Input-Output Analysis of Channel Flows

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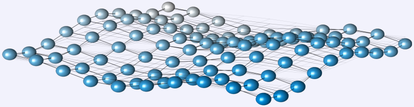
Support:

Energy, Power & Adaptive Systems Program (ECCS)

Control Systems (CMMI)

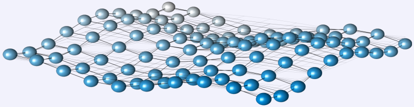
Dynamics & Control Program

Context: Networked Systems and DPS

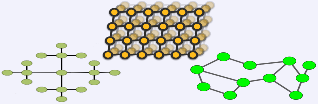
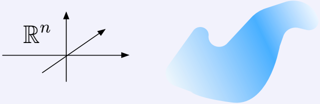
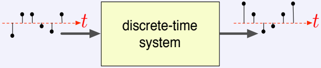
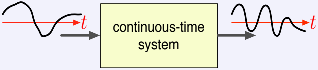
Spatially Distributed Systems	
Networked/Cooperative/Distributed Control	Distributed Parameter Systems
	

Context: Networked Systems and DPS

SPATIALLY DISTRIBUTED SYSTEMS

Networked/Cooperative/Distributed Control	Distributed Parameter Systems
	

ANALOGY WITH TEMPORAL SYSTEMS (Systems & Controls perspective)

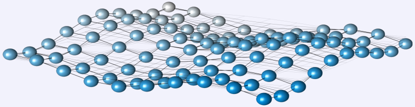
discrete space described by graph structure	*continuum space*
	
	

UNIFYING PERSPECTIVE: *Spatio-temporal* systems over discrete or continuum space

- Signals over continuous and/or discrete time and space
- Investigate systems properties (e.g. system norms & responses)

Context: Case Studies

Spatially Distributed Systems

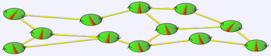
Networked/Cooperative/Distributed Control	Distributed Parameter Systems
	

Look at specific problems

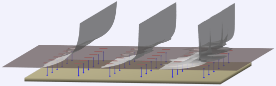
- Vehicular Strings and Consensus

- Structured Control Design

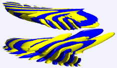
- Synchrony in AC Power Networks



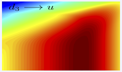
- Flow Turbulence & Control



- Spatio-temporal



Impulse Responses



Frequency Responses

Context: Emerging Common Themes

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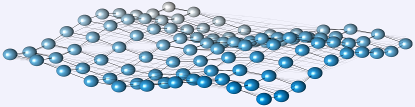
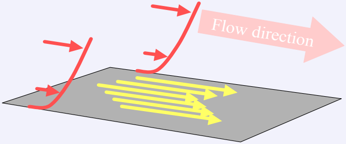
	

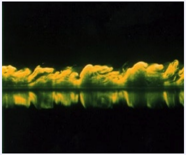
Some Common Themes Emerge

- *The use of system norms and responses*
- *Large-scale (even linear) systems exhibit some surprising phenomena*
- *Large-scale & Regular Networks $\longrightarrow$ Asymptotic statements (in system size)*
- *Network topology imposes asymptotic "hard performance limits"*

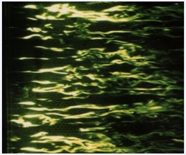
Turbulence in Streamlined Flows (Boundary Layers)



boundary layer turbulence

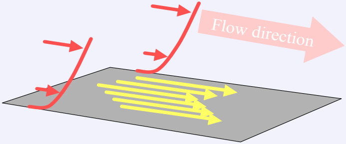


side view



top view

Turbulence in Streamlined Flows (Boundary Layers)



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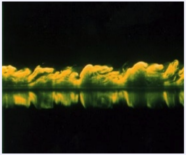

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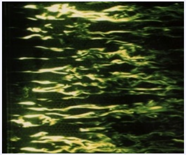

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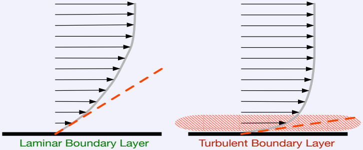
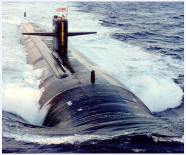


skin-friction drag: laminar vs. turbulent

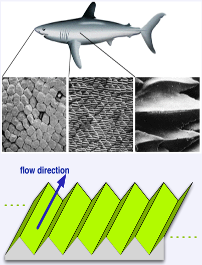


- Streamlining a vehicle reduces *form drag*
- Still stuck with: *Skin-Friction Drag* (higher in *Turbulent BL* than in *Laminar BL*)
- Same in pipe flows *(increases required pumping power)*

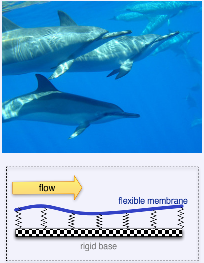
Control of Boundary Layer Turbulence

in nature: “passive” control

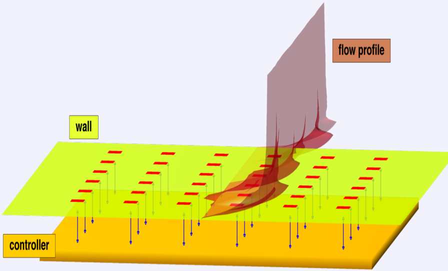
active control with sensor/actuator arrays



corrugated skin



compliant skin

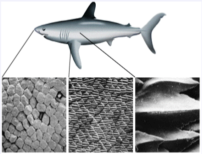


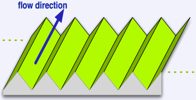
- Intuition: must have ability to actuate at spatial scale comparable to flow structures

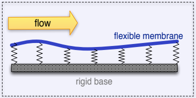
spatial-bandwidth of controller $\geq$ plant’s bandwidth

Control of Boundary Layer Turbulence

in nature: "passive" control







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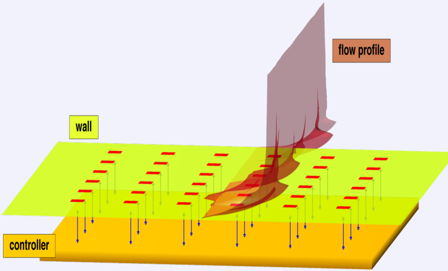


- Intuition: must have ability to actuate at spatial scale comparable to flow structures

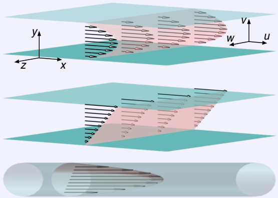
  spatial-bandwidth of controller $\geq$ plant's bandwidth

- Caveat: *Plant's dynamics are not well understood*

  obstacles { not only device technology
  also: *dynamical modeling and control design*

Mathematical Modeling of Transition: Hydrodynamic Stability

The Navier-Stokes (NS) equations:



$$
\begin{aligned}
\partial_t \mathbf{u} &= -\nabla_{\mathbf{u}}\mathbf{u} - \text{grad}\, p + \frac{1}{R}\Delta \mathbf{u} \\
0 &= \text{div}\, \mathbf{u}
\end{aligned}
$$

- Hydrodynamic Stability: view NS as a dynamical system
- *laminar flow* $\bar{\mathbf{u}}_R :=$ a stationary solution of the NS equations (an *equilibrium*)

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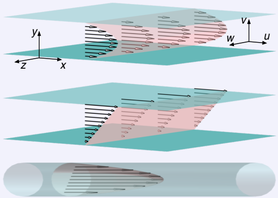

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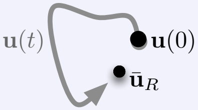
- *laminar flow* $\bar{\mathbf{u}}_R :=$ a stationary solution of the NS equations (an *equilibrium*)

laminar flow $\bar{\mathbf{u}}_R$ stable $\longleftrightarrow$ i.c. $\mathbf{u}(0) \neq \bar{\mathbf{u}}_R$, $\mathbf{u}(t) \xrightarrow{t \to \infty} \bar{\mathbf{u}}_R$

- typically done with dynamics linearized about $\bar{\mathbf{u}}_R$
- various methods to track further "non-linear behavior"



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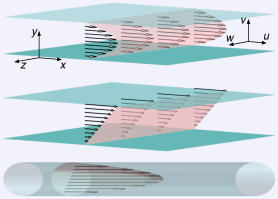


- Hydrodynamic Stability: view NS as a dynamical system
- A very successful (*phenomenologically predictive*) approach for many decades
- However: *it fails badly in the special (but important) case of streamlined flows*

Mathematical Modeling of Transition: Adding Signal Uncertainty

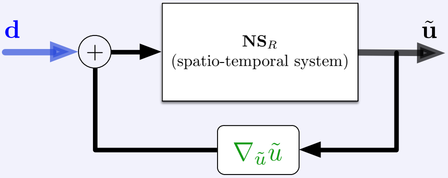
- Decompose the fields as

$$\mathbf{u} \;=\; \underset{\text{laminar}}{\underset{\uparrow}{\bar{\mathbf{u}}_R}} \;+\; \underset{\text{fluctuations}}{\underset{\uparrow}{\tilde{\mathbf{u}}}}$$

- Fluctuation dynamics: In *linear* hydrodynamic stability, $-\nabla_{\tilde{u}}\tilde{u}$ is ignored

$$\begin{aligned} \partial_t \tilde{\mathbf{u}} &= -\nabla_{\bar{\mathbf{u}}_R}\tilde{\mathbf{u}} - \nabla_{\tilde{\mathbf{u}}}\bar{\mathbf{u}}_R - \text{grad}\,\tilde{p} + \tfrac{1}{R}\Delta\tilde{\mathbf{u}} - \nabla_{\tilde{u}}\tilde{u} + \mathbf{d} \\ 0 &= \text{div}\,\tilde{\mathbf{u}} \end{aligned}$$

  - a time-varying *exogenous disturbance* field $\mathbf{d}$ (e.g. random body forces)



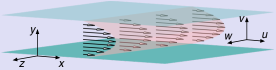
Input-Output view of the Linearized NS Equations

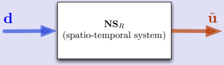
Jovanovic, BB, '05 JFM

Input-Output Analysis of the Linearized NS Equations

$$\partial_t \begin{bmatrix} \Delta \tilde{v} \\ \tilde{\omega} \end{bmatrix} = \begin{bmatrix} U''\partial_x - U\Delta\partial_x + \frac{1}{R}\Delta^2 & 0 \\ -U'\partial_z & -U\partial_x + \frac{1}{R}\Delta \end{bmatrix} \begin{bmatrix} \tilde{v} \\ \tilde{\omega} \end{bmatrix} + \begin{bmatrix} -\partial_{xy} & \partial_x^2 + \partial_z^2 & -\partial_{zy} \\ \partial_z & 0 & -\partial_x \end{bmatrix} \begin{bmatrix} d_x \\ d_y \\ d_z \end{bmatrix}$$

$$\begin{bmatrix} \tilde{u} \\ \tilde{v} \\ \tilde{w} \end{bmatrix} = \left(\partial_x^2 + \partial_z^2\right)^{-1} \begin{bmatrix} \partial_{xy} & -\partial_z \\ \partial_x^2 + \partial_z^2 & 0 \\ \partial_{zy} & \partial_x \end{bmatrix} \begin{bmatrix} \tilde{v} \\ \tilde{\omega} \end{bmatrix}$$





$$\begin{aligned} \partial_t \Psi &= \mathcal{A}\,\Psi + \mathcal{B}\,\mathbf{d} \\ \tilde{\mathbf{u}} &= \mathcal{C}\,\Psi \end{aligned}$$

Input-Output Analysis of the Linearized NS Equations

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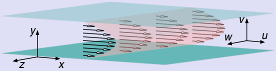

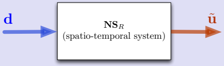

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- eigs $(\mathcal{A})$: determine stability

  (standard technique in *Linear Hydrodynamic Stability*)

- Transfer Function $\mathbf{d} \longrightarrow \tilde{\mathbf{u}}$: determines response to disturbances

  (uncommon in Fluid Mechanics
  an "open system")

Input-Output Analysis of the Linearized NS Equations

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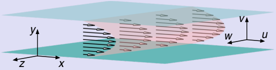

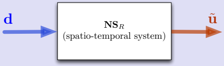

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Surprises:

- Even when $\mathcal{A}$ is stable — the gain $\mathbf{d} \longrightarrow \tilde{\mathbf{u}}$ can be very large

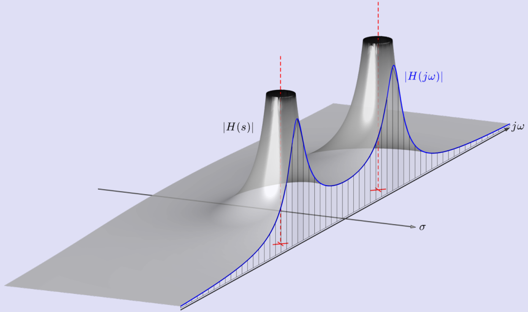
  ($(H^2$ norm$)^2$ scales with R^3)

- Input-output resonances — very different from least-damped modes of $\mathcal{A}$

Modal vs. Input-Output Response

Typically: underdamped poles $\longleftrightarrow$ frequency response peaks

cf. The "rubber sheet analogy":



Modal vs. Input-Output Response

However: Pole Locations $\not\leftrightarrow$ Frequency Response Peaks

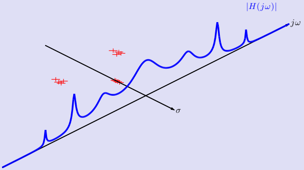
Theorem: *Given any desired pole locations*

$$z_1, \ \ldots, \ z_n \ \in \mathbb{C}_- \text{ (LHP)},$$

and any stable frequency response $H(j\omega)$, arbitrarily close approximation is achievable with

$$\left\| H(s) \; - \; \left(\sum_{i=1}^{N_1} \frac{\alpha_{1,i}}{(s-z_1)^i} \; + \; \cdots \; + \; \sum_{i=1}^{N_n} \frac{\alpha_{n,i}}{(s-z_n)^i} \right) \right\|_{\mathcal{H}^2} \leq \epsilon$$

by choosing any of the N_k's large enough



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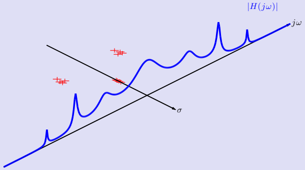


Remarks:

- No necessary relation between *pole locations* and *system resonances*
- ($\epsilon \to 0 \Rightarrow N_k \to \infty$), i.e. this is a *large-scale systems* phenomenon
- Large-scale systems: IO behavior not always predictable from modal behavior

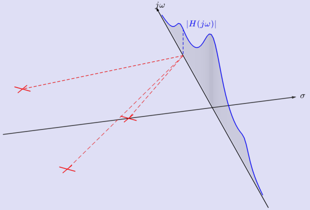
Modal vs. Input-Output Response

However: Pole Locations $\not\leftrightarrow$ Frequency Response Peaks

MIMO case: $H(s) = (sI - A)^{-1}$

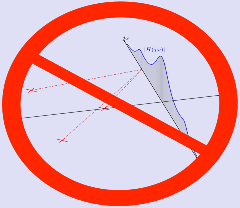
- *If A is normal* (has orthogonal eigenvectors), then

$$\sigma_{\max}\left((j\omega I - A)^{-1}\right) = \frac{1}{\text{distance}\left(j\omega, \text{nearest pole}\right)}$$



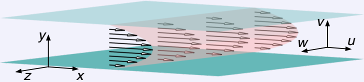
- *If A is non-normal* : no clear relation between

singular value plot $\not\leftrightarrow$ *eigs(A)*



Translation invariance in x & z implies

- *Impulse Response* *(Green's Function)*



$$\tilde{\mathbf{u}}(t,x,y,z) \;=\; \int G(t-\tau, x-\xi, \text{y,y'}, z-\zeta)\; \mathbf{d}(\tau,\xi,y',\zeta)\; d\tau d\xi dy' d\zeta$$

$$\tilde{\mathbf{u}}(t,x,.,z) \;=\; \int \mathcal{G}(t-\tau, x-\xi, z-\zeta)\; \mathbf{d}(\tau,\xi,.,\zeta)\; d\tau d\xi d\zeta$$

$\mathcal{G}(t,x,z)$: *Operator-valued* impulse response

- *Frequency Response*

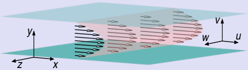
$$\tilde{\mathbf{u}}(\omega,k_x,k_z) \;=\; \mathcal{G}(\omega,k_x,k_z)\; \mathbf{d}(\omega,k_x,k_z)$$

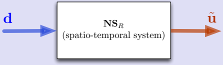
$\mathcal{G}(\omega,k_x,k_z)$: *Operator-valued* frequency response (Packs lots of information!)

- *Spectrum of $\mathcal{A}$*:

$$\sigma(\mathcal{A}) \;=\; \overline{\bigcup_{k_x,k_z} \sigma\left(\hat{\mathcal{A}}(k_x,k_z)\right)}$$

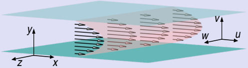
Modal vs. Input-Output Analysis

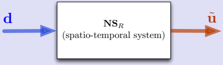




- $$\begin{aligned} \partial_t \Psi &= \mathcal{A}\,\Psi + \mathcal{B}\,\mathbf{d} \\ \tilde{\mathbf{u}} &= \mathcal{C}\,\Psi \end{aligned}$$
- IR: $\mathcal{G}(t, x, z)$
- FR: $\mathcal{G}(\omega, k_x, k_z)$

Modal vs. Input-Output Analysis





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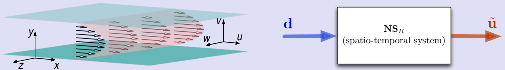
Modal Analysis: Look for unstable eigs of $\mathcal{A}$ $\left(\bigcup_{k_x,k_z} \sigma\left(\hat{A}(k_x, k_z)\right)\right)$

Flow type	Classical linear theory R_c	Experimental R_c
Channel Flow	5772	$\approx$ 1,000-2,000
Plane Couette	∞	$\approx$ 350
Pipe Flow	∞	$\approx$ 2,200-100,000

Modal vs. Input-Output Analysis



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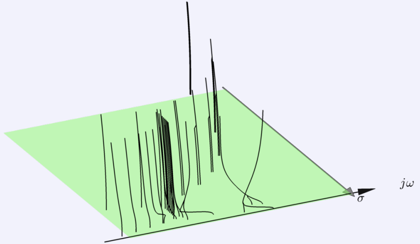
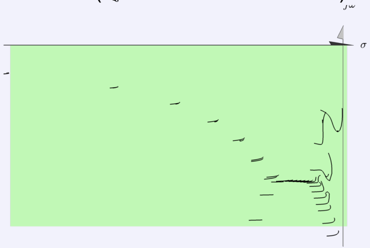
Modal Analysis: Look for unstable eigs of $\mathcal{A}$ $\left(\bigcup_{k_x,k_z} \sigma\left(\hat{\mathcal{A}}(k_x, k_z)\right)\right)$

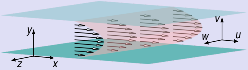
- Channel Flow @ $R = 2000$, $k_x = 1$, (k_z = vertical dimension):

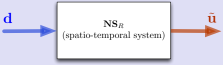




top view

Modal vs. Input-Output Analysis





- $$\begin{aligned} \partial_t \Psi &= \mathcal{A}\,\Psi + \mathcal{B}\,\mathbf{d} \\ \tilde{\mathbf{u}} &= \mathcal{C}\,\Psi \end{aligned}$$
- IR: $\mathcal{G}(t, x, z)$
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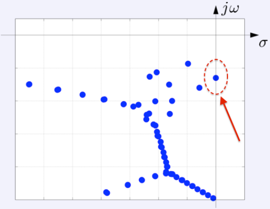
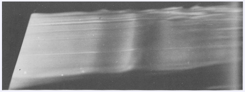
Modal Analysis: Look for unstable eigs of $\mathcal{A}$ $\left(\bigcup_{k_x,k_z} \sigma\left(\hat{\mathcal{A}}(k_x, k_z)\right)\right)$

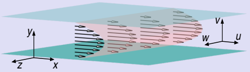
- Channel Flow @ $R = 6000$, $k_x = 1$, $k_z = 0$:

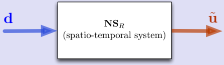


- Flow structure of corresponding eigenfunction: Tollmein-Schlichting (TS) waves



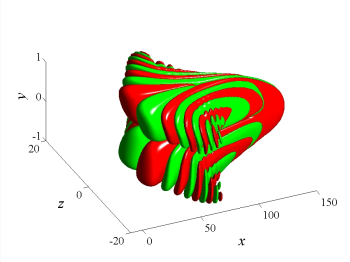
Modal vs. Input-Output Analysis

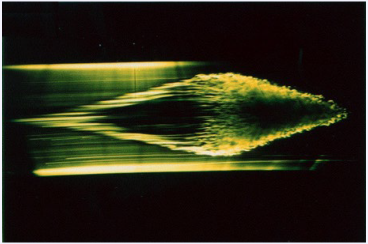




- $$\begin{aligned} \partial_t \Psi &= \mathcal{A}\,\Psi + \mathcal{B}\,\mathbf{d} \\ \tilde{\mathbf{u}} &= \mathcal{C}\,\Psi \end{aligned}$$
- IR: $G(t, x, y, -1, z)$
- FR: $\mathcal{G}(\omega, k_x, k_z)$

Impulse Response Analysis: Channel Flow @ $R = 2000$





similar to "turbulent spots"

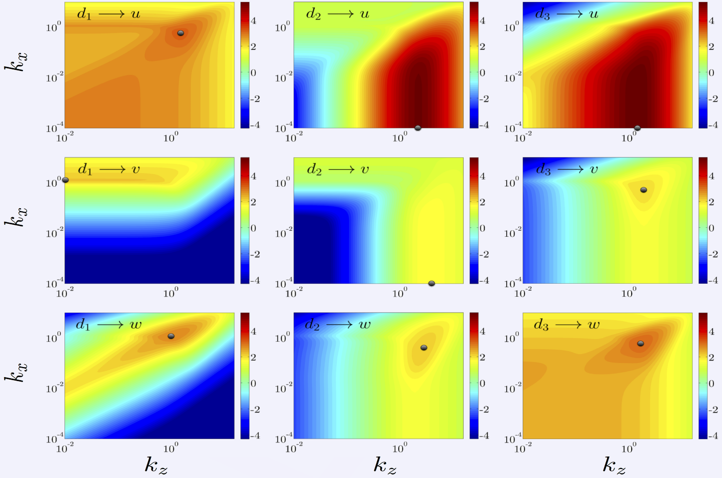
Jovanovic, BB, '01 ACC,

more movies and pics at `http://engineering.ucsb.edu/~bamieh/pics/impulse_page.html`

Spatio-temporal Frequency Response

$\mathcal{G}(\omega, k_x, k_z)$ is a *LARGE* object! (very "data rich"!)

one visualization method: $\sup_\omega \sigma_{\max}\left(\mathcal{G}(\omega, k_x, k_z)\right)$



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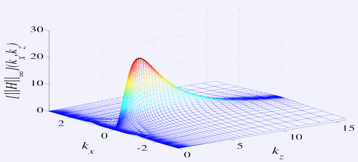
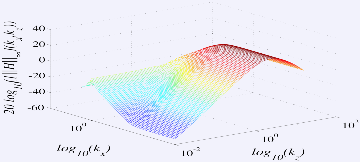
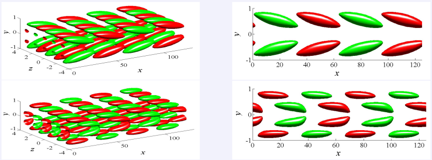
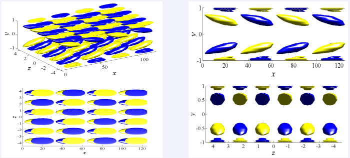




What do the corresponding flow structures look like?



streamwise velocity isosurfaces



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Spatio-temporal Frequency Response

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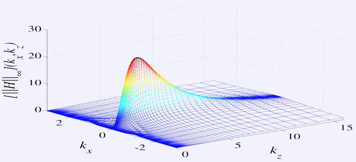
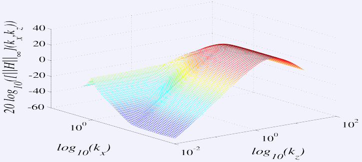
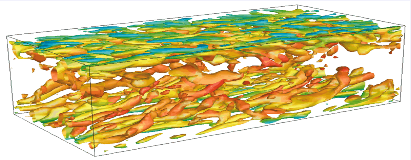
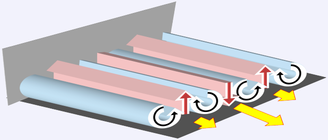




What do the corresponding flow structures look like?

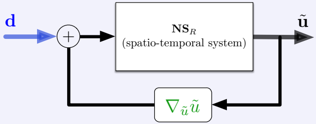
much closer (than TS waves) to structures seen in turbulent boundary layers



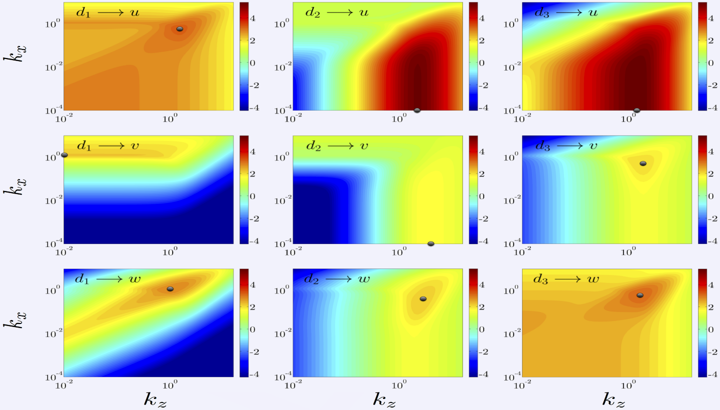


Spatio-temporal Frequency Response

How to view of $\mathcal{G}(\omega, k_x, k_z)$?



bring $\nabla_{\tilde{u}}\tilde{u}$ back in through IQCs?



Collaborators

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- S. Patterson
- J.C. Doyle (CDS)
- B. Mckeon (CDS)
- M. Dahleh (CDS)
- P. Mitra (CDS)
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Support:

Energy, Power & Adaptive Systems Program (ECCS)

Control Systems (CMMI)

Dynamics & Control Program