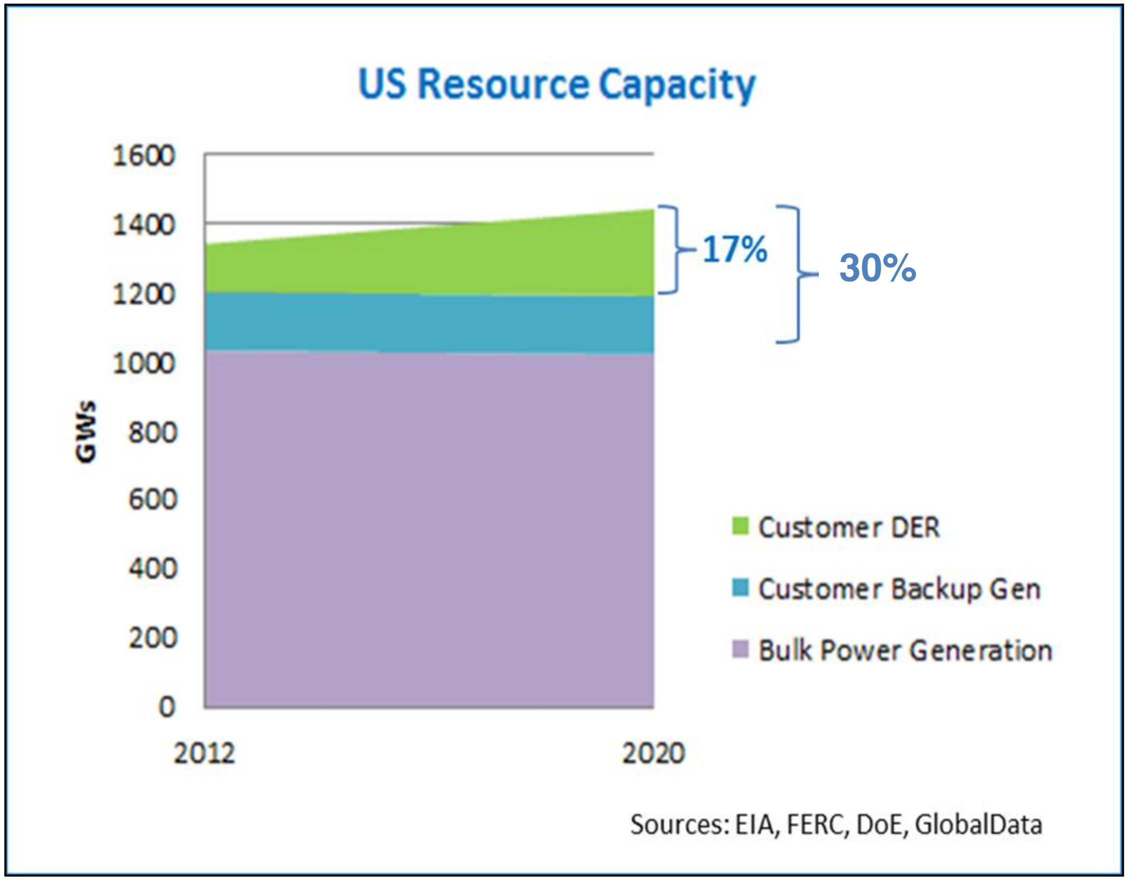
Area to power the world by solar



- power: electricity, machines, transportations
- 2030 usage: 44% greater than 2008 usage
- solar: 1kW/m^2, 20% efficiency, 2000 hrs/yr

DER will reach 30% of Installed US Capacity by 2020

Effectively all incremental growth in capacity will come from customers

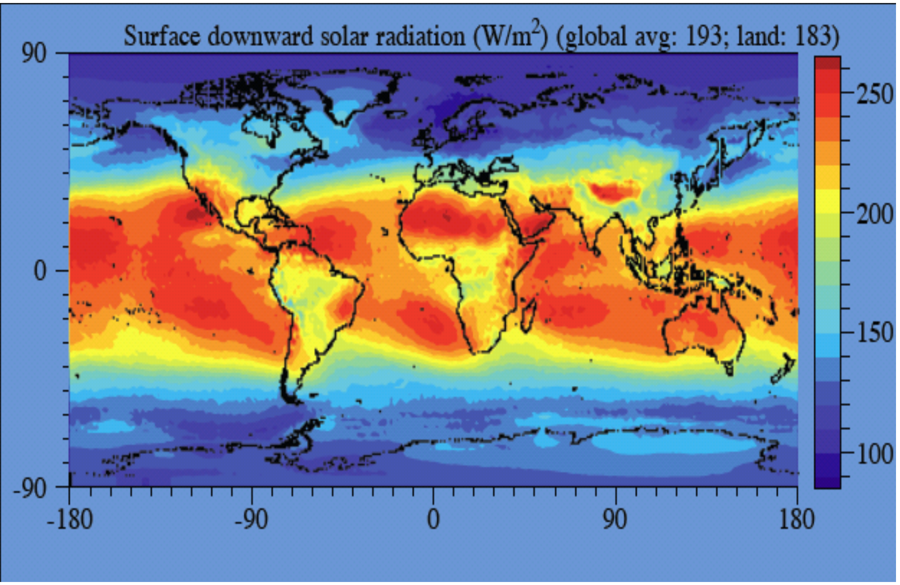


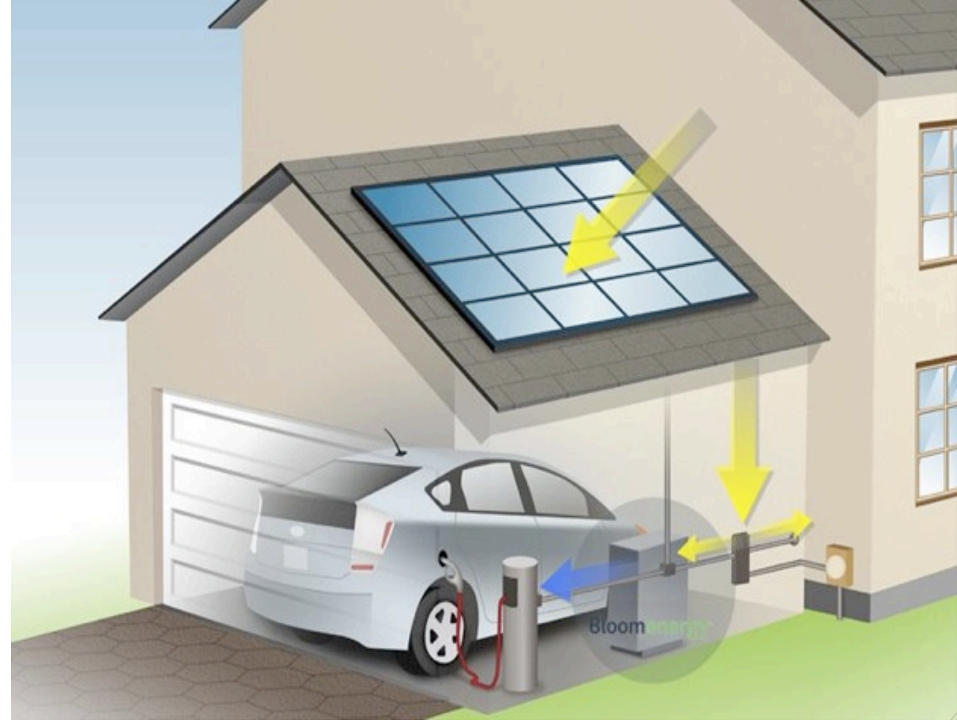
Backup Generation:	**225 GW**
CHP:	**122 GW**
Demand Response:	**90 GW**
Solar PV:	**50 GW**
Other DG:	**25 GW**
Dist. Storage:	**3 GW**

Potential DER Total: 515 GW

Jeff Taft, PNNL, Nov 2013

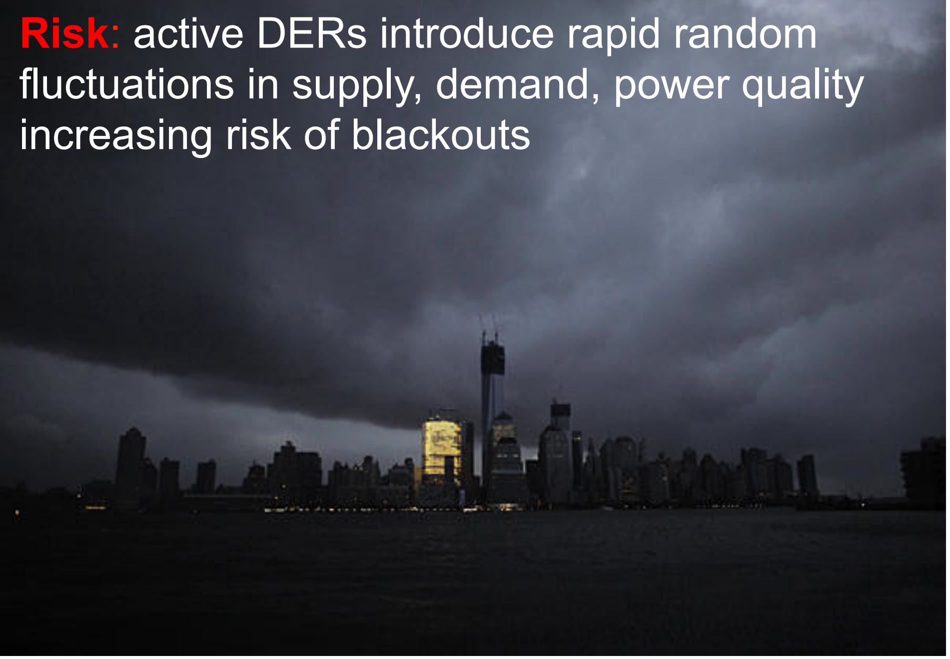
Technical potential of solar power:
> 200x world energy demand





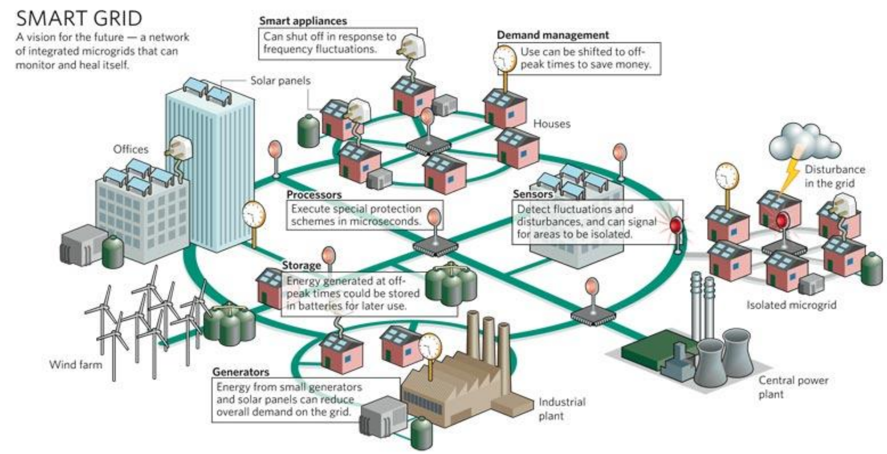
network of billions of active distributed energy resources (DERs)

DER: PV, wind tb, EV, storage, smart bldgs/appls

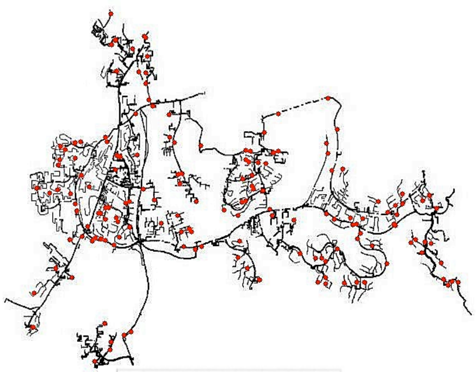


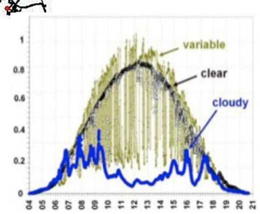
Risk: active DERs introduce rapid random fluctuations in supply, demand, power quality increasing risk of blackouts

Opportunity: active DERs enables realtime dynamic network-wide feedback control, improving robustness, security, efficiency



Caltech research: distributed control of networked DERs

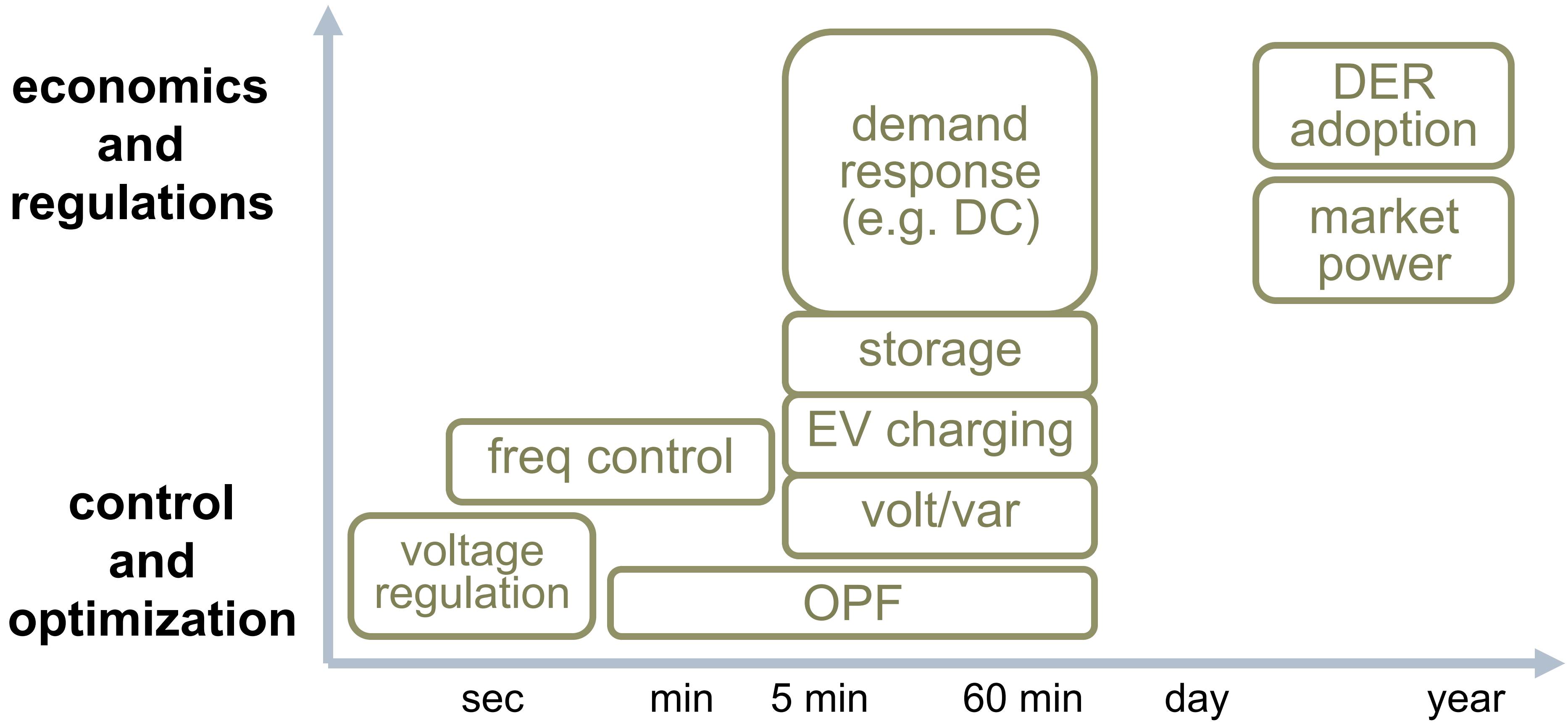




1. **Endpoint based control**
   Self-manage through local sensing, comm, control
2. **Local algorithms with global perspective**
   Decompose global objectives into local algorithms
3. **CDS tools provide**
   Structure, clarity, systematic algorithm design

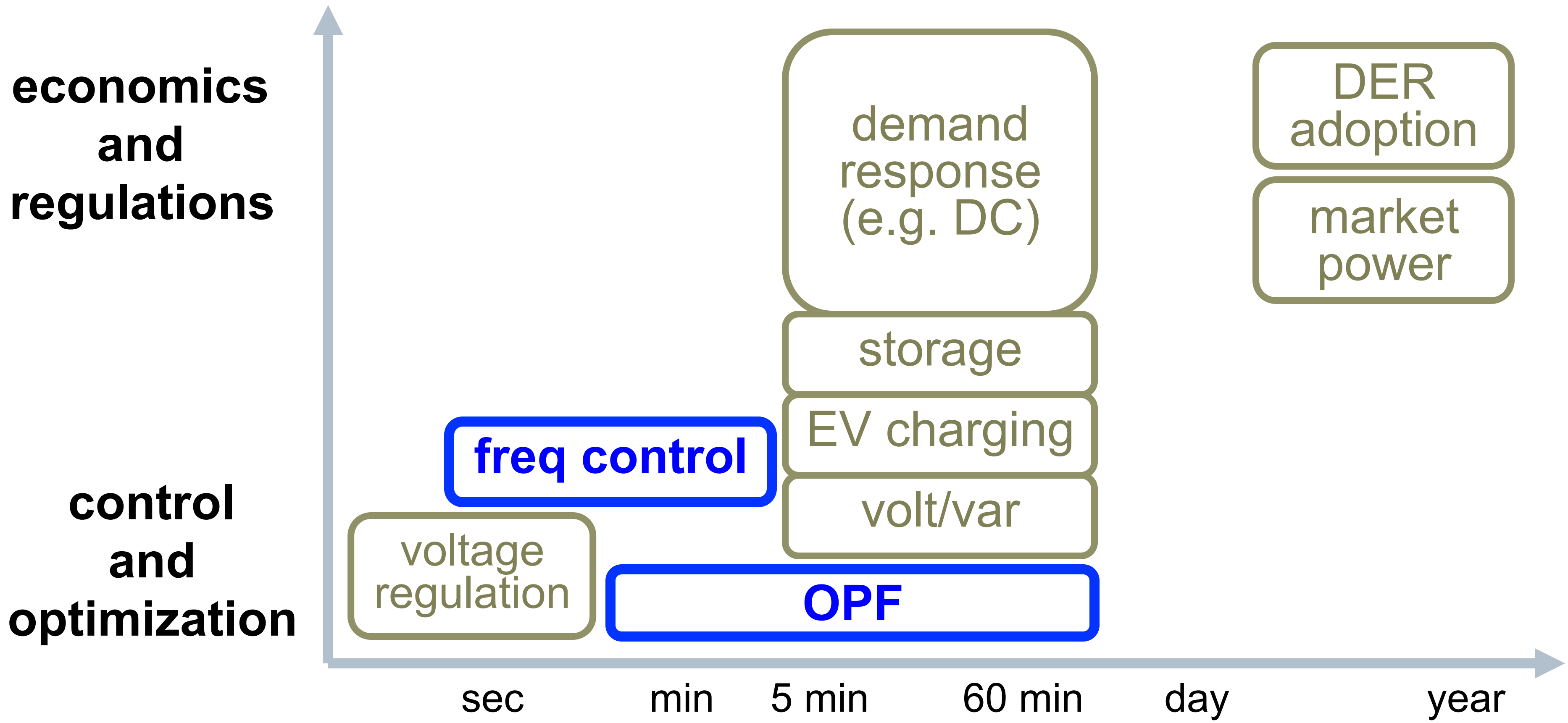
SOUTHERN CALIFORNIA EDISON
RESNICKINSTITUTE science + energy + sustainability
CISCO SYSTEMS

Caltech research



Caltech research





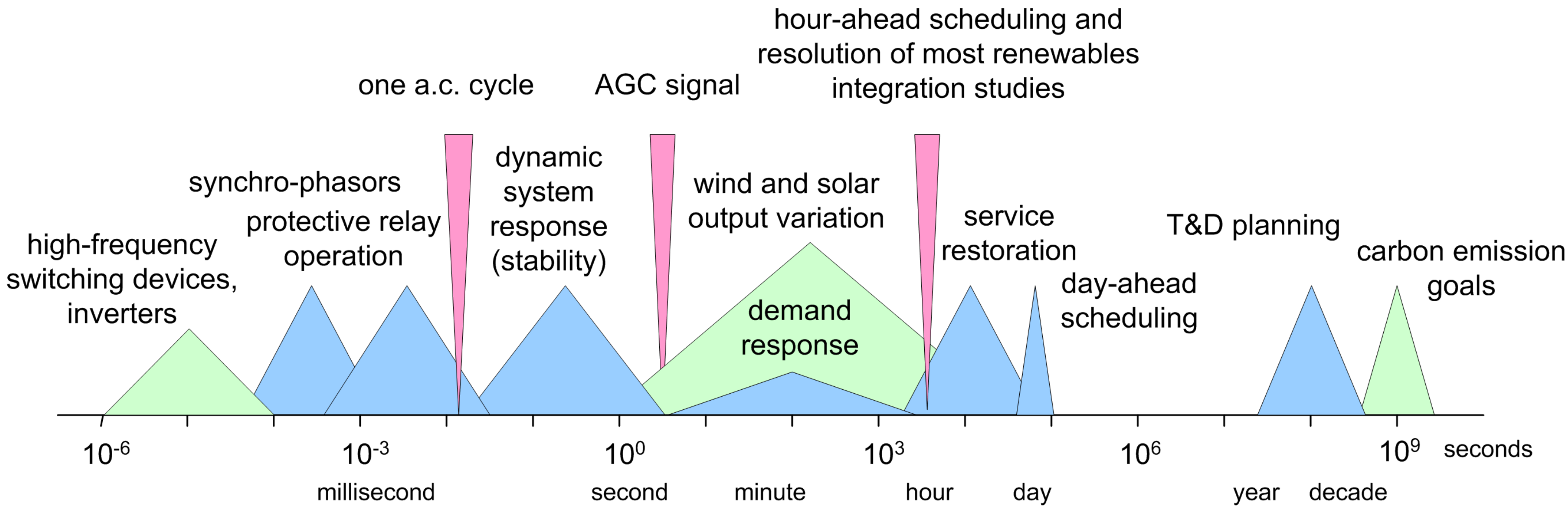



Key challenges

- multiple timescales
- uncertainty
- large scale
- nonconvexity
- ……

Multiple timescale

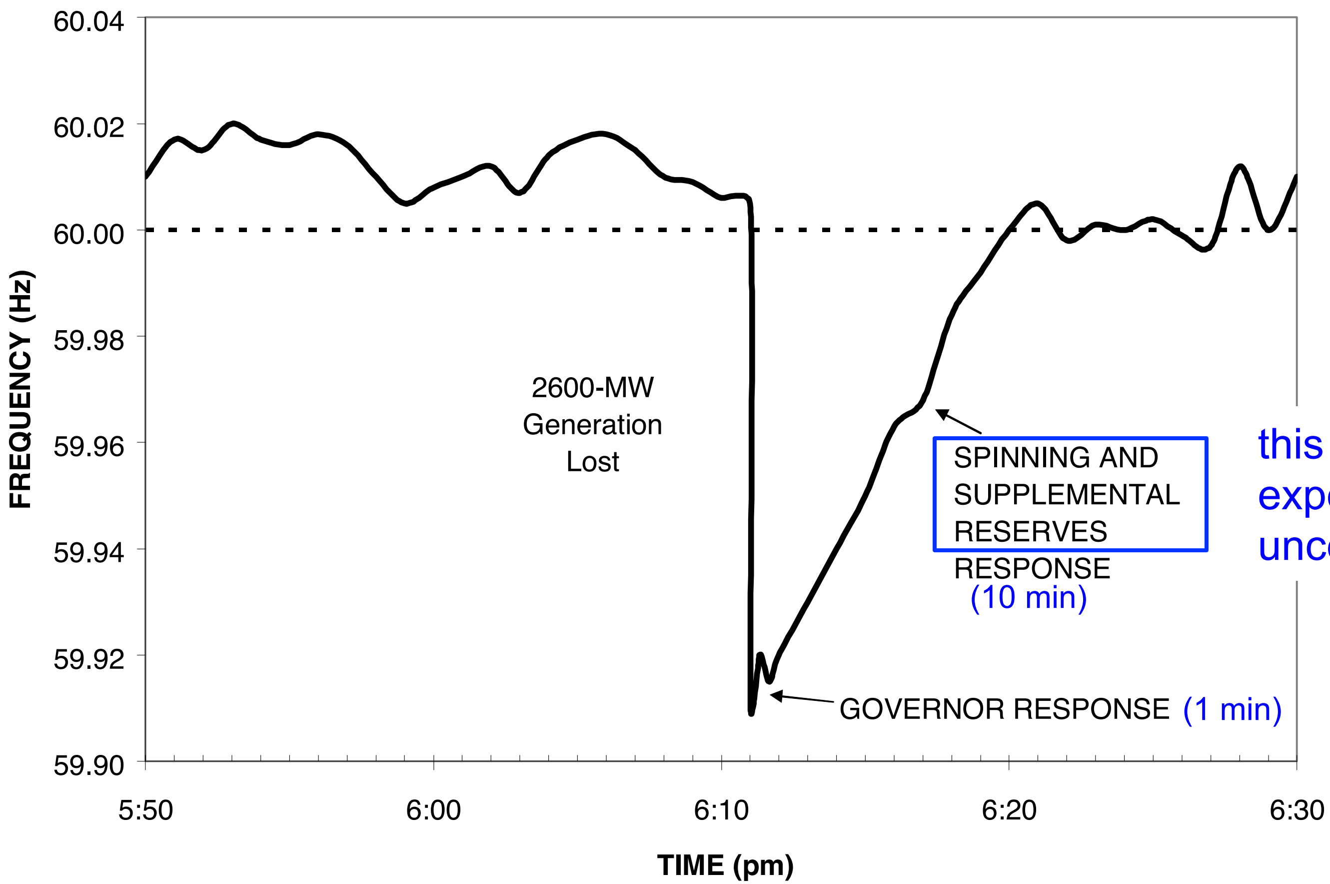
System dynamics and controls at different timescales
- require different models
- they interact



Sean Meyn, 2010

Uncertainty

Uncertainty creates difficulty in both control and markets

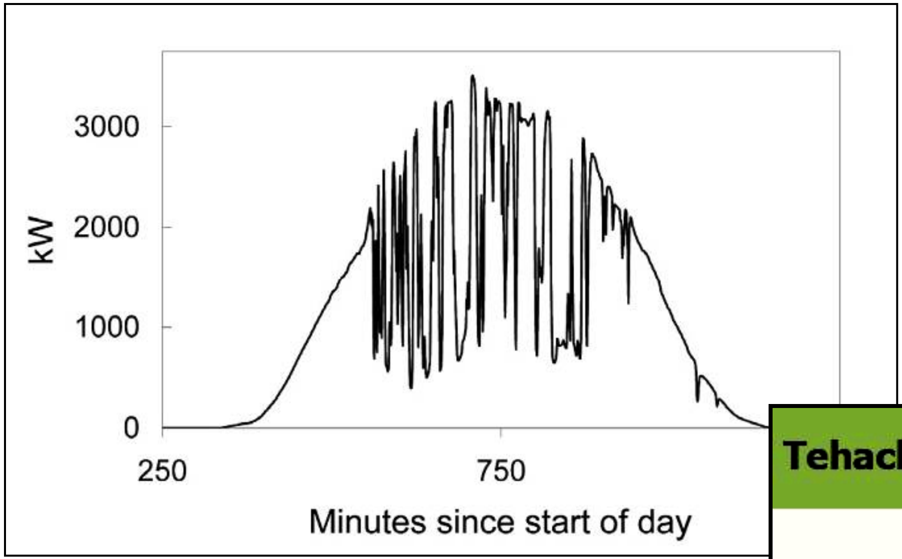


this can be very expensive as uncertainty grows

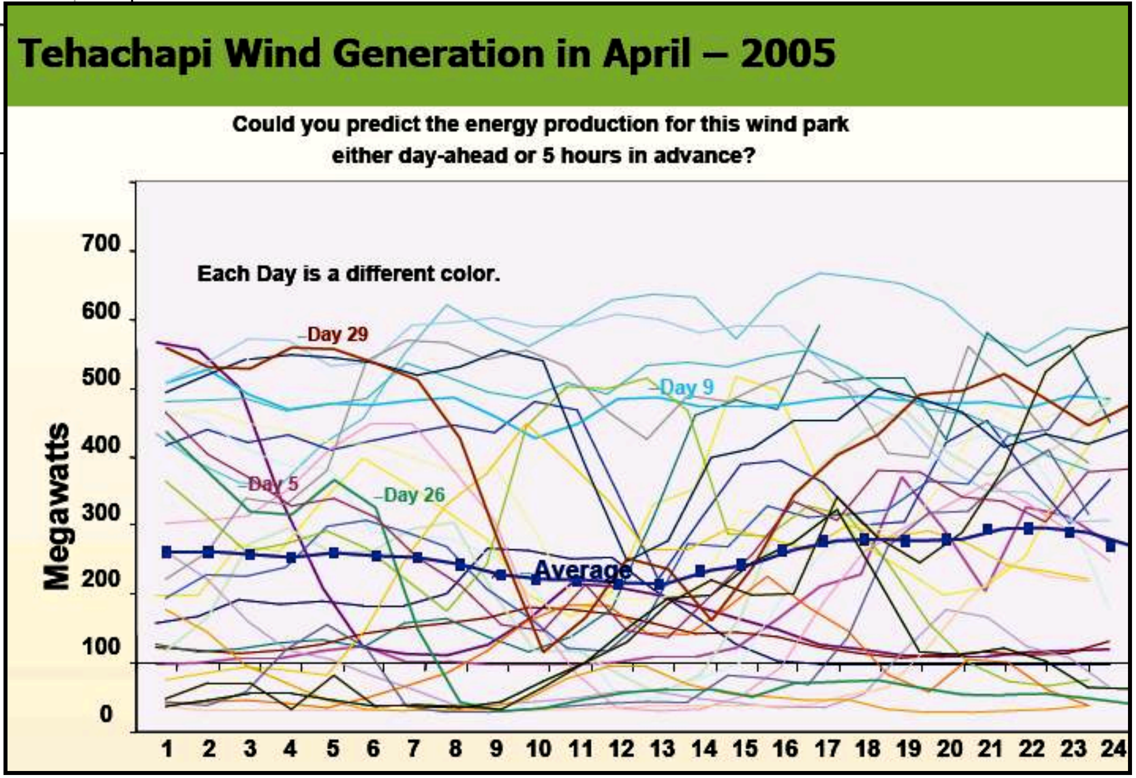
Loss of 2 nuclear plants in ERCOT

Kirby 2003 [ORNL/TM-2003/19]

Uncertainty



Imagine when we have 33%+ renewable generation …



How can load help?

- ubiquitous
- continuous
- fast-acting
- distributed

Uncertainty

Real-time price can be more than 100x the average price !

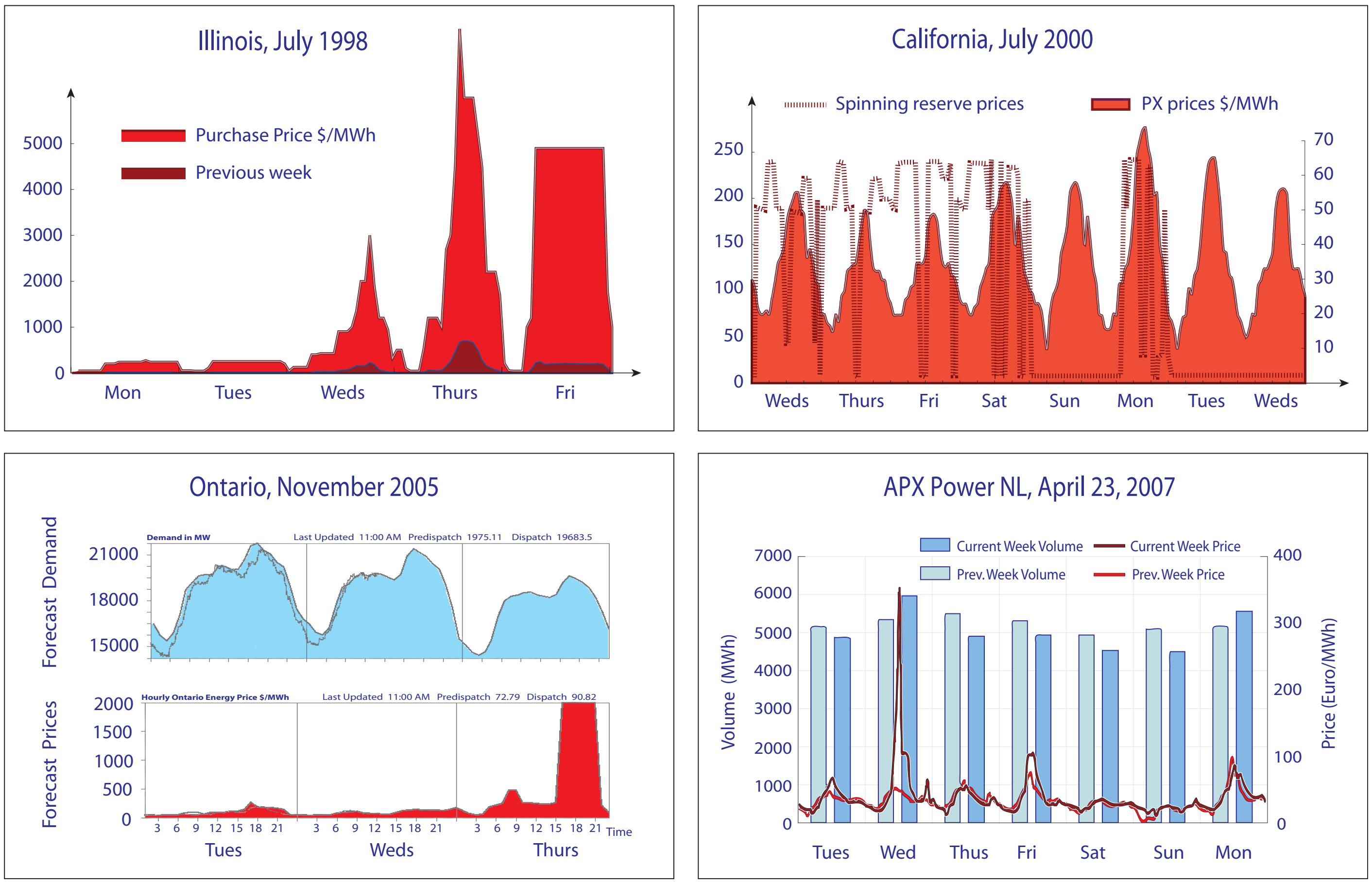


Figure: Real-world price dynamics

Sean Meyn, 2010

Large scale

Example: Southern California Edison

- 4-5 million customers

SCE Rossi feeder circuit

- #houses: 1,407; #commercial/industrial: 131
- #transformers: 422
- #lines: 2,064 (multiphase, inc. transfomers)
- peak load: 3 – 6 MW
- #optimization variables: 50,000

SCE has 4,500 feeders

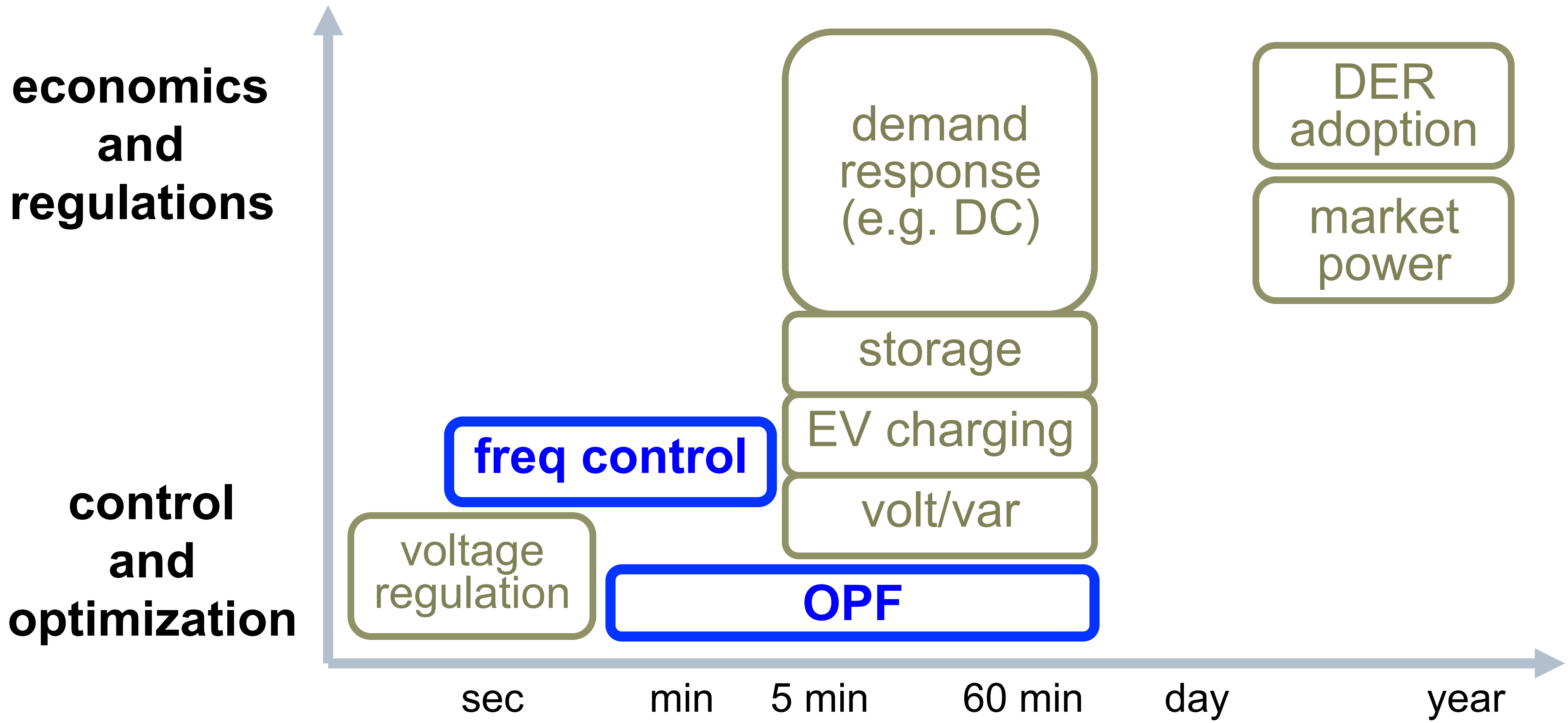
- ~100M variables

United States

- 131M customers, 300K miles of transmission & distr lines, 3,100 utilities

… much more DERs in the future

Caltech research



Optimal power flow (OPF)

OPF is solved routinely to determine

- How much power to generate where
- Parameter setting, e.g. taps, VARs
- Market operation & pricing

Non-convex and hard to solve

- Huge literature since 1962
- Common practice: DC power flow (LP)

OPF: bus injection model

$$\min \quad \mathrm{tr}\, C V V^*$$

$$\text{subject to} \quad \underline{s}_j \leq \boxed{\mathrm{tr}\left(Y_j V V^*\right)} \leq \bar{s}_j \qquad \underline{v}_j \leq |V_j|^2 \leq \bar{v}_j$$

- nonconvex (QCQP)
- due to Kirchhoff's laws
  - cannot be designed away
- should exploit hidden convexity structure
  - not just for speed and scale

Feasible sets

$$\min \quad \operatorname{tr} CVV^*$$

$$\text{subject to} \quad \underline{s}_j \le \boxed{\operatorname{tr}\left(Y_j VV^*\right)} \le \bar{s}_j \qquad \underline{v}_j \le |V_j|^2 \le \bar{v}_j$$

quadratic in V
linear in W !!

Equivalent problem:

$$\min \quad \operatorname{tr} CW$$

$$\text{subject to} \quad \boxed{\underline{s}_j \le \operatorname{tr}\left(Y_j W\right) \le \bar{s}_j \qquad \underline{v}_i \le W_{ii} \le \bar{v}_i}$$

$$W \ge 0, \quad \operatorname{rank} W = 1$$

convex in W
except this constraint

But SDP is not scalable enough

Feasible set

Consider

dec
\# vars

- full matrix W
- partial matrix $W_{c(G)}$ defined on a chordal ext of G
- partial matrix W_G defined on G

C1: $$W \succeq 0, \ \text{rank } W = 1$$

Feasible set

Consider

dec
vars

- full matrix W
- partial matrix $W_{c(G)}$ defined on a chordal ext of G
- partial matrix W_G defined on G

C1: $$W \succeq 0, \ \ \text{rank } W = 1$$

C2: $$W_{c(G)} \succeq 0, \ \ \text{rank } W_{c(G)} = 1$$

Feasible set

dec
vars

Consider

- full matrix W
- partial matrix $W_{c(G)}$ defined on a chordal ext of G
- partial matrix W_G defined on G

C1: $$W \succeq 0, \ \text{rank}\ W = 1$$

2x2
rank-1

C2: $$W_{c(G)} \succeq 0, \ \text{rank}\ W_{c(G)} = 1$$

C3: $$\begin{cases} W_G(j,k) \succeq 0, \ \text{rank}\ W_G(j,k) = 1, & (j,k) \in E \\ \sum_{(j,k)\in c} \angle [W_G]_{jk} = 0 \mod 2\pi \end{cases}$$

cycle
condition

Feasible set

Theorem

C1 = C2 = C3

C1: $$W \succeq 0, \ \text{rank}\ W = 1$$

C2: $$W_{c(G)} \succeq 0, \ \text{rank}\ W_{c(G)} = 1$$

C3: $$\begin{cases} W_G(j,k) \succeq 0, \ \text{rank}\ W_G(j,k) = 1, \quad (j,k) \in E \\ \displaystyle\sum_{(j,k)\in c} \angle [W_G]_{jk} = 0 \mod 2\pi \end{cases}$$

cycle condition

Feasible set

Theorem

C1 = C2 = C3

Moreover, given W_G that satisfies C3, there is a unique completion W that satisfies C1

C1: $$W \succeq 0, \ \text{rank} \ W = 1$$

C2: $$W_{c(G)} \succeq 0, \ \text{rank} \ W_{c(G)} = 1$$

C3: $$\begin{cases} W_G(j,k) \succeq 0, \ \text{rank} \ W_G(j,k) = 1, \quad (j,k) \in E \\ \sum_{(j,k)\in c} \angle [W_G]_{jk} = 0 \mod 2\pi \end{cases}$$

cycle condition

Implication: feasible sets

$$\mathbf{W}_G := \left\{ \begin{matrix} W_{jj}, W_{jk} : (j,k) \text{ in } G \\ \text{satisfy } \underline{\text{linear}} \text{ constraints} \end{matrix} \right\} \cap \left\{ \begin{matrix} W(j,k) \geq 0 \ \cancel{\text{rank-1}}, \\ \cancel{\text{cycle cond on } \angle W_{jk}} \end{matrix} \right\}$$

$$\text{idea: } W_G = \left(VV^* \text{ only on } G \right)$$

$$\mathbf{W}_{c(G)} := \left\{ \begin{matrix} W_{jj}, W_{jk} : (j,k) \text{ in } c(G) \\ \text{satisfy } \underline{\text{linear}} \text{ constraints} \end{matrix} \right\} \cap \left\{ W_{c(G)} \geq 0 \ \cancel{\text{rank-1}} \right\}$$

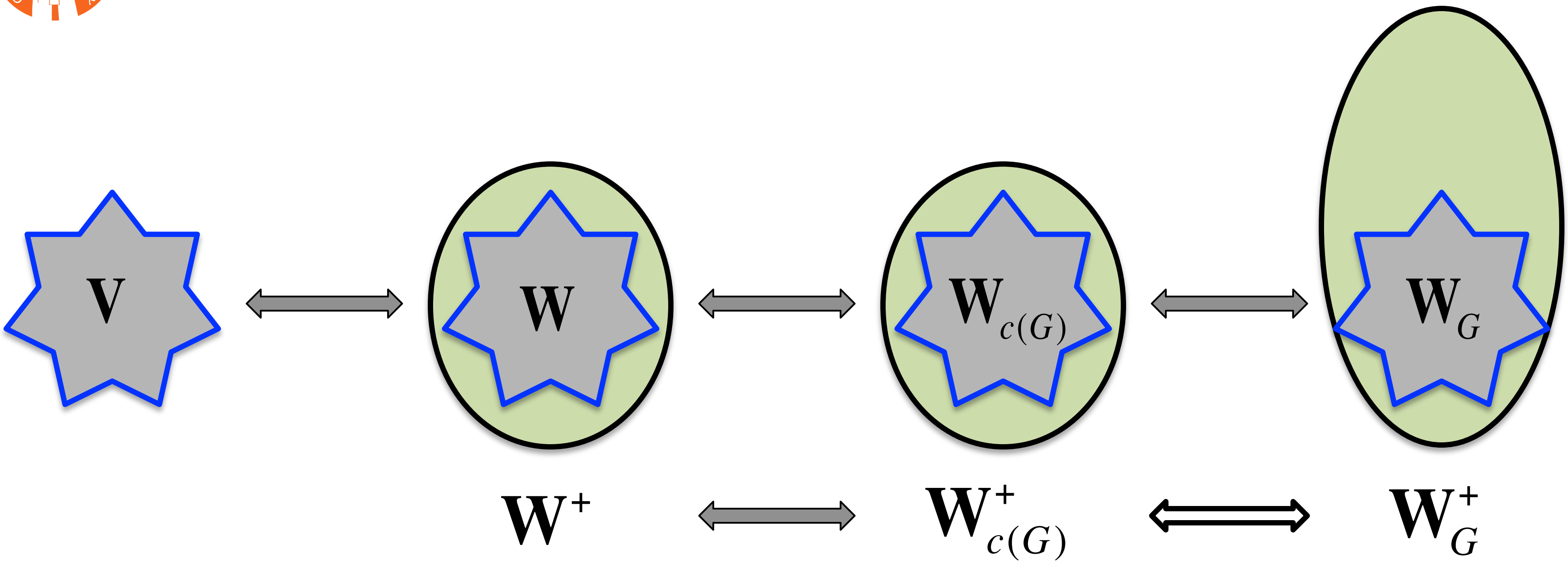
$$\text{idea: } W_{c(G)} = \left(VV^* \text{ on } c(G) \right)$$

matrix completion [Grone et al 1984]

$$\mathbf{W} := \left\{ W : \text{satisfies } \underline{\text{linear}} \text{ constraints} \right\} \cap \left\{ W \geq 0 \ \cancel{\text{rank-1}} \right\}$$

$$\text{idea: } W = VV^*$$

Feasible sets



<u>Theorem</u>

- Radial G : $\mathbf{V} \subseteq \mathbf{W}^+ \cong \mathbf{W}^+_{c(G)} \cong \mathbf{W}^+_G$
- Mesh G : $\mathbf{V} \subseteq \mathbf{W}^+ \cong \mathbf{W}^+_{c(G)} \subseteq \mathbf{W}^+_G$

But even SOCP is not scalable enough

Examples: radial unbalanced

network	BIM-SDP	
	time	ratio
13-bus	1.7s	5.7e-11
34-bus	–	–
37-bus	4.6s	1.0e-11
123-bus	9.3s	9.5e-8
1982-bus	–	–

Table 1: Simulation results using convex programming solver *sedumi*.

network	BIM-SDP	
	time	ratio
13-bus	2.6s	1.1e-8
34-bus	–	–
37-bus	4.6s	1.9e-8
123-bus	8.4s	1.1e-8
1982-bus	–	–

Table 2: Simulation results using convex programming solver *sdpt3*.

OPF: branch flow model

$$\min \quad f(x)$$

$$\text{over} \quad x := (S, I, V, s)$$

$$\text{s. t.} \quad \underline{s}_j \le s_j \le \overline{s}_j \qquad \underline{v}_j \le v_j \le \overline{v}_j$$

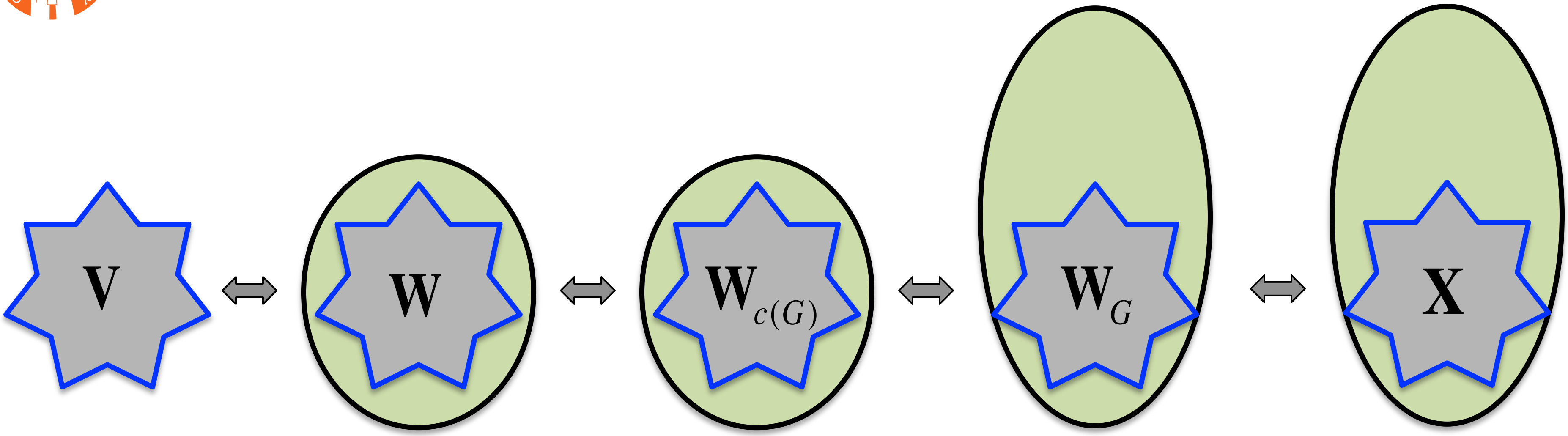
branch flow model

$$\sum_{i \to j} \left(S_{ij} - z_{ij} \left| I_{ij} \right|^2 \right) - \sum_{j \to k} S_{jk} = s_j$$

$$V_j = V_i - z_{ij} I_{ij} \qquad S_{ij} = V_i I_{ij}^*$$

- numerically more stable
- better linear approximation for tree networks

SOCP in branch flow model



$$\mathbf{W}^+ \Leftrightarrow \mathbf{W}^+_{c(G)} \Leftrightarrow \mathbf{W}^+_G \Leftrightarrow \mathbf{X}^+$$

Theorem

$$\mathbf{W}_G \equiv \mathbf{X} \quad \text{and} \quad \mathbf{W}^+_G \equiv \mathbf{X}^+$$

Examples: radial unbalanced

network	BIM-SDP		BFM-SDP	
	time	ratio	time	ratio
13-bus	1.7s	5.7e-11	1.5s	8.2e-11
34-bus	–	–	3.1s	6.6e-12
37-bus	4.6s	1.0e-11	2.7s	3.8e-12
123-bus	9.3s	9.5e-8	6.8s	6.1e-12
1982-bus	–	–	320s	4.9e-8

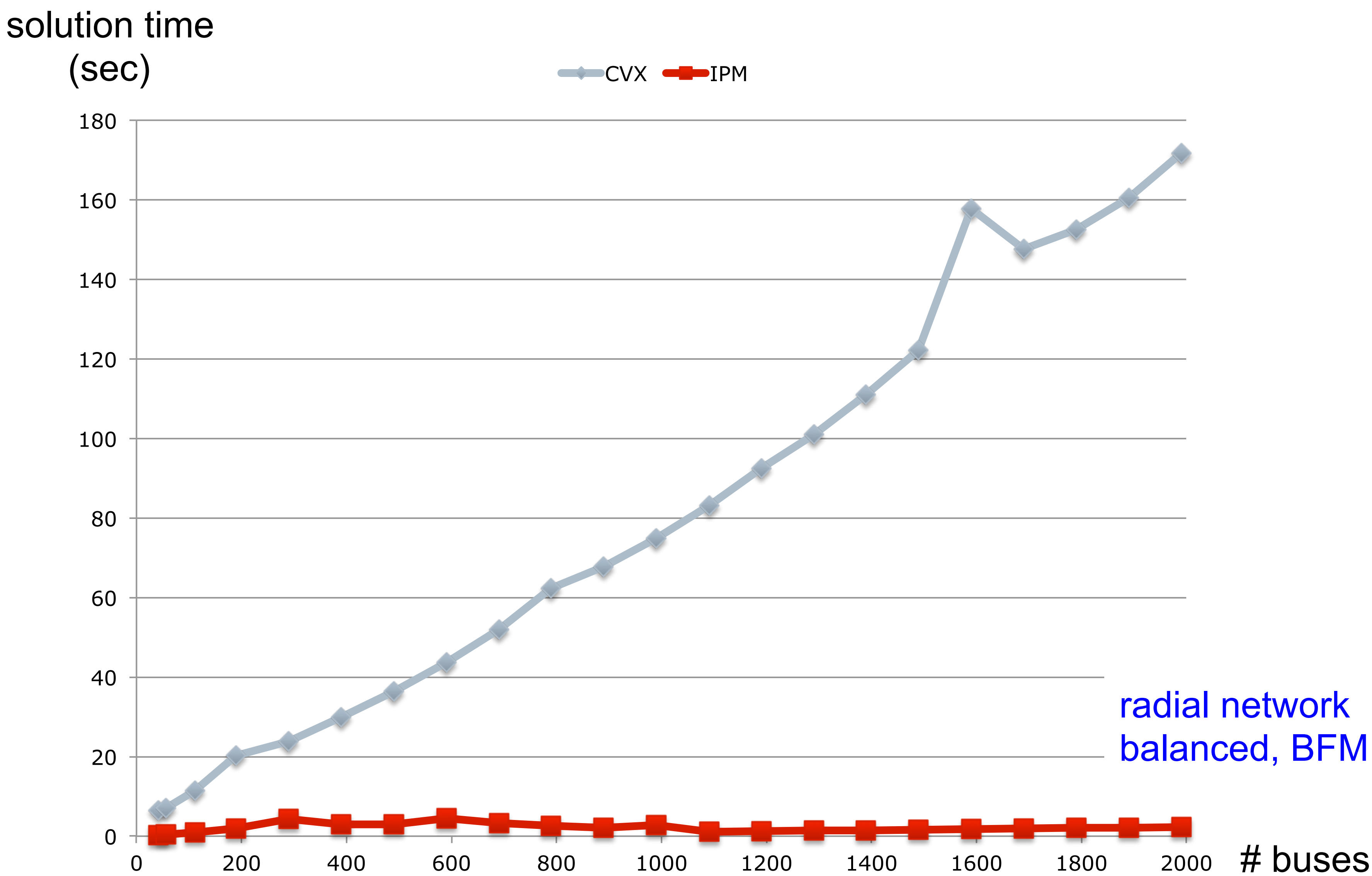
Table 1: Simulation results using convex programming solver *sedumi*.

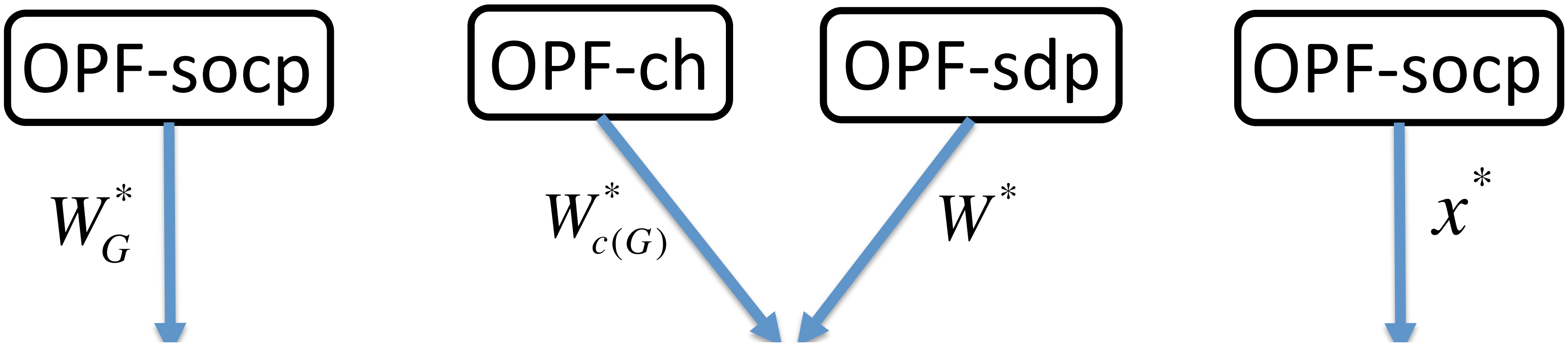
network	BIM-SDP		BFM-SDP	
	time	ratio	time	ratio
13-bus	2.6s	1.1e-8	2.6s	4.6e-8
34-bus	–	–	4.6s	1.2e-8
37-bus	4.6s	1.9e-8	5.5s	4.5e-9
123-bus	8.4s	1.1e-8	8.1s	4.0e-9
1982-bus	–	–	398s	5.0e-11

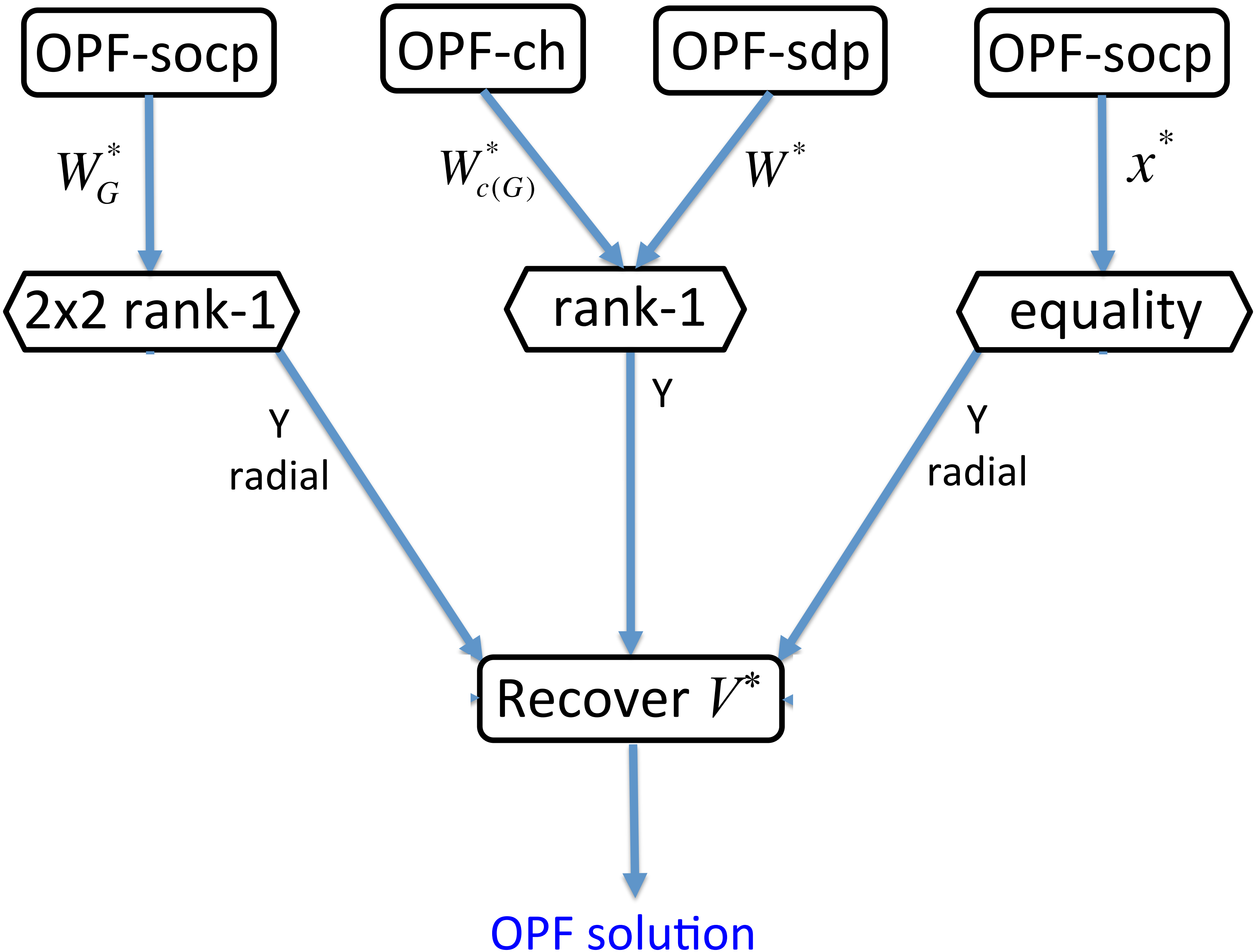
Table 2: Simulation results using convex programming solver *sdpt3*.

Branch flow model is much more numerically stable, but more variables !

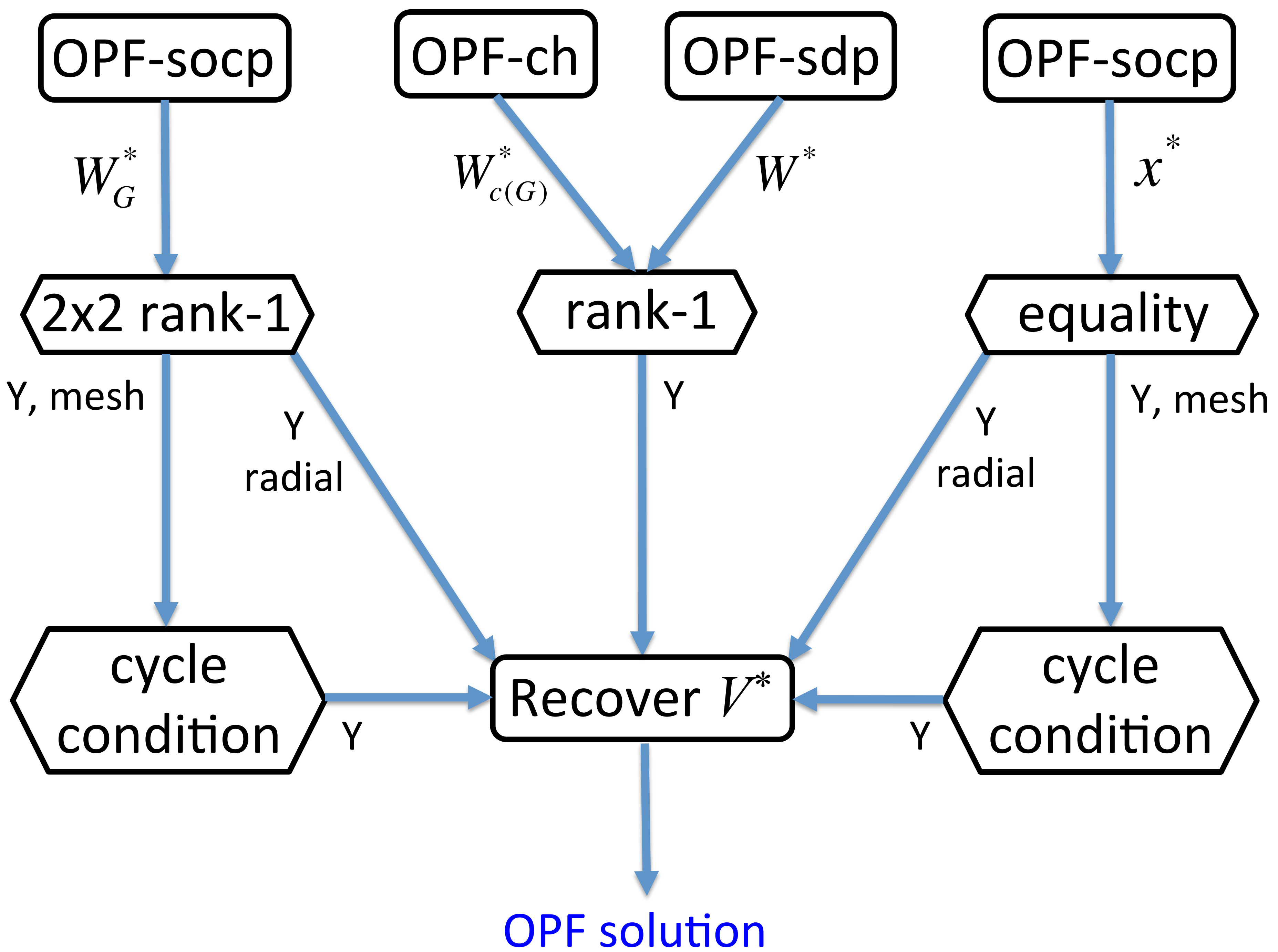
Examples: radial balanced




OPF-socp
OPF-ch
OPF-sdp
OPF-socp
W_G^*
$W_{c(G)}^*$
W^*
x^*



OPF-socp
OPF-ch
OPF-sdp
OPF-socp
W_G^*
$W_{c(G)}^*$
W^*
x^*
2x2 rank-1
rank-1
equality
Y
radial
Y
Y
radial
Recover V^*
OPF solution


OPF-socp
OPF-ch
OPF-sdp
OPF-socp
W_G^*
$W_{c(G)}^*$
W^*
x^*
2x2 rank-1
rank-1
equality
Y, mesh
Y
radial
Y
Y
radial
Y, mesh
cycle condition
Recover V^*
cycle condition
Y
Y
OPF solution

When will SOCP be exact?

Exactness

- For tree networks, SOCP always exact practically
- For general networks, often exact empirically but no theory

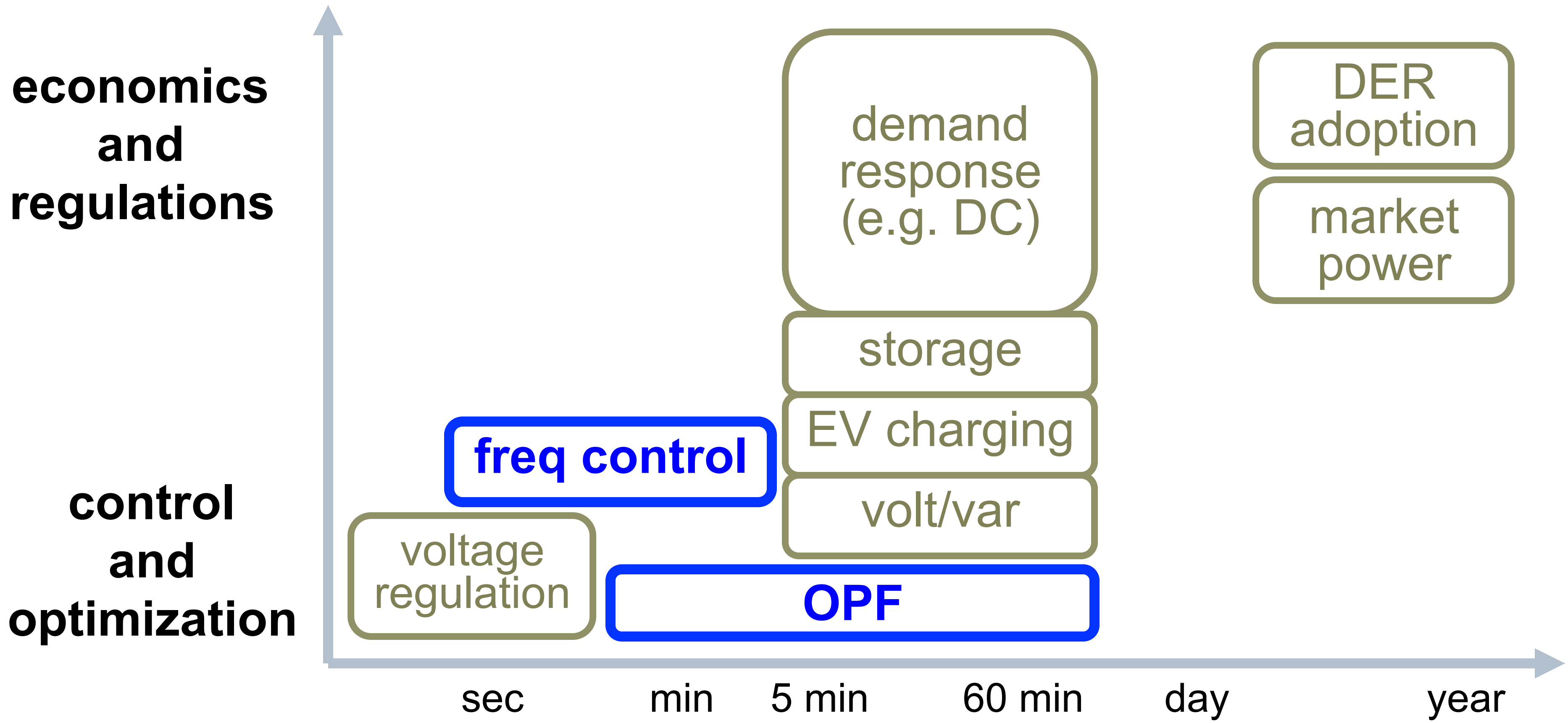
Bus injection model

- Jabr 2006, Bai et al 2008, Lavaei & Low 2012
- Bose et al 2011, Zhang & Tse 2011, Sojoudi & Lavaei 2012, Bose et al 2012, …
- Lesieutre et al 2011, …

Branch flow model

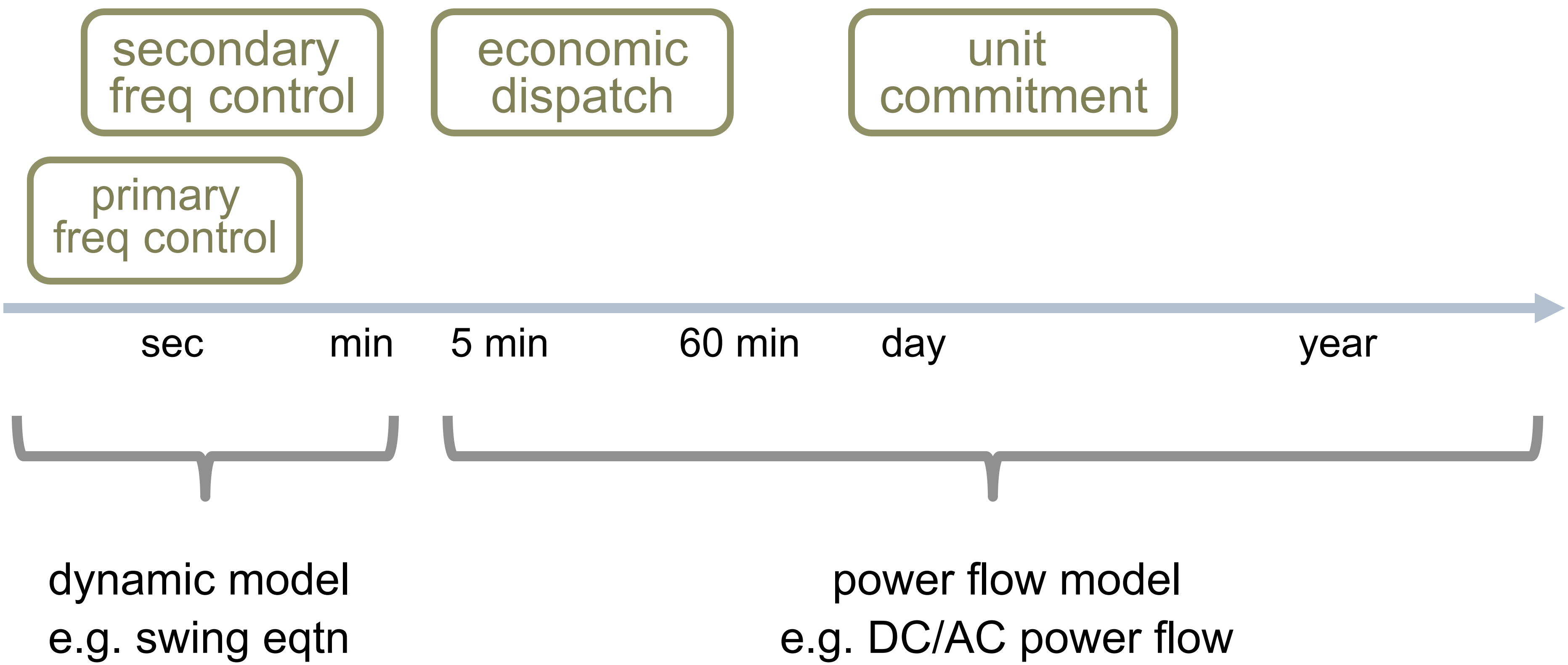
- Baran & Wu 1989, Chiang & Baran 1990, Taylor 2011, Farivar et al SGC2011, …
- Farivar et al TPS2013, Gan et al TAC2014, Bose et al TAC2014

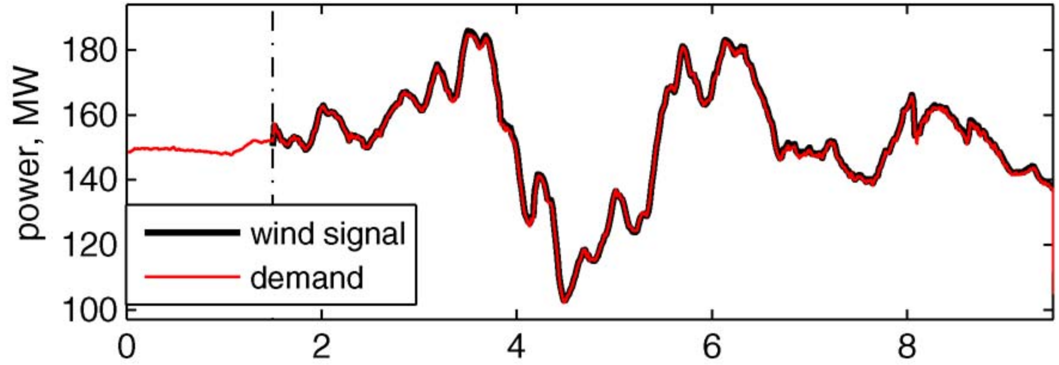
Caltech research



Frequency control

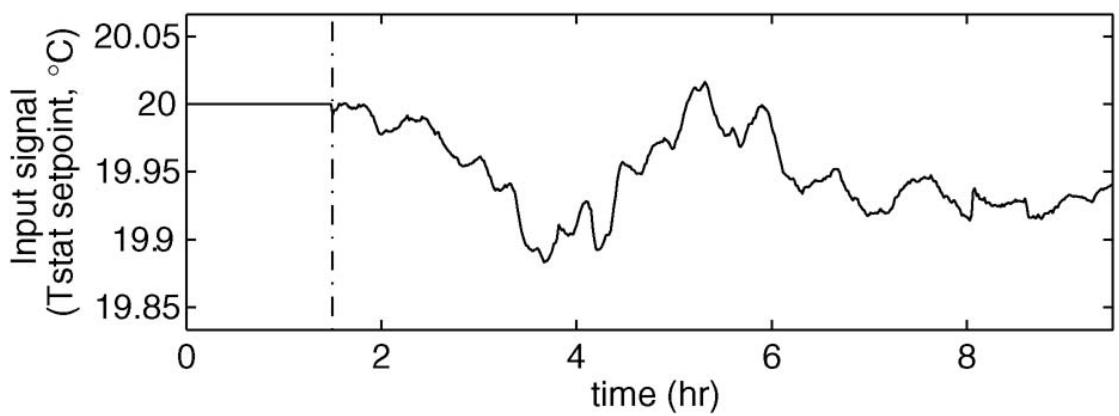
Frequency control is traditionally done on generation side





Can household Grid Friendly appliances follow its own PV production?

- 60,000 AC
- avg demand ~ 140 MW
- wind var: +- 40MW
- temp var: 0.15 degC



Dynamically adjust thermostat setpoint

Fig. 7. ***Load control example for balancing variability from intermittent renewable generators, where the end-use function—in this case, thermostat setpoint—is used as the input signal.***

Callaway, Hiskens (2011)
Callaway (2009)

Network model

Generator bus (may contain load):

$$\dot{\omega}_i = -\frac{1}{M_i}\left(d_i + D_i\omega_i - P_i^m + \sum_{i \to j} P_{ij} - \sum_{j \to i} P_{ji} \right)$$

Load bus (no generator):

$$0 = d_i + D_i\omega_i - P_i^m + \sum_{i \to j} P_{ij} - \sum_{j \to i} P_{ji}$$

Real branch power flow:

$$\dot{P}_{ij} = b_{ij}\left(\omega_i - \omega_j\right) \qquad \forall\ i \to j$$

swing dynamics

Frequency control

$$\dot{\omega}_i = -\frac{1}{M_i}\left(d_i + D_i\omega_i - P_i^m + \sum_{i\to j} P_{ij} - \sum_{j\to i} P_{ji} \right)$$

$$0 = d_i + D_i\omega_i - P_i^m + \sum_{i\to j} P_{ij} - \sum_{j\to i} P_{ji}$$

$$\dot{P}_{ij} = b_{ij}\left(\omega_i - \omega_j\right) \qquad \forall\ i \to j$$

Suppose the system is in steady state

$$\dot{\omega}_i = 0 \qquad \dot{P}_{ij} = 0$$

and suddenly …

Frequency control

Given: disturbance in gens/loads

Current: adapt remaining generators P_i^m

- to re-balance power
- restore nominal freq and inter-area flows (zero ACE)

Our goal: adapt controllable loads d_i

- … same as above …
- while minimizing disutility of load control

Frequency control

$$\dot{\omega}_i = -\frac{1}{M_i}\left(d_i + D_i\omega_i - P_i^m + \sum_{i\to j} P_{ij} - \sum_{j\to i} P_{ji}\right)$$

$$0 = d_i + D_i\omega_i - P_i^m + \sum_{i\to j} P_{ij} - \sum_{j\to i} P_{ji}$$

$$\dot{P}_{ij} = b_{ij}\left(\omega_i - \omega_j\right) \qquad \forall\ i \to j$$

proposed approach

current approach

Load-side controller design

$$\dot{\omega}_i = -\frac{1}{M_i}\left(d_i + D_i\omega_i - P_i^m + \sum_{i\to j} P_{ij} - \sum_{j\to i} P_{ji} \right)$$

$$0 = d_i + D_i\omega_i - P_i^m + \sum_{i\to j} P_{ij} - \sum_{j\to i} P_{ji}$$

$$\dot{P}_{ij} = b_{ij}\left(\omega_i - \omega_j\right) \qquad \forall\ i \to j$$

How to design feedback control law

$$d_i = F_i\left(\omega(t), P(t)\right)$$

Load-side controller design

$$\dot{\omega}_i = -\frac{1}{M_i}\left(d_i + D_i\omega_i - P_i^m + \sum_{i\to j} P_{ij} - \sum_{j\to i} P_{ji} \right)$$

$$0 = d_i + D_i\omega_i - P_i^m + \sum_{i\to j} P_{ij} - \sum_{j\to i} P_{ji}$$

$$\dot{P}_{ij} = b_{ij}\left(\omega_i - \omega_j\right) \qquad \forall\ i \to j$$

Control goals

- Rebalance power
- Resynchronize/stabilize frequency
- Restore nominal frequency
- Restore scheduled inter-area flows

Load-side controller design

$$\dot{\omega}_i = -\frac{1}{M_i}\left(d_i + D_i\omega_i - P_i^m + \sum_{i \to j} P_{ij} - \sum_{j \to i} P_{ji}\right)$$

$$0 = d_i + D_i\omega_i - P_i^m + \sum_{i \to j} P_{ij} - \sum_{j \to i} P_{ji}$$

$$\dot{P}_{ij} = b_{ij}\left(\omega_i - \omega_j\right) \qquad \forall\ i \to j$$

Design approach: forward engineering

- formalize control goals as OLC
- derive local control as distributed solution

Optimal load control (OLC)

$$\min_{d,\hat{d},P,R} \quad \sum_i \left(c_i(d_i) + \frac{1}{2D_i}\hat{d}_i^2 \right)$$

$$\text{s. t.} \quad d_i + \hat{d}_i = P_i^m - \sum_{e \in E} C_{ie} P_e$$

demand = supply
per bus

$$d_i = P_i^m - \sum_j L_{ij} v_j$$

restore nominal
frequency

$$\hat{C} v = \hat{P}$$

restore inter-area
flows

Summary

Primary frequency control $d_i = F_i\left(\omega_i(t)\right)$

- Completely decentralized load control works
- network dynamics + active load control

  = primal-dual algorithm for OLC
- Feedback system is GAS

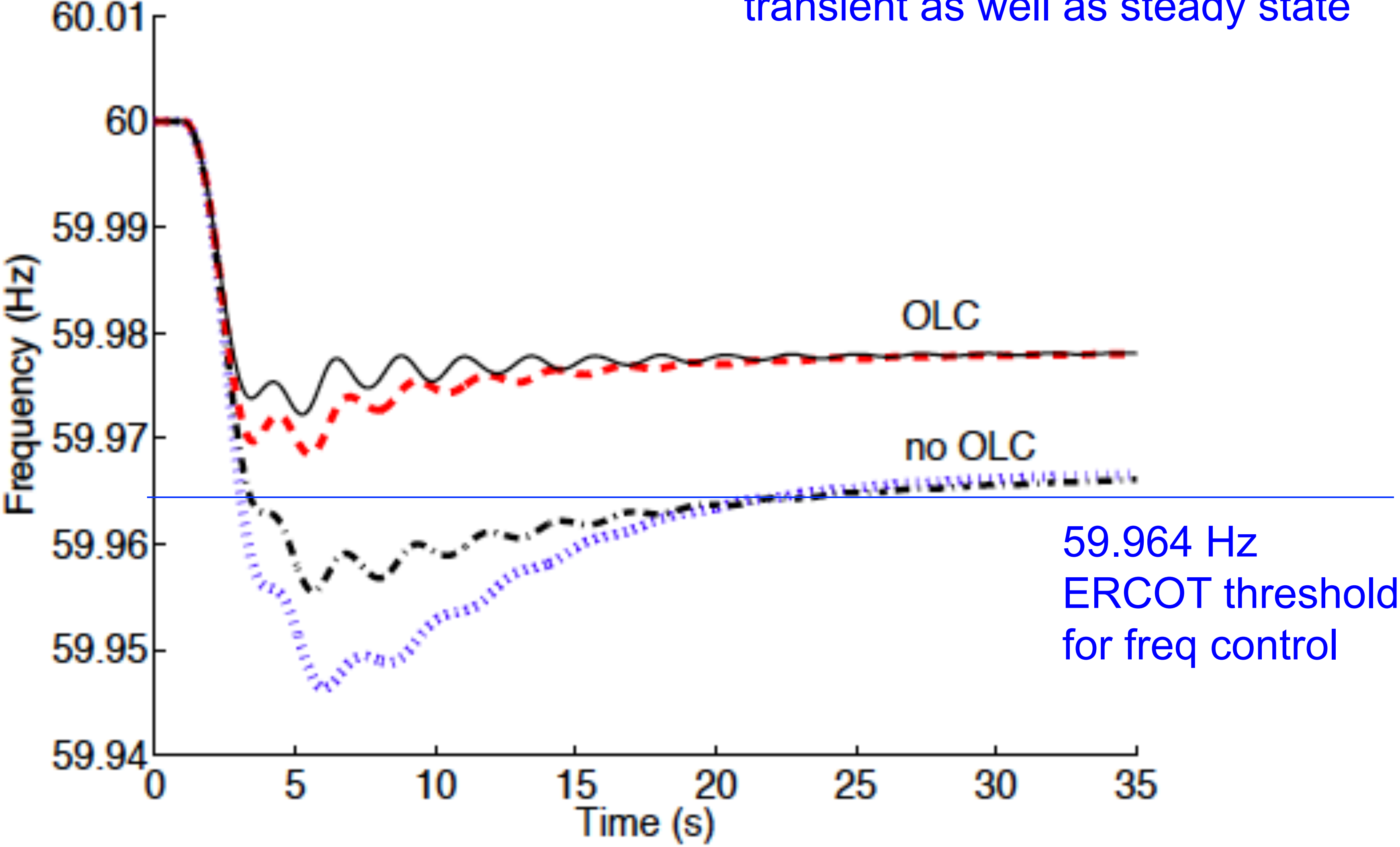
Secondary frequency control

- Each load maintains internal dynamic vars and communicates with neighbors
- … same as above

Load-side frequency control works !

Simulations

load-side participation improves transient as well as steady state



Simulations