Restricted nonlinear (RNL) turbulence models

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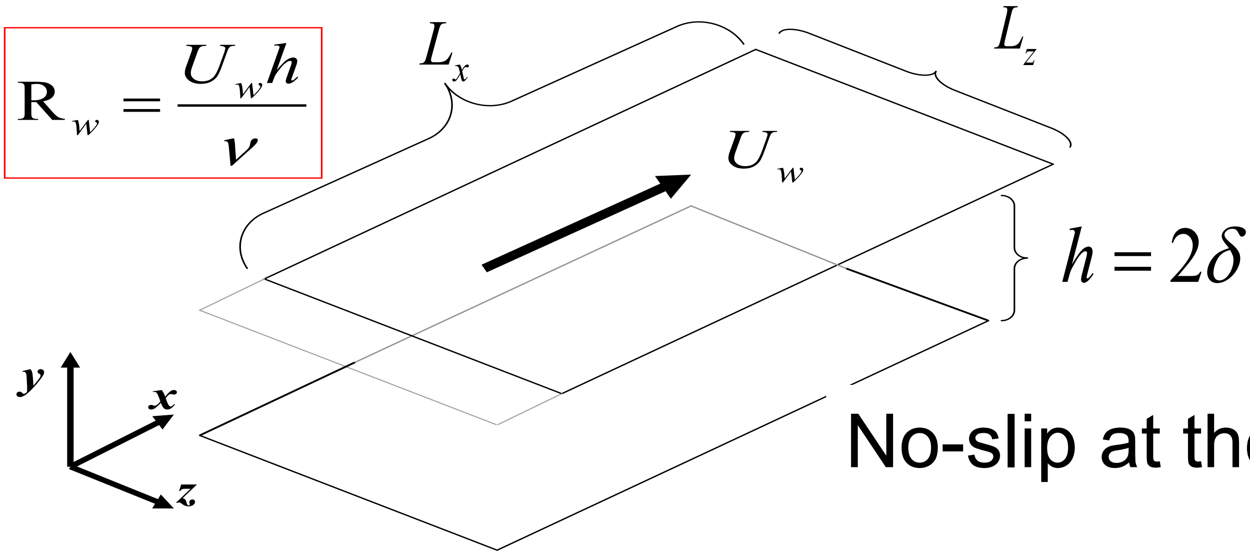
Flow configuration

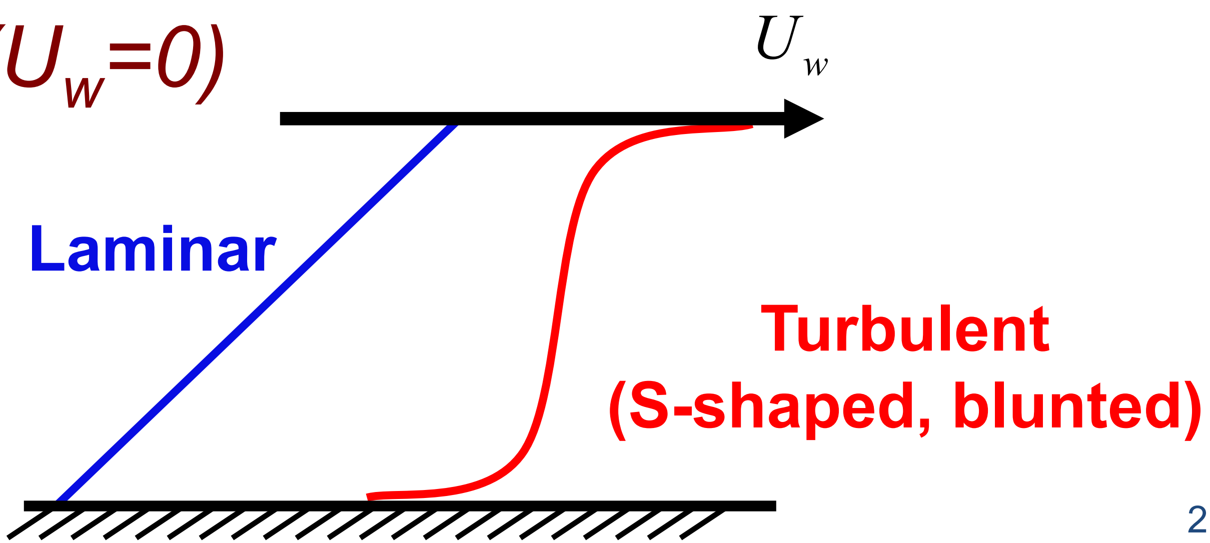
$$R_w = \frac{U_w h}{\nu}$$



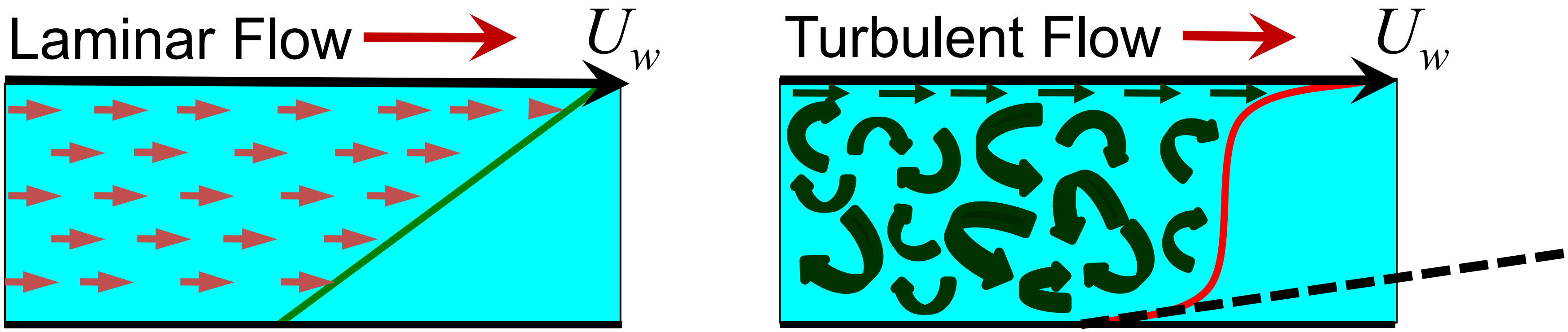
No-slip at the walls

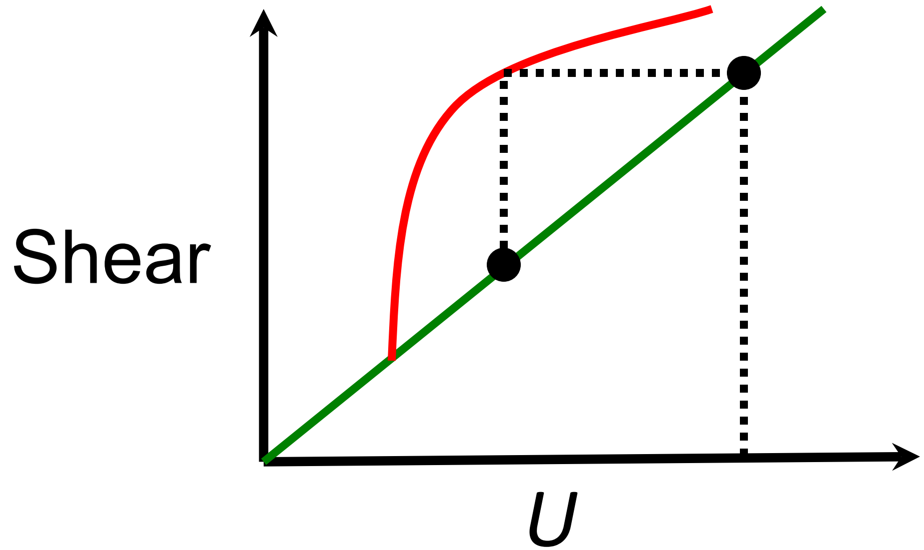
Channel flow *(U_w=0)*

Couette Flow



Drag and profile blunting





Equal shear laminar

more mass flow

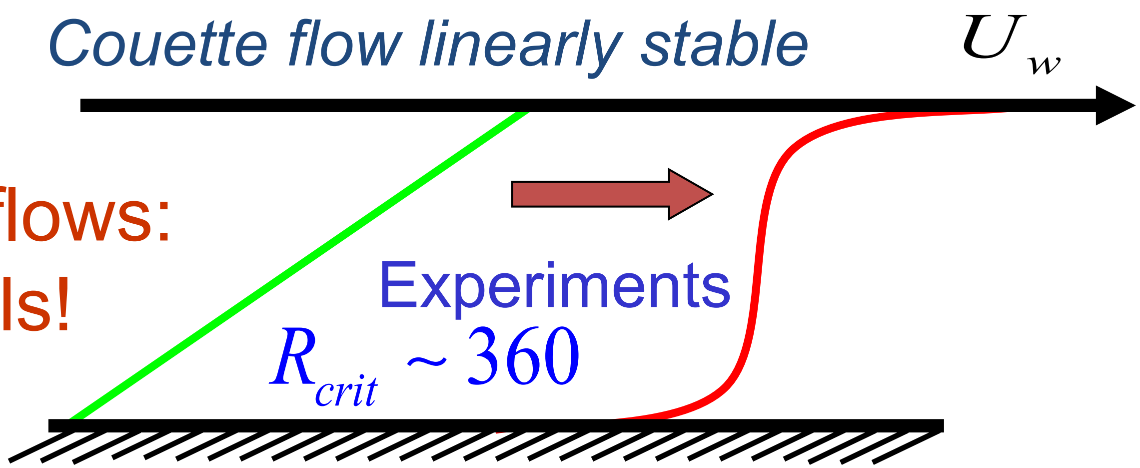
Turbulent flow has less efficient transport

Increased shear stress at wall (blunting) is key!

How does the profile blunt?

- Transition as a linear stability problem

Wall bounded shear flows:
Linear analysis fails!



[Romanov 73, Tillmark & Alfredsson 92]

Why does it fail?

- Two schools of thought
  - Form of the linear operator
  - Nonlinear phenomena are important

Linear Models and Energy

- Non-normality (i.e $\mathcal{A}\mathcal{A}^T \neq \mathcal{A}^T\mathcal{A}$) of linear operator leads to large transient growth

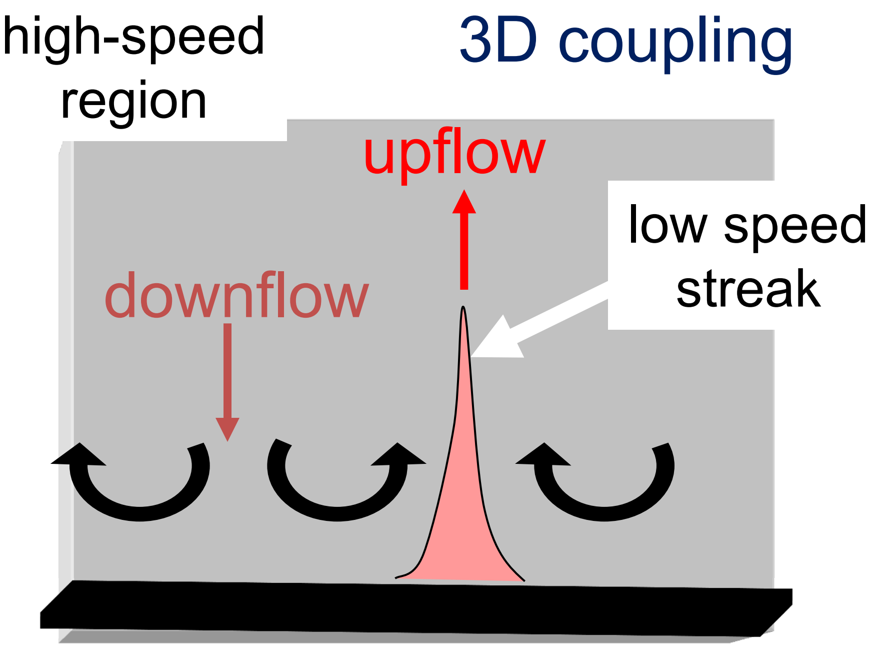
$$\begin{bmatrix} \dot{v} \\ \dot{\eta}_y \end{bmatrix} = \begin{bmatrix} * & 0 \\ -\dfrac{dU}{dy}\dfrac{\partial}{\partial z} & * \end{bmatrix} \begin{bmatrix} v \\ \eta_y \end{bmatrix}$$

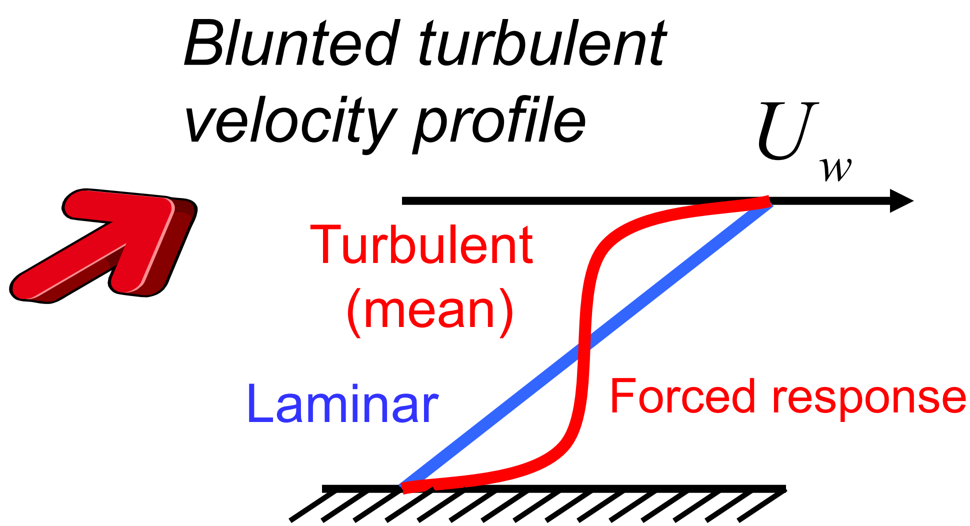
- Linear energy growth turns out to be key aspect of subcritical transition (i.e. the failure of linear theory)
  - Coupling operator plays a crucial role in transition and maintenance of wall-turbulence

Not robust to disturbances/uncertainty!

Why a restricted nonlinear model

- Basic linear models do not reproduce blunted turbulent profile
- Momentum redistribution comes from nonlinear interactions



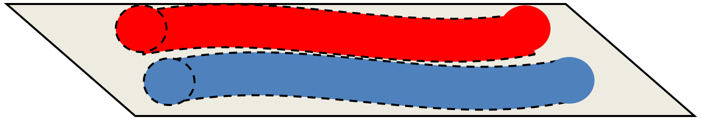


Question of which nonlinearity?

Coherent structures in turbulence

Linear Energy Growth

- Parallel flows streamwise constant disturbances amplified $O(R^3)$ versus $(R^{3/2})$ [Farrell & Ioannou 93]
- Largest growth when initially seeded with streamwise vortices [Butler & Farrell 92]
- In the unstable regime of channel flows streamwise structures have more energy than unstable modes [Jovanović & Bamieh 04]

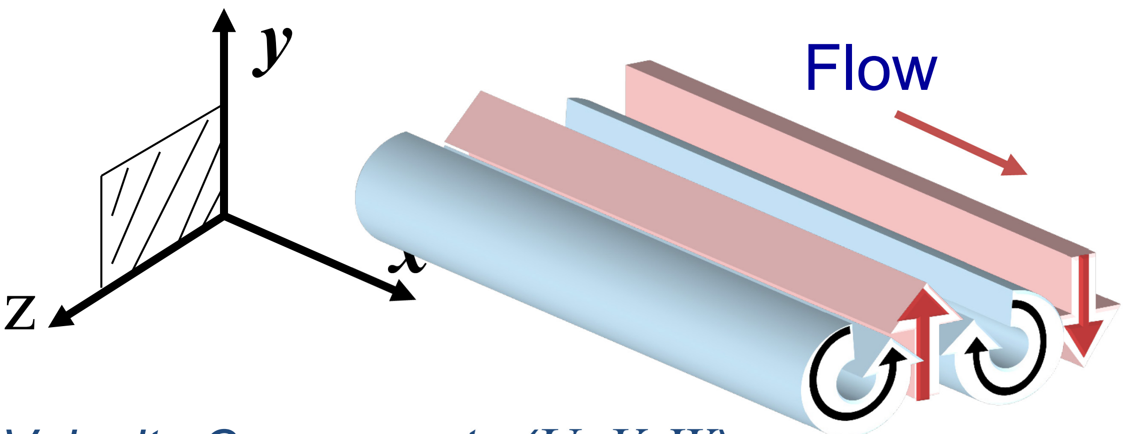


Full Simulations and Experiments

- Near wall dominated by elongated streaks/vortices
- Longer structures throughout the height of the channel [eg. Kim & Adrian 99, Morrison et al. 02, Guala et al. 06, Hutchins & Marusic 07 …]
- Couette flow: Core (channel center) structures longer than other flows [Lee & Kim 91, Kitoh et al. 05, Tillmark & Alfredsson 92]

Streamwise constant (2D/3C) mean flow

IDEA: MEAN flow is 2D, Use all 3 velocity components (3C), to capture 3D nature of turbulence.



Velocity Components (U, V, W)

Stream function

$$V = \frac{\partial \psi}{\partial z}, \quad W = -\frac{\partial \psi}{\partial y}$$

Hypothesis: Such a nonlinear mean flow model will capture blunting of profile

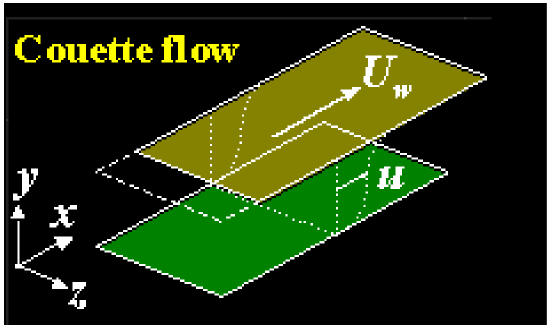
The 2D/3C model

$$U = u'_{sw} + U_{lam}, \quad V = \frac{\partial \psi}{\partial z}, \quad W = -\frac{\partial \psi}{\partial y}$$

$$\boxed{\begin{aligned} \frac{\partial u'_{sw}}{\partial t} &= -\frac{\partial \psi}{\partial z}\frac{\partial (u'_{sw} + U_{lam})}{\partial y} + \frac{\partial \psi}{\partial y}\frac{\partial u'_{sw}}{\partial z} + \frac{1}{\mathrm{R}_w}\Delta u'_{sw} + \eta_u \\ \frac{\partial \Delta \psi}{\partial t} &= -\frac{\partial \psi}{\partial z}\frac{\partial \Delta \psi}{\partial y} + \frac{\partial \psi}{\partial y}\frac{\partial \Delta \psi}{\partial z} + \frac{1}{\mathrm{R}_w}\Delta^2 \psi + \eta_\psi \end{aligned}}$$

This model rigorously connects observed flow features and linear energy growth to the blunting of the profile

Properties of 2D/3C



Theorem

Plane Couette flow $U(y) = y$ is (conditionally) globally asymptotically stable for all Reynolds numbers [Bobba et. al. 04]

$$\frac{\partial}{\partial t}\begin{bmatrix}\psi \\ u'_{sw}\end{bmatrix} = \begin{bmatrix} \Delta^{-1}\frac{\Delta^2}{R} & 0 \\ -\frac{\partial U_{lam}}{\partial y}\frac{\partial}{\partial z} & \frac{\Delta}{R}\end{bmatrix}\begin{bmatrix}\psi \\ u'_{sw}\end{bmatrix}$$

Maintains coupling operator

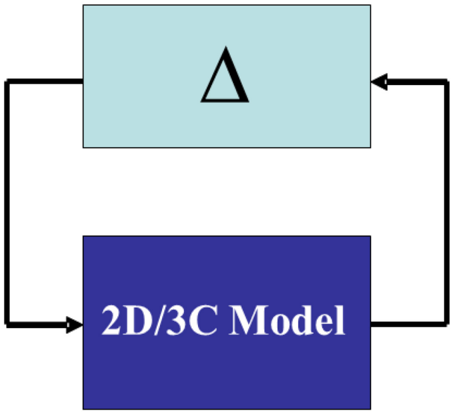
Laminar is only solution of the 2D/3C model

No nonlinear instability transition scenarios

Robustness problem is preserved

We demonstrate how this links structural features & linear mechanisms to the nonlinearity that drives profile blunting

Simulating the 2D/3C System



Understand large scale behavior and interpret the small stuff and unmodeled effects as a bounded perturbation

Small Amplitude White Noise → **2D/3C Model**

Linearized

$$\boxed{\frac{\partial \Delta \psi}{\partial t} = \frac{1}{R_w}\Delta^2 \psi + \eta(y,z,t)}$$

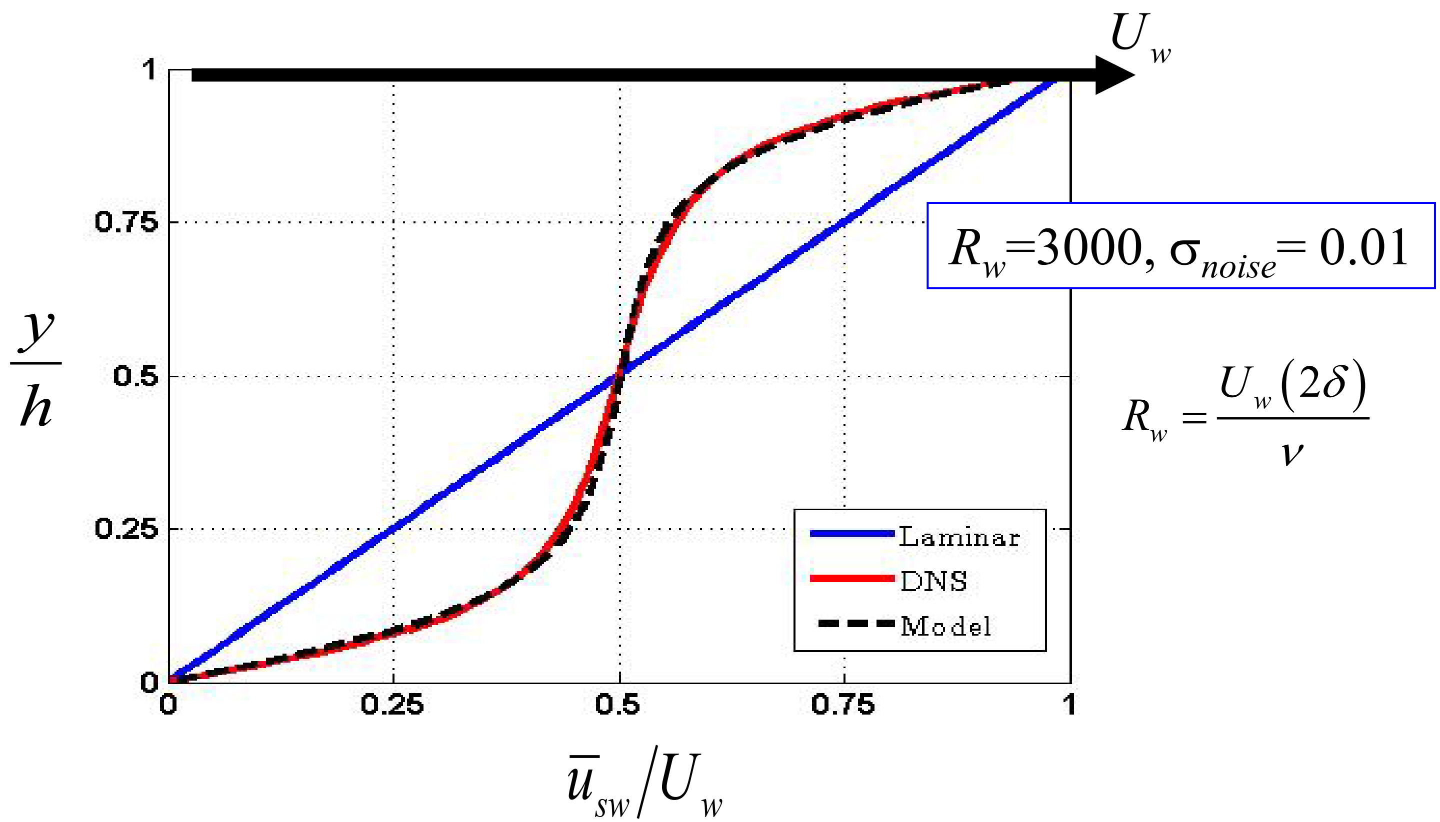
Zero mean Gaussian

$$\boxed{\frac{\partial u'_{sw}}{\partial t} = -\frac{\partial \psi}{\partial z}\frac{\partial u'_{sw}}{\partial y} - \frac{\partial \psi}{\partial z}\frac{\partial U}{\partial y} + \frac{\partial \psi}{\partial y}\frac{\partial u'_{sw}}{\partial z} + \frac{1}{R_w}\Delta u'_{sw}}$$

Nonlinear

Hypothesis: nonlinearity in u'_{sw} leads to blunting

Blunting of the mean velocity profile



DNS* simulates full Navier Stokes

*[Tsukahara et al., 2006]

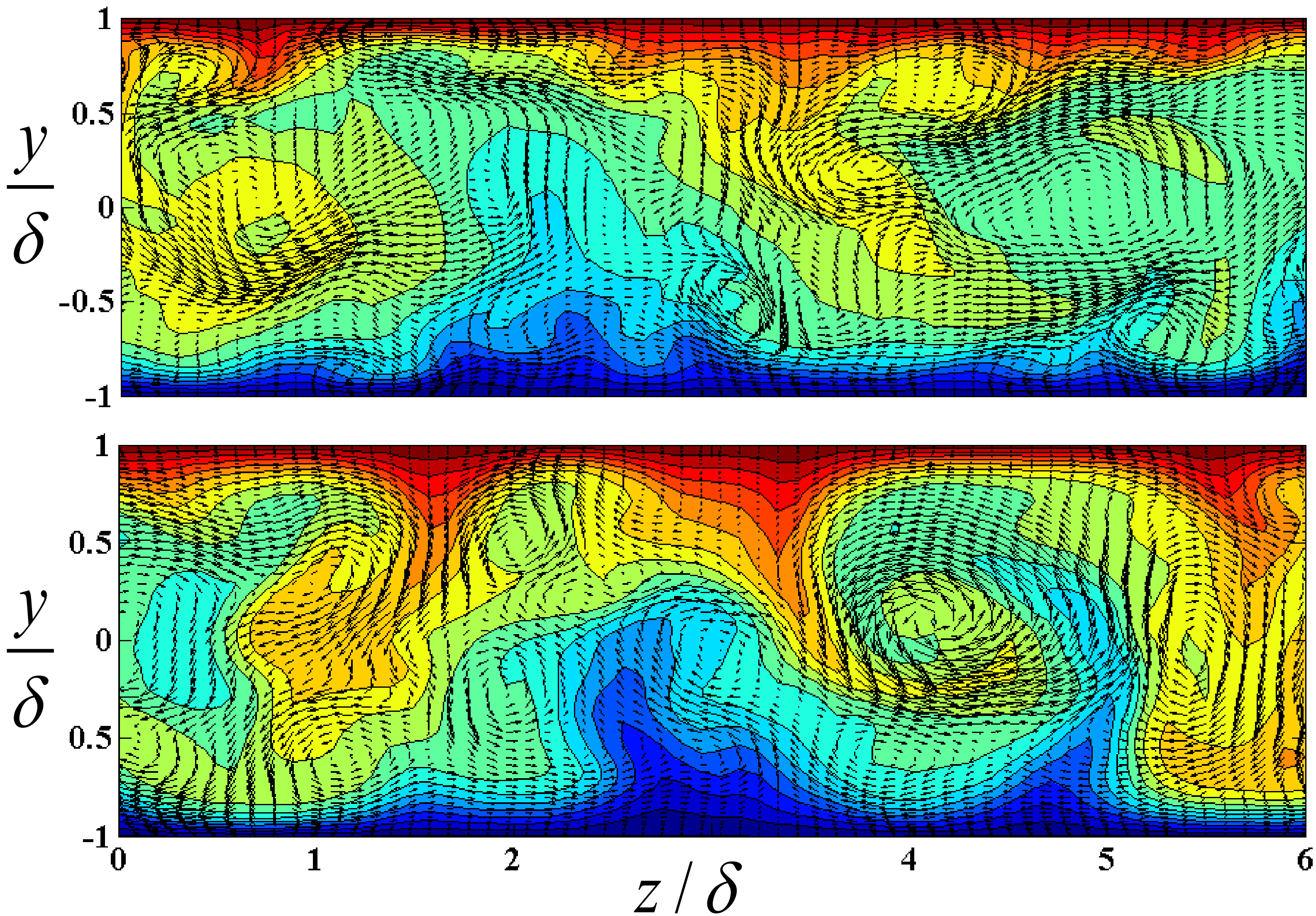
The u'_{sw} nonlinearity captures blunting (i.e. more drag)

[Gayme et al. 2010]

Flow fields (DNS top, 2D/3C bottom)

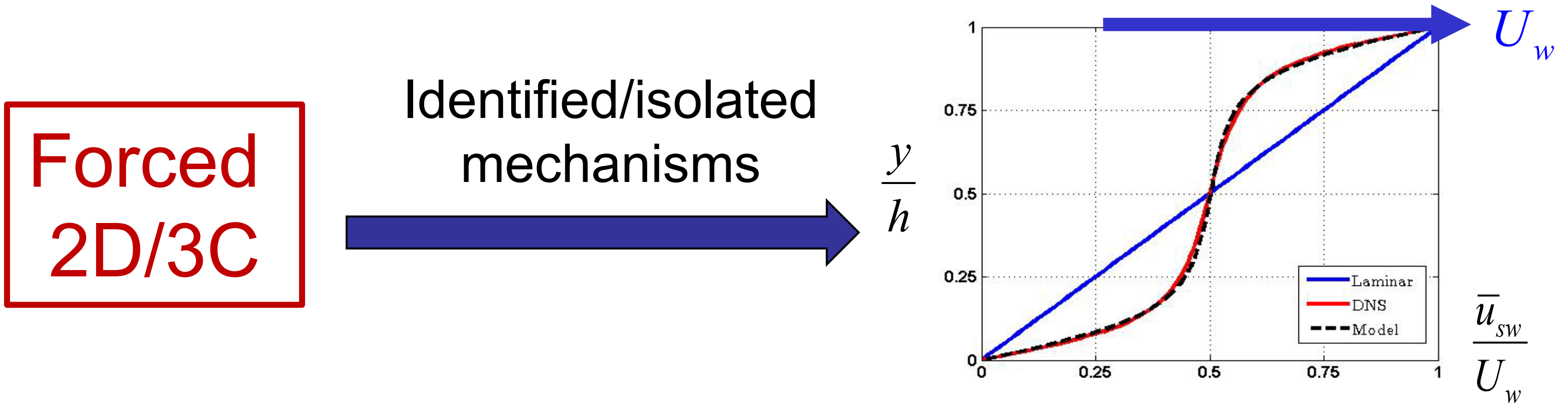
DNS

2D/3C



Implications

- Connected structures to profile blunting



- Results imply that mean flow is primarily determined by the streamwise constant interactions

BUT

- Stability of laminar solution implies turbulence is not self-sustaining
- Turbulence is not 2D (cannot do a 2D experiment)

Restricted Nonlinear (RNL) Model

Adding some streamwise variation

$\mathbf{U}(t) = (U, V, W)$ Streamwise constant mean flow

$\mathbf{u}(t) = (u, v, w)$ Streamwise varying perturbations about that mean flow

$$\mathbf{U}_t + \mathbf{U}\cdot\nabla\mathbf{U} + \nabla P - \frac{\Delta\mathbf{U}}{R} = -\langle \mathbf{u}\cdot\nabla\mathbf{u} \rangle$$

$$\mathbf{u}_t + \mathbf{U}\cdot\nabla\mathbf{u} + \mathbf{u}\cdot\nabla\mathbf{U} + \nabla p - \frac{\Delta\mathbf{u}}{R} = e$$

Depends on the instantaneous $U(y, z, t)$

Based on a second order closure of the dynamics of the statistical state

$\langle \cdot \rangle$ Denotes a streamwise average

RNL Simulation

LTV

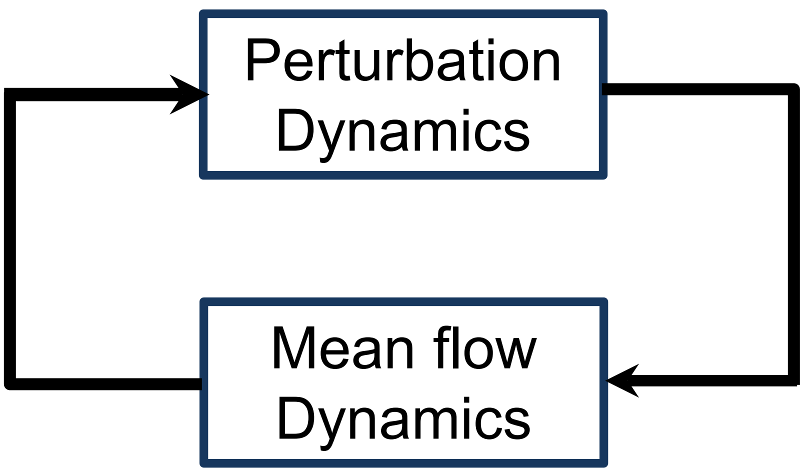
$$\mathbf{u}_t = A(\mathbf{U}(t))\mathbf{u} + \ldots$$

Zero mean Gaussian

$$\mathbf{U}_t = G(\mathbf{U}) + \mathcal{L}C$$

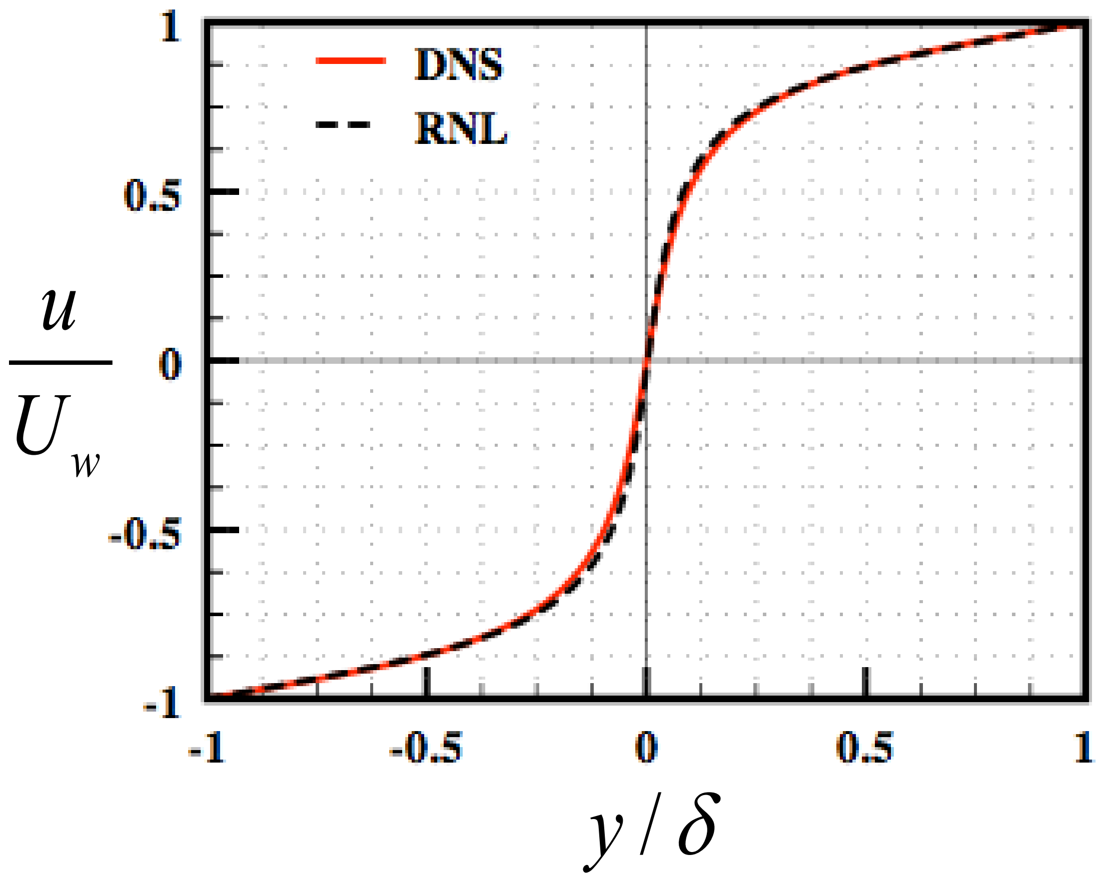
Nonlinear 2D/3C

Mean flow $\mathbf{U}(t)$ regulates $\mathbf{u}(t)$



Perturbations $\mathbf{u}(t)$ drive mean flow $\mathbf{U}(t)$

Velocity profiles for R=1000



$$R = \frac{U_w \delta}{\nu}$$

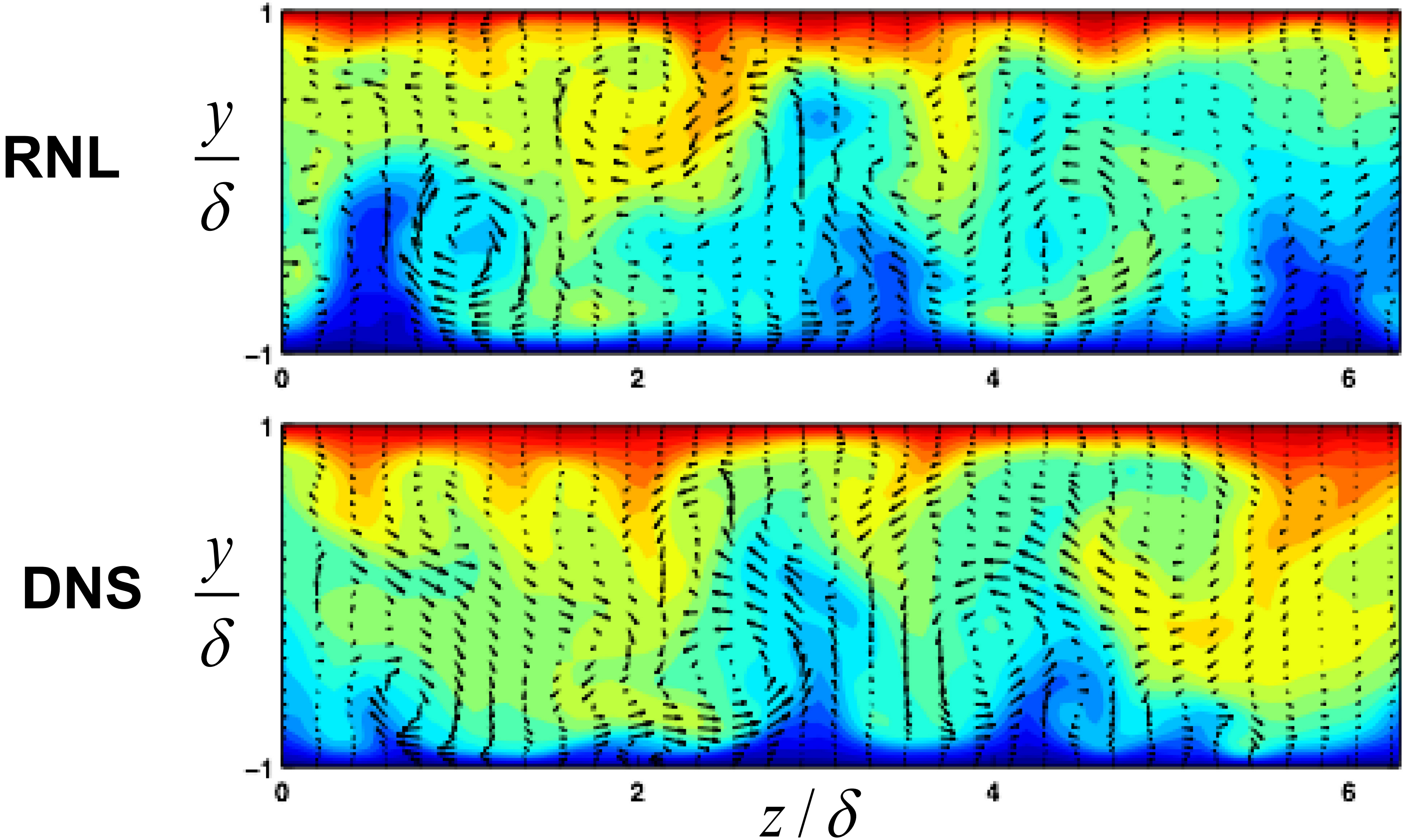
δ:= channel half height

U_w:= plate velocity

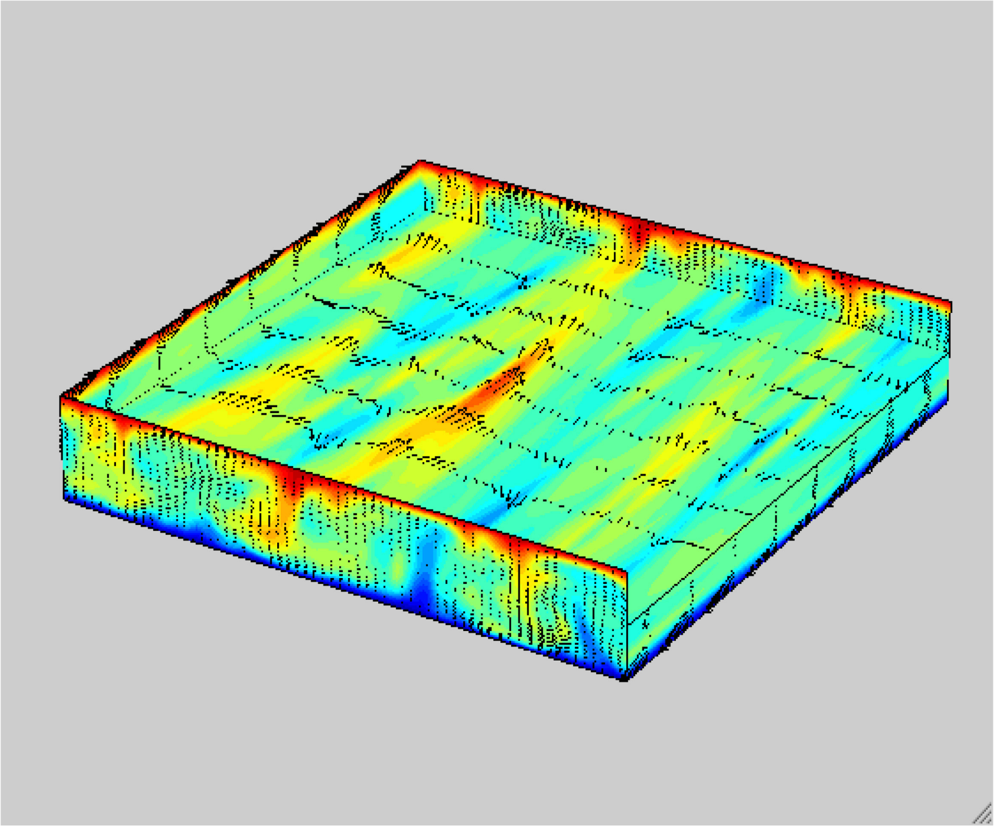
DNS Gibson 2012

	x	y	z	$M_x \times M_y \times M_z$
DNS	$[0, 4\pi]$	$[-1, 1]$	$[0, 4\pi]$	$83 \times 65 \times 41$
RNL	$[0, 4\pi]$	$[-1, 1]$	$[0, 4\pi]$	$9 \times 65 \times 41$

Flow fields



Self sustaining turbulent activity in RNL

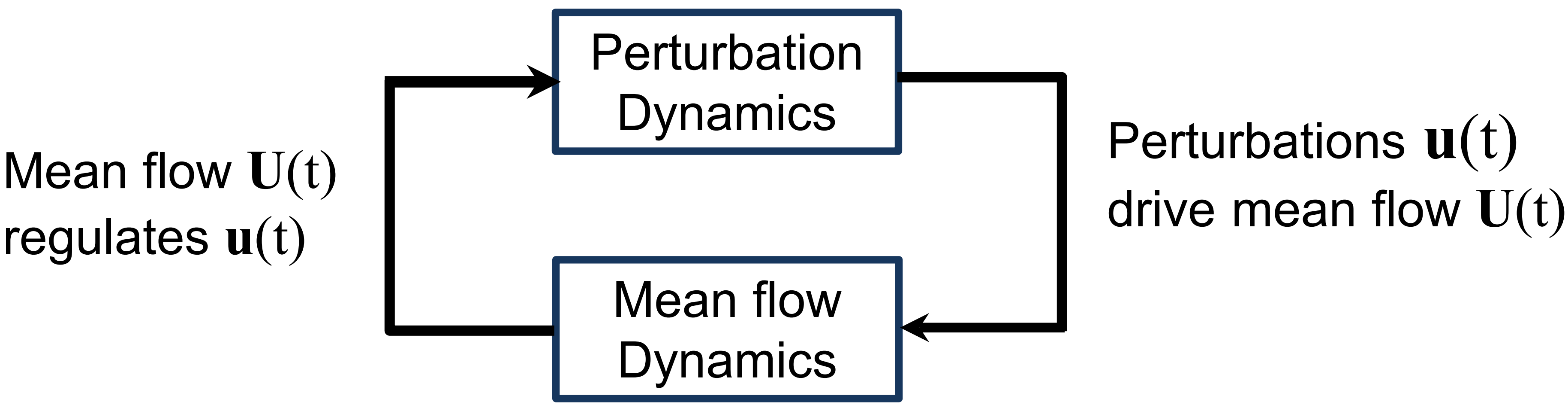


The self sustaining process

- Critical interaction of the turbulent mean flow to the perturbations (regulates the perturbation amplitude)
  - Maintains mean flow forcing (internal to the model)

$$\mathbf{U}_t + \mathbf{U}\cdot\nabla\mathbf{U} + \nabla P - \frac{\Delta\mathbf{U}}{R} = \boxed{-\langle \mathbf{u}\cdot\nabla\mathbf{u}\rangle}$$

$$\mathbf{u}_t = \boxed{-\mathbf{U}\cdot\nabla\mathbf{u} - \mathbf{u}\cdot\nabla\mathbf{U}} - \nabla p + \frac{\Delta\mathbf{u}}{R}$$



Ongoing work

Goal: Identify the pathways that can be manipulated to alter the turbulent state

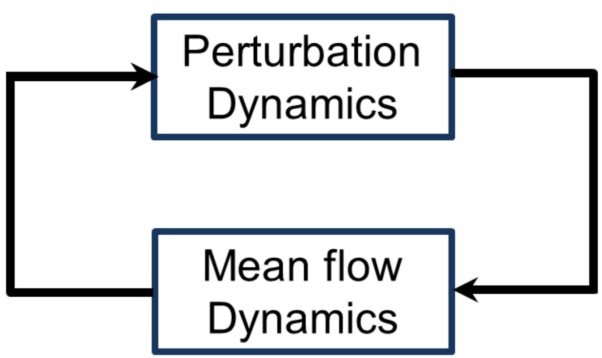
Modified Restricted Nonlinear Model

$$\tilde{\mathbf{U}} = (\mathbf{U} - \mathbf{U}_{\mathbf{lam}})$$

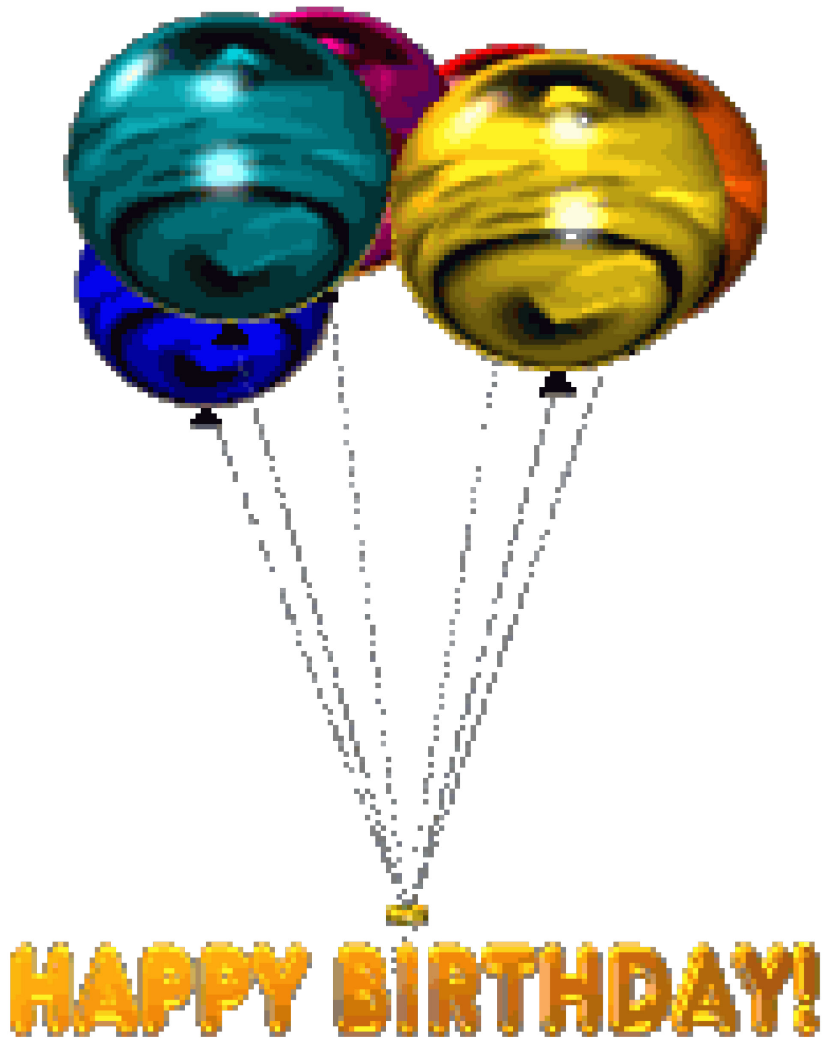
$$\mathbf{u}_t + \left(\mathbf{U}_{laminar} + \varepsilon\tilde{U}\right)\cdot\nabla\mathbf{u} + \mathbf{u}\cdot\nabla\left(\mathbf{U}_{laminar} + \varepsilon\tilde{U}\right) + \nabla p - \frac{\Delta\mathbf{u}}{R} + e$$

$$\mathbf{U}_t + \mathbf{U}\cdot\nabla\mathbf{U} + \nabla P - \frac{\Delta\mathbf{U}}{R} = -\langle\mathbf{u}\cdot\nabla\mathbf{u}\rangle$$

Mean flow $\mathbf{U}(\mathrm{t})$ regulates $\mathbf{u}(\mathrm{t})$



Perturbations $\mathbf{u}(\mathrm{t})$ drive mean flow $\mathbf{U}(\mathrm{t})$



Karl, John, Richard and CDS